University of Fribourg / Faculty of Science and Medicine / Department of Mathematics

Moduli Spaces of Riemannian Metrics of Positive Ricci and Non-Negative Sectional Curvature on 5, 7 and 15-dimensional Manifolds

Jonathan Wermelinger

In this thesis, we study the topology of the moduli spaces of Riemannian metrics of non-negative sectional or positive Ricci curvature on certain 5, 7 and 15-dimensional manifolds.
Specifically, using the relative eta-invariant of the Atiyah-Patodi-Singer index theory, we show that the moduli space of non-negative sectional curvature metrics on orientable, closed, smooth non-spin 5-manifolds with universal cover S^3xS^2 and fundamental group $\mathbb{Z}_2$ has infinitely many path components.
Furthermore, we show that the moduli space of positive Ricci curvature metrics on the total space of linear S^7-bundles over S^8 which are rational cohomology 15-spheres has infinitely many path components. In addition, we carry out the diffeomorphism classification of certain homotopy $\mathbb{R}P^7$ which arise as the quotient of a Milnor sphere by a specific involution to show that their moduli space of non-negative sectional curvature has infinitely many path components. Similarly, we show that there are only finitely many diffeomorphism types of certain homotopy $\mathbb{R}P^{15}$ which arise as the quotient of a Shimada sphere by a specific involution to show that for those diffeomorphism types which can be described by an infinite family of pairwise diffeomorphic manifolds, the moduli space of positive Ricci curvature has infinitely many path components. These results are all obtained with the help of a relative index invariant due to Gromov and Lawson.

Jury:
Prof. Dr. Anand Dessai (thesis supervisor)
Prof. Dr. Uwe Semmelmann (external co-examiner)
PD. Dr. Michael Wiemeler (external co-examiner)
Prof. Dr. Christian Mazza (president of the jury)