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Intersection and Contact Graphs of Paths on a Grid

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An *edge intersection graph of paths on a grid* (*EPG graph* for short) is a graph for which there exists a family of paths on a grid in one-to-one correspondence with its vertex set such that two vertices are adjacent if and only if the corresponding paths share at least one grid-edge. A *vertex intersection graph of paths on a grid* (*VPG graph* for short) is defined similarly with the exception that two vertices are adjacent if and only if the corresponding paths intersect. In this thesis we are primarily concerned with the contact counterpart of these graph classes, namely *Contact graphs of paths on a grid* (*CPG graphs* for short) which are defined as follows. A graph $G = (V, E)$ is a CPG graph if there exists a family of pairwise interior-disjoint paths on a grid (that is, for any two paths, the intersection of their interiors is empty) in one-to-one correspondence with V such that two vertices are adjacent if and only if the corresponding paths touch. If furthermore every path has at most k bends (turns) then G is a B_k-CPG graph (we define mutatis mutandis a B_k-EPG graph and a B_k-VPG graph).

We first show that for each $k \geq 0$, recognizing B_k-CPG graphs is NP-complete and remains NP-complete within the class of planar graphs for $k \geq 3$. We then show that the class of planar graphs and CPG graphs are incomparable, and by further investigating the relation between these two graph classes, we exhibit for any $k \geq 0$, a planar B_{k+1}-CPG graph which is not a B_k-CPG graph.

We then consider circular-arc graphs, that is, intersection graphs of arcs in a circle, and investigate their relation to EPG and CPG graphs. In particular, we characterize by a family of minimal forbidden induced subgraphs B_1-EPG graphs within the class of proper circular-arc graph, a well-studied subclass of circular-arc graphs, and provide an analogous characterization of the related B_1-EPR graphs (i.e., edge intersection graphs of paths on a rectangle where each path has at most one bend) within the class of proper circular-arc graphs. Finally, we characterize B_0-CPG graphs within the class of circular-arc graphs. We note that all three characterizations lead to polynomial-time recognition algorithms.

We further show that the 3-Colorability problem for B_0-CPG graphs is NP-complete which implies that this problem is NP-complete when restricted to B_k-EPG graph with $k \geq 2$ as any B_0-CPG graph is also a B_2-EPG graph. On the other hand, this problem is known to be solvable in polynomial time for B_0-EPG as this class coincides with that of interval graphs; we complete the picture by showing that this problem is NP-complete for B_1-EPG graphs. We additionally provide tight upper bounds on both the chromatic and clique chromatic number for (some subclasses of) CPG graphs. These bounds are in part obtained by showing that CPG graphs cannot contain cliques of size larger than 6.

Finally, we show that Independent Set and Clique Cover remain NP-complete when restricted to B_0-CPG graphs. We then provide polynomial-time approximation schemes for Independent Set and Dominating Set on a nontrivial subclass of VPG graphs in which, as we show, both problems remain NP-hard.

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