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Analytical Results of *k*-core Pruning Process on Complex Networks

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k-core pruning process (or *k*-core decomposition, *k*-shell decomposition) is a well-known algorithm that has been used in thousands of papers by scientists from a broad range of research fields. Scientists use it to identify the most important nodes in a social network; find the most effective countries for crisis spreading in a global economic crisis; predict the structural collapse in mutualistic eco-systems; and locate the most influential spreaders in an epidemic process, etc.

Despite the wide applications in dealing with the real-world problems, it also exhibits interesting critical behaviors that contain different kinds of phase transitions in the pruning process so that it is also appealing for many theoretical physicists. Physicists have obtained the exact solution of the final state of the process on large uncorrelated random networks and consider it as a solvable example of critical phenomena in disordered systems.

Due to the intrinsic mathematical complexity, yet there exist no clear theoretical results to the question of what the network is like during the pruning process. For the first time, we solve the mathematics and obtain the exact analytical results of the network at any given pruning step. With these exact analytical results, we can clearly depict even the finest details of the critical behavior in the process. In addition, we propose a simple and intuitive solution of the *k*-core pruning process. It not only greatly simplifies the difficulty of mathematical calculations, but also can be naturally extended to analyze the *k*-core pruning process on correlated networks and multi-layer networks.

In conclusion, this thesis thoroughly studies the *k*-core pruning process with several theoretical approaches and obtains the analytical results of *k*-core pruning process on uncorrelated networks, correlated networks as well as multi-layer networks for the first time. From the analysis of criticality to the entire description of the process, using a wide range of existing tools or developing new methods, this thesis provides several important contributions to the body of knowledge on the *k*-core pruning process.

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