The Critical Random-Cluster Model in Two Dimensions : RSW Theory and how Boundary Conditions Influence It

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Bernoulli percolation is one of the most studied probabilistic models. The model is quite simple: Every edge of the square lattice is open with probability p and closed with probability $1-p$, independently of all other edges. The random-cluster model is a generalisation of Bernoulli percolation: edges depend on each other via an additional parameter q, and the model depends on boundary conditions.
The random-cluster model is interesting due to its phase transition, and depending on the value of q, its phase transition looks quite different. One way of studying the phase transition is via crossings: knowing how rectangles are crossed, one can deduce global properties of the model. For Bernoulli percolation, the RSW theory provides bounds on the probability of such crossing events and in recent years, an analoguous theory has been developed for the random-cluster model. For $q \in [1;4]$, this theory yields bounds on the probability of crossing events which are uniform in the boundary conditions.
In this thesis, we discuss the influence of boundary conditions on these bounds. In particular, we prove that the boundary conditions do have a certain influence, in the case $q \in (1;4)$.

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