New Contributions to Groups of Hyperbolic Isometries

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This thesis contains several new results on hyperbolic geometry.

The main outcome is the identification of the smallest non-arithmetic cusped hyperbolic 3-orbifold as the quotient of hyperbolic 3-space by the Coxeter group $[5, 3, 6]$ using geometric and combinatorial considerations on horoball packings (joint with Ruth Kellerhals).

The same tools are used to establish that the Coxeter group $[5, \infty]$ yields the smallest non-arithmetic cusped hyperbolic 2-orbifold. This gives a Siegel type result by providing a list of all cusped hyperbolic 2-orbifolds with area less than $\frac{\pi}{2}$.

Another perspective concerns compact hyperbolic 2-orbifolds realising minimal area hyperbolic n-gons as a fundamental polygon for $n > 6$. We consider stabiliser subgroups of the Coxeter groups $[n, 3]$ acting on appropriately chosen graphs. Where possible, we describe 2-manifolds by means of torsion-free subgroups thereof. In this way we complete previous work of Matthieu Jacquemet.

These methods are also used to demonstrate that smallest non-arithmetic cusped hyperbolic 3-manifolds covering a Coxeter orbifold cannot be based on $[5, 3, 6]$. In fact, the Coxeter group $[(3^3, 6)]$ yields a smaller manifold cover.

In higher dimensional hyperbolic space we construct right-angled p-gons by utilising the upper half space model based on Clifford vectors (joint with Edoardo Dotti). The main idea of analysing orthogonal geodesics using cross ratios is helpful through most parts of this thesis.

Jury

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