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Caractérisation des automorphismes CR d'une classe d'hypersurfaces dans $\mathbb{C}^4$: le problème PQR

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This dissertation is about the stability group Aut(M_g), which is the set of all the automorphisms (or biholomorphisms stabilizing a point) of a real and smooth hypersurface M_g. In order to study Aut(M_g), one can classify real-analytic infinitesimal CR-automorphisms hol(M) of an easier homogeneous polynomial and model hypersurface M. M is associated with M_g in the sense that its defining equation is constituted only of the first terms of Taylor's development of the equation of M_g. Some general results exist in $\mathbb{C}^{n+1}$; more detailed results appear in $\mathbb{C}^3$; and this dissertation will provide new results in $\mathbb{C}^4$. Hypersurfaces that are relevant to study are real, holomorphically nondegenerate, of finite type and Levi degenerate.

Chapter 1 introduces the elementary notions, the study question as well as all the main results. Chapter 2 is about the general framework in $\mathbb{C}^{n+1}$. Based on "Chern-Moser operators and polynomial models in CR geometry", this chapter introduces the complete approach that uses the Chern-Moser operator, supported by more precise explanations and examples. Chapter 3 has a dual purpose. To begin with, it serves as a springboard before generalizing in $\mathbb{C}^4$. In this sense, it rephrases the results of "Infinitesimal CR automorphisms for a class of polynomial models" and offers a description of the real-analytic infinitesimal CR-automorphisms of a homogeneous and model hypersurface of $\mathbb{C}^3$, M, described by the following equation :

$$\operatorname{Im} w = P\overline{Q} + \overline{Q}P,$$

where $(z, w) = (z_1, z_2, w) \in \mathbb{C}^3$, and P and Q are polynomials in z. In addition, this chapter provides important specifications on this case, particularly regarding the decomposition of rotations. Chapter 4 introduces the newest contributions on a homogeneous hypersurface M of $\mathbb{C}^4$ described as follows :

$$\operatorname{Im} w = P\overline{Q} + \overline{Q}P + R\overline{R},$$

where $(z, w) = (z_1, z_2, z_3, w) \in \mathbb{C}^4$, and P, Q and R are polynomials in z. This case is called the PQR problem. A model case serves as an introduction to this chapter in section 4.1. A decoupled case (for which a variable is strictly reserved to R) gives interesting results in 4.2. However, as presented in 4.3, it is the general case, although monomial, that seems important : the theorem that describes the decomposition of rotations proves to be the most important result. The dimension of the set of the real-analytic infinitesimal CR-automorphisms of the hypersurface are detailed. Chapter 5 broadens the perspectives and offers new leads, while keeping in mind the fundamental link between M and M_g, that is between real-analytic infinitesimal CR-automorphisms described from the model hypersurface, and the automorphisms of the general hypersurface.

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