

Minimal surfaces of higher topology in metric spaces

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Assume you are given a rectifiable Jordan curve Γ in \mathbb{R}^n . The famous problem of Plateau asks if there exists a surface of disc-type and minimal area spanning Γ . Its solution was obtained independently by Douglas and Radó in the early 1930's. Over the decades, the formulation and solution to the problem have gradually been extended in various aspects. In the thesis at hand, we solve a very general version of Plateau's problem: we show that a collection of several, possibly intersecting, rectifiable closed curves Γ in a proper metric space X bounds a minimal surface of fixed genus. Our methods are mostly based on generalizing preexisting ideas within the context of different, yet similar non-degeneracy conditions.

In the first part, we solve the problem assuming Courant's condition of cohesion and Iseri's condition of adhesion for a minimal sequence of surfaces of prescribed topology. In a joint collaboration with Stefan Wenger, the solution under the condition of cohesion on surfaces spanning a configuration Γ of disjoint Jordan curves was first published in [4] and [3]. The respective result for a configuration of possibly singular curves needs the additional assumption of the condition of adhesion and was established in a joint work with Paul Creutz in [2].

In the second part, the existence of a solution is shown under the so-called Douglas condition. Chronologically, we first solved the problem for a topologically regular collection Γ in a proper metric space X admitting an isoperimetric inequality (see [4]). Consequently, the problem was treated in the most general setting described above ([2]), where our approach is based on extension and thickening techniques of the curves in Γ and the ambient space X in order to reduce the problem back to the setting in [4]. Here however, we first solve the minimization problem for a regular collection of curves in a general proper metric space X (using the aforementioned thickening technique *inter alia*), before we address the case of a possibly singular configuration of curves in X (based on Creutz' extension idea first appearing in [1]).

We emphasize that our techniques allow to conclude even new results if the ambient space is a general complete Riemannian manifold.

Jury:

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References

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