An analysis of differentiability, rectifiability, and currents in and outside Euclidean space.

Denis Roger Marti

Master of Science in Mathematics

Differentiability, rectifiability, and currents are three important aspects of geometric measure theory. In Euclidean space, these concepts are well understood. In a more general setting such as Banach spaces or metric spaces, the situation is more complicated.
This thesis investigates the problems that arise when one attempts to generalize results concerning differentiability, rectifiability, and currents to a more general setting. To this end, we focus on three classical results in geometric measure theory, each of which is important for the study of the three different concepts.
The first result is the Rademacher theorem. It plays a fundamental role in the connection between differentiability and Lipschitz functions in Euclidean space. We introduce the notion of Lipschitz function and discuss how to generalize the concept of differentiability. We conclude this section with our own proof of a generalization of Rademacher's theorem to separable Banach spaces with the Radon-Nikodym property.
Rectifiable sets are the fundamental class of geometric objects in geometric measure theory. The second result, the Besicovitch-Federer projection theorem, characterizes these sets in Euclidean space. Recently, D. Bate has published a result which qualifies as a generalization of the Besicovitch-Federer projection theorem. We discuss the connection between these two results and address the question of whether D. Bate's result can be considered a generalization.
The third result is the closure theorem. It plays an important role in solving the Plateau problem. This is a classical problem of geometric measure theory and was solved in 1960 by H. Federer and W. Fleming using the theory of currents. Later, L. Ambrosio and B. Kirchheim extended the theory of currents to metric spaces. They solved a generalization of the Plateau problem, also using a closure theorem. However, their approach to the proof of the closure theorem is very different from the classical proof. We present an attempt to prove the closure theorem for metric currents using the same approach as in the classical proof. Here the result of D. Bate plays an important role. Thus, we investigate at the same time the connection between this result and the Besicovitch-Federer projection theorem and gain a better understanding of the difference between the theory of currents in and outside Euclidean space.

Prof. Dr. Stefan Wenger