Fujimoto’s theorem about complete minimal surfaces

Christoph Jutzet

Master thesis in mathematics

In my thesis I give a proof of Fujimoto’s theorem, which states that a regular complete minimal surface in the 3-dimensional euclidean space is either flat or its Gauss map omits at most four points. This is in fact the best upper bound, as there are known examples of complete minimal surfaces whose Gauss map omits exactly four points. It is possible to weaken the requirement of regularity if further assumptions are made. In the context of minimal surfaces, the Enneper and Weierstrass theorem yields an important geometric view for the Gauss map and the line element of such surfaces. Using the uniformization theorem, we deduce that minimal surfaces are either of hyperbolic or parabolic type, where Picard’s theorem immediately shows that the latter case satisfies Fujimoto’s upper bound. Hence, in order to prove Fujimoto, the study of hyperbolic metrics becomes the main concern.

Supervisor: Patrick Ghanaat