# Lawson-Yau Theorem on positive scalar curvature through Cheeger deformations

Séverine Oppliger

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In the early 70's, Cheeger used an isometric action of a compact Lie group $G$ with a biinvariant metric $b$ on a Riemannian manifold $(M,h)$ to create a parametrized family of metrics $h_t$ on $M$ which shrinks the orbits $G \cdot p$. According to the Gray-O'Neill Formula applied to the orbital submersions $\rho : (M \times G, h + (1/t) \cdot b) \rightarrow (M, h)$, the Cheeger metrics $h_t$ don't carry a lower sectional curvature than the respective ones on $M \times G$. This construction discloses new non-negatively or even positively curved manifolds.

This thesis first details the technical aspects of the Cheeger deformation following Müter's approach and, as an illustration of this process, we explore the example of the rotation of $\mathbb{C}$ through an $S^1$-action. We then expose some properties of the sectional curvatures $sec_{h_t}$ compared to the initial one $sec_h$. In the last chapter, we discuss the Lawson-Yau Theorem (1974) stating that a compact manifold with a non-abelian symmetry always admits a Riemannian metric of strictly positive scalar curvature. In 2018, Cavenaghi and Sperança found a more intuitive proof of this result by using Cheeger deformations. A concrete formula for the scalar curvatures $scal_{h_t}$ developped through all the accumulated knowledge plays a crucial role in their argumentation.

Supervisor : Professeur Anand Dessai