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# Risky human capital investment, income distribution, and macroeconomic dynamics

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## Abstract

This paper examines the implications of human capital risk for the relationship between inequality and economic development. It argues that due to missing insurance markets for human capital risk, the initial distribution of family wealth may play an important role for an economy's process of development fueled by human capital accumulation. The analysis suggests that, in the absence of credit constraints, higher inequality tends to increase the aggregate human capital stock and per capita income, under conditions which are supported empirically for advanced countries. Taking additionally into account that, due to borrowing constraints, higher inequality impedes human capital investment in poorer economies, this suggests a non-linear relationship between inequality and economic development.

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## 1. Introduction

Schooling decisions are made under a substantial degree of uncertainty. First, individual ability and thus performance in school are imperfectly known to students *ex ante*. Since

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school performance is an important determinant of labor market performance, this source of uncertainty transmits into earnings risk. Second, there is uncertainty about the quality of schooling, due to substantial heterogeneity among schooling institutions which may only partly be known *ex ante*. Third, and maybe most important, one's relative position in the post-school earnings distribution is uncertain because unforeseen patterns of technological change and product demand shifts are not neutral across industries. Since skills are to a large degree specific to industries, this leads to uncertainty about relative labor demand within groups of workers with similar education levels (e.g., a college degree).

Empirical evidence indeed strongly suggests that “there is a great deal of uncertainty regarding the returns to schooling” (Carneiro et al., 2003, p. 362).<sup>1</sup> Since human capital risk is typically neither insurable nor diversifiable, uncertainty in the returns to education may be a dominant concern for individuals in their schooling decision, in addition to expected returns.

This paper builds on the premise that risk-averse individuals face idiosyncratic, non-diversifiable and uninsurable labor income risk associated with human capital investments and analyzes its implications for the interaction between intergenerational wealth transmission of heterogeneous individuals and economic development. It is argued that due to missing insurance markets for human capital risk, the initial distribution of family wealth (or parental income, respectively) may play an important role for an economy's process of development fueled by human capital accumulation.

To focus on the role of missing insurance markets for human capital risk, the analysis mostly abstracts from constraints to borrow for educational purposes. This seems a reasonable modelling device for the analysis of advanced countries, where extensive provisions of college financial aid (like in the US) or public education finance (prevalent in Europe) tend to remove credit constraints for human capital investments for the bulk of individuals. In fact, recent studies find no evidence for the relevance of educational borrowing constraints in the US (see e.g. Cameron and Taber, 2004, and the references therein). Nevertheless, empirical evidence strongly suggests that parents' income is an important determinant of human capital investments. For instance, Taubman (1989) reviews estimates for the elasticity of years of schooling with respect to parental income. These are generally positive and range from 3% to 80%, after controlling for parents' education, father's occupation, and/or children's test scores on mental ability tests. Similarly, Solon (1999, p. 1789) concludes: “Most of the evidence [...] indicates that intergenerational earnings elasticities are substantial and are larger than we used to think.” Sacerdote (2002) finds that the effect of socioeconomic status on children's college attendance is just as large for adoptees as for children raised by biological parents, suggesting no significance of genetic factors. Plug and Vijverberg (2003) report higher effects of genetic factors (measured by parents' IQ) on the children's years of schooling and college attainment, although family income still has a large effect. The present framework is consistent with such evidence. It is shown that educational investment at the individual level positively depends on family wealth under standard

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<sup>1</sup> Carneiro et al. (2003) as well as Cunha et al. (2005) develop and apply a procedure to separate uncertainty in labor earnings from unobserved heterogeneity in earnings regressions. Cunha et al. (2005) find that a fraction of about 40% of the variability of earnings is unpredictable to agents. Similarly, Hartog et al. (2004) identify a substantial risk component in the distribution of returns to attend university. Other empirical contributions examine the link between the mean and the variance of returns to education (e.g. Pereira and Martins, 2002; Palacios-Huerta, 2003).

assumptions, even in the absence of credit constraints or heterogeneity in ability. This suggests a potential role of wealth inequality for macroeconomic dynamics.

The main purpose of this study is to characterize this role in terms of observable micro-relations. Therefore, apart from some illustrative examples, the utility function and education technology are held quite general. For instance, the analysis suggests that higher inequality tends to increase the aggregate human capital stock and per capita income when the individual marginal propensity to save (MPS) for intergenerational transfers is increasing in individual income and the expected return to education is non-diminishing. These conditions are supported empirically for the US. For instance, in a recent paper, [Dynan et al. \(2004\)](#) show that the MPS is increasing in lifetime income (see also [Menchik and David, 1983](#)). Moreover, the bulk of individuals does not seem to acquire human capital at levels where returns to education are diminishing, according to a survey by [Card \(1999\)](#).

For developing countries, there is wide agreement that credit constraints are an important obstacle to human capital formation. Taking this additionally into account, the macroeconomic predictions of the model are consistent with evidence by [Barro \(2000\)](#). According to his study, there is no significant relationship between inequality and growth in a broad sample of countries. However, splitting up the sample in relatively poor and relatively rich countries, the relationship is negative in the former and positive in the latter subsample.<sup>2</sup>

The paper is organized as follows. Section 2 discusses the related literature. Section 3 sets up the model. Section 4 analyzes individual decisions and discusses existence and stability of stationary equilibria of the dynamical system. Section 5 examines the role of inequality for aggregate income dynamics in terms of observable micro-relations. It distinguishes the short-run, medium-run and long-run, and provides illustrative examples of the general analysis. Section 6 discusses the results of the paper in the light of empirical evidence on micro- and macro-relations relevant in the present context. The last section concludes. All proofs are relegated to an appendix.

## 2. Related literature

There is a well-developed literature on human capital formation under heterogeneity of agents. Most closely related to the present paper are studies which examine the link from inequality to economic development which is driven by private education investments. In their seminal contribution, [Galor and Zeira \(1993\)](#) show that inequality typically slows down growth because poor individuals cannot borrow sufficiently high amounts to finance schooling investments. Some contributions demonstrate that the relationship between inequality and human capital-based growth can be positive in initially very poor economies, where higher inequality can overcome poverty traps (e.g., [Perotti, 1993](#); [Galor and Tsiddon, 1997](#); [Moav, 2002](#)). [Tamura \(1991, 1992\)](#) shows that there is long-run income convergence of individuals initially differing in their human capital level under decreasing returns to human capital production. [Tamura \(1992, 1996\)](#) introduces coordination costs of market

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<sup>2</sup> Non-linearity may also contribute to an understanding why, generally, empirical evidence on the inequality–growth relationship is mixed. Earlier empirical evidence suggests a negative link between inequality and growth (e.g., [Alesina and Rodrik, 1994](#); [Persson and Tabellini, 1994](#); [Perotti, 1996](#)). However, using a new and comprehensive high-quality data set which allows to study panels, [Deininger and Squire \(1998\)](#) and [Banerjee and Duflo \(2003\)](#) find practically none, whereas [Li and Zou \(1998\)](#) and [Forbes \(2000\)](#) report a positive relationship.

integration in a model where market size gives rise to specialization gains. Conditional on whether coordination costs depend on the distribution of human capital within a market (in addition to the number of market participants), heterogeneity may raise or inhibit economic development. [De la Croix and Doepke \(2003\)](#) analyze a framework in which poor families have more children and invest less in their children's education than rich families; consequently, aggregate human capital investment is negatively related to inequality. [Galor and Moav \(2004\)](#) argue that historically inequality has depressed growth in mature stages of development when human capital accumulation became the prime engine of growth, whereas in early stages inequality may foster growth. In contrast to these contributions, the link between inequality and human capital accumulation in the present paper derives from uncertainty in the return to educational investments.

Relatively few theoretical studies in the human capital literature explicitly consider risk aspects. [Wildasin \(2000\)](#) shows in a framework where skill demand is subject to asymmetric shocks across regions, that regional labor migration reduces uncertainty with respect to returns to private education and therefore boosts efficiency. [Gould et al. \(2001\)](#) argue that an increasing variance of sectoral shocks increase educational attainment of workers because general education reduces the costs of moving across sectors. Their paper is concerned with the effects of technical progress (which varies across sectors) on the evolution of wage inequality, rather than with the role of inequality on development. Closer to the present paper is the study by [Bénabou \(2002\)](#), who assumes that labor income risk magnifies with the level of human capital investment. He shows that efficiency and growth-maximization requires some degree of progressive income taxation by indirectly providing insurance, in the absence of private insurance markets for human capital risk. In contrast, this paper is concerned with the role of the initial distribution of market income for macroeconomic dynamics and does not consider effects of distortionary redistribution. It is thus suited to address cross-country evidence on the inequality–growth relationship like provided by [Barro \(2000\)](#). Moreover, an important channel how changes in equality affect development derives from intergenerational wealth transmission, which is not considered in [Bénabou \(2002\)](#). [Krebs \(2003\)](#) shows that a reduction of labor income risk fosters human capital formation, in turn raising growth and welfare, in a framework with ex ante identical agents. In contrast, this paper holds fixed the degree of human capital risk and varies initial inequality in the wealth distribution. Finally, rather than uncertainty with respect to the return to education, [Tamura \(2006\)](#) considers the role of uncertainty with respect to survival of adults to old age and its role for human capital investments and economic growth. Human capital accumulation reduces young adult mortality and thereby enhances educational investments while reducing fertility. The model fits well data on global income inequality along with the timing and speed of demographic transitions.

Whereas the literature reviewed so far focusses on private education, there also exist important linkages between inequality and aggregates through public education finance. [Glomm and Ravikumar \(1992\)](#) show that, under decreasing returns of parental human capital for human capital formation of children, a more unequal initial income distribution is associated with lower future per capita income in a public education system. [Fernandez and Rogerson \(1995\)](#) demonstrate in a model where education subsidies are determined by majority vote that a small middle class (high inequality) gives rise to an equilibrium in which the credit constrained poor are effectively excluded from education. A series of papers demonstrates that, if education is locally financed by the public sector and community composition is endogenous (i.e., there is choice of residency), community spillovers in

education (like peer effects, social networks, tax base effects) typically lead to socioeconomic stratification and leave the distribution of human capital and income suboptimally unequal (Bénabou, 1993, 1996a,b; Fernandez and Rogerson, 1996, 1997). As a consequence, moving to centralized education finance or reducing heterogeneity may raise growth or welfare. Tamura (2001) shows however that even when public education is a locally financed and thus not of equal quality, poor districts and rich districts may converge over time. In his model, this occurs when teacher quality is relatively more important than class size for human capital production. In an interesting recent study, Canaday and Tamura (2006) argue that, due to migration possibilities of blacks out of discriminatory plantations or towns, convergence of human capital levels between whites and blacks eventually occurs. However, discrimination and its induced inequality is harmful for growth in the process of development. In the present paper which deals with private education finance, convergence between rich and poor agents is not guaranteed. For instance, it is shown that a convexity of the individual saving schedule acts as force towards divergence.

Some brief remarks on the literature where inequality affects macroeconomic dynamics through other channels than human capital are in order. First, according to the classical view, with a modern foundation provided by Bourguignon (1981), inequality positively affects physical capital accumulation, fueled by domestic savings, when the marginal propensity to save is increasing in income. Also in the present paper inequality may positively affect macroeconomic performance when the MPS is rising; however, this possibility arises for a very different reason than in the classical view. The present framework is one of a small open economy in which national savings are unrelated to physical capital investments. Savings matter for human capital investments at the individual level because of uninsurable human capital risk, not because they affect per capita income through physical capital accumulation. Other literature typically suggests that higher inequality depresses growth. Murphy et al. (1989a) show that both extreme inequality and extreme equality hinder industrialization since it implies too small a market for firms to cover fixed cost investments necessary for mass production of manufacturing goods which the poor cannot afford. At intermediate levels of inequality, industrialization can be promoted by a larger middle class. Murphy et al. (1989b) argue that wage differences between cottage producers and workers employed in mass production give rise to possibilities for a big push with respect to industrialization. Alesina and Rodrik (1994) and Persson and Tabellini (1994) argue that inequality is positively related to demand for redistributive and growth-depressing taxation in the political process. This result is modified by Bénabou (1996c), who allows for a bias in the political system. Alesina and Perotti (1996) provide evidence for a negative effect of inequality on social stability and discuss growth consequences. Zweimüller (2000) develops a model in which inequality depresses innovation-based growth by reducing aggregate demand for R&D-intensive products. Finally, Fishman and Simhon (2002) argue that inequality is an obstacle for the degree of specialization of labor.

### 3. The model

Consider a small open overlapping-generations economy with uninsurable risk of educational investments.<sup>3</sup>

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<sup>3</sup> The basic structure of the model follows Galor and Moav (2004), who however consider a closed economy without human capital risk.

### 3.1. Production of final output

In every period, a single homogenous consumption good is produced according to a neoclassical, constant-returns-to-scale production technology. Output at time  $t$ ,  $Y_t$ , is

$$Y_t = F(K_t, H_t) \equiv H_t f(k_t), \quad k_t \equiv K_t/H_t, \quad (1)$$

where  $K_t$  and  $H_t$  are amounts of physical capital and human capital employed in period  $t$ , the latter being measured in efficiency units.  $f(\cdot)$  fulfills the standard properties which lead to an interior solution of the profit maximization problem.

Output is sold in a perfectly competitive environment, with output price normalized to unity. The rate of return to capital,  $\bar{r}$ , is internationally given and time-invariant. Thus, profit maximization of the representative firm in any period  $t$  implies that  $k_t$  is given by  $\bar{r} = f'(k_t)$ . Thus,  $k_t = (f')^{-1}(\bar{r}) \equiv \bar{k}$ . Consequently, the wage rate per efficiency unit of human capital reads  $w = f(\bar{k}) - \bar{k}f'(\bar{k})$ . Moreover,  $Y_t = H_t f(\bar{k})$  grows at the same rate as the aggregate human capital stock  $H_t$ .

The small open economy assumption allows us to focus on the link between income distribution and macroeconomic dynamics which arises from uninsurable human capital risk. This is because it excludes any effect of domestic accumulation of physical capital on this relationship which arises once we allow for non-linear saving functions. It ensures that results on the role of inequality for aggregates are not driven by the mechanism suggested by the classical view, highlighted in Bourguignon (1981).

### 3.2. Individuals and education technology

In each period, there is a unit mass of individuals with two-period lives. They are identical with respect to preferences and their ability to acquire human capital, but may differ in wealth holding. Initially, there are two groups of dynasties. A fraction  $\lambda \in (0, 1)$  of (“rich”) young individuals in  $t = 0$  is endowed with (inherited) wealth  $b_0^R > 0$  and a fraction  $1 - \lambda$  (the “poor”) with  $b_0^P \in [0, b_0^R)$ . Thus, total wealth in period 0 is  $B_0 \equiv \lambda b_0^R + (1 - \lambda)b_0^P$ . In the first period, an individual  $i$  born in period  $t$  (a member  $i$  of generation  $t$ ) chooses educational investment  $e_t^i$  (in units of the consumption good) and saves  $s_t^i$  for future wealth. Human capital investments can be thought of both schooling and non-schooling forms of training. For simplicity, there is no consumption in the first period. With wealth endowment  $b_t^i$ , the budget constraint of a young member  $i$  of generation  $t$  thus is  $e_t^i + s_t^i = b_t^i$ . In the second period (adulthood), individuals supply human capital  $h_{t+1}^i$  (generated by investment  $e_t^i$ ) to the labor market and allocate their income between consumption,  $c_{t+1}^i$ , and transfers to their offspring (bequests or inter vivos transfers),  $b_{t+1}^i$ . Thus, the budget constraint of adult  $i$  in  $t + 1$  reads  $c_{t+1}^i + b_{t+1}^i = I_{t+1}^i$ , where lifetime income  $I_{t+1}^i = wh_{t+1}^i + Rs_t^i$ ,  $R \equiv 1 + \bar{r}$ . Adult individuals possess an aggregate human capital stock  $H_0$ , i.e., initial output is  $Y_0 = H_0 f(\bar{k})$ .

It remains to specify the education technology and preferences. Member  $i$  of generation  $t$  with educational investment  $e_t^i$  obtains

$$h_{t+1}^i = h(e_t^i, \tilde{a}) \quad (2)$$

efficiency units of human capital.  $\tilde{a}$  is a random variable which follows an i.i.d. process and is drawn each period from a (cumulative) distribution function  $\Phi(\tilde{a})$  with a bounded

support  $\mathcal{A}$ . The random shock realizes after investment decisions are made, i.e., at the end of the first period of life.

It is assumed that an insurance market for the idiosyncratic human capital risk is missing (e.g. Arrow, 1971). To focus the analysis on this market failure, suppose that individuals can freely borrow for educational purposes, e.g., due to public provision of financial college aid. (Section 6 discusses implications when relaxing this assumption.) Function  $h(e, \tilde{a})$  fulfills the following properties. ( $h_e$  denotes the first partial derivative of  $h$  with respect to  $e$ , etc.)

**A1.** Suppose  $h_{\tilde{a}} > 0$ ,  $h_e > 0$ ,  $h_{ee} \leq 0$  and  $h_{e\tilde{a}} > 0$ .

$h_e > 0$  implies that the expected marginal return to educational investment is positive. Moreover, given that  $h_{\tilde{a}} > 0$ , which merely serves as a convention,  $h_{e\tilde{a}} > 0$  implies that the variance of earnings increases with human capital investment  $e$ .<sup>4</sup>

Three remarks are in order. First, empirical evidence suggests that human capital risk is substantial (see e.g. Carneiro et al., 2003; Hartog et al., 2004; Cunha et al., 2005, and the references therein). Moreover, by assuming that the return to physical capital is certain, the framework captures the standard notion that human capital investment is riskier than physical capital investment (Krebs, 2003).<sup>5</sup> Second, also the property that the variance of earnings rises with the level of education is well-supported empirically (see e.g. Levhari and Weiss, 1974, and, more recently, Pereira and Martins, 2002, 2004). Third, the analysis explicitly allows for the case of non-diminishing (expected) returns to schooling ( $h_{ee} = 0$ ). This does not deny that the return to schooling is ultimately diminishing due to physical constraints of human brain capacity. However, it will become apparent that – unlike in a deterministic framework – diminishing returns are not necessary for obtaining an interior solution of the educational choice problem under uncertainty.

Each member  $i$  of generation  $t$  maximizes expected utility  $E(U_t^i)$ , where for simplicity  $U_t^i$  is additively separable in consumption and bequests:

$$U_t^i = u(c_{t+1}^i) + v(b_{t+1}^i). \quad (3)$$

That intergenerational transfers,  $b_{t+1}^i$ , enter the utility function reflects a “joy of giving” saving motive, which has received strong empirical support (see e.g. Wilhelm, 1996; Altonji et al., 1997; Carroll, 2000). Functions  $u$  and  $v$  have the following properties.

**A2.** Let  $u' > 0$ ,  $u'' < 0$ ,  $v' > 0$ ,  $v'' < 0$ ,  $\lim_{I \rightarrow \infty} u'(I) < v'(0)$ ,  $u''' u' > (u'')^2$  and  $(v'')^3 [u' u''' - (u'')^2] + (u'')^3 [v' v''' - (v'')^2] > 0$ .

As will be shown, Assumption A2 implies several plausible properties. First, strict concavity of functions  $u$  and  $v$  gives rise to risk aversion of individuals. The final two assumptions in A2 imply decreasing absolute risk aversion, consistent with observed behavior in the context of portfolio decisions in financial markets (e.g. Carroll, 2002), occupational choice, demand for insurance, and other household decisions (e.g. Gollier, 2001). In an

<sup>4</sup> This is shown in supplementary material to this paper, available on request. There it is also argued formally that the education technology captures uncertainty about the demand for specific skills.

<sup>5</sup> First, human capital risk is non-diversifiable since embodied in individuals, whereas diversified portfolios of financial capital can be held. Second, many forms of financial assets are indeed almost risk-free (e.g. government bonds), at least in advanced countries.

interesting empirical study, Guiso and Paiella (2001) present survey evidence which clearly rejects the hypothesis that the degree of absolute risk aversion is non-decreasing. In the model, this implies that educational investment  $e_t^i$  is an increasing function of inherited wealth,  $b_t^i$ , a property which is well-supported empirically even in the US where credit constraints are found to play a negligible role for human capital investments. Moreover, A2 implies that intergenerational transfer,  $b_{t+1}^i$ , is increasing in income, which also is the empirically relevant case.

#### 4. Equilibrium

We first derive individual decisions and then discuss long-run wealth distributions.

##### 4.1. Individual decisions

In their educational investment decision, individuals maximize expected lifetime utility arising from income

$$I_{t+1}^i = wh(e_t^i, \tilde{a}) + R(b_t^i - e_t^i), \quad (4)$$

which is uncertain due to human capital risk. (Recall  $I_{t+1}^i = wh_{t+1}^i + Rs_t^i$  and substitute human capital from (2) and budget constraint  $s_t^i = b_t^i - e_t^i$ .) We thus solve backwards: first, the decision of adults to allocate a given income  $I_{t+1}^i$  on consumption and bequests, and second, the decision how much of wealth  $b_t^i$  to invest in acquisition of human capital and how much in the financial market.

Using  $c_{t+1}^i = I_{t+1}^i - b_{t+1}^i$ , optimal savings of an adult (bequests) in  $t + 1$  are given by

$$b(I_{t+1}^i) \equiv \arg \max_{b_{t+1}^i \geq 0} u(I_{t+1}^i - b_{t+1}^i) + v(b_{t+1}^i). \quad (5)$$

This gives us indirect life-time utility,

$$V(I_{t+1}^i) \equiv u(I_{t+1}^i - b(I_{t+1}^i)) + v(b(I_{t+1}^i)). \quad (6)$$

Thus, using (4), the optimal human capital investment is given by

$$e(b_t^i) \equiv \arg \max_{e_t^i \geq 0} E[V(wh(e_t^i, \tilde{a}) + R(b_t^i - e_t^i))]. \quad (7)$$

Throughout, the analysis exclusively focusses on an interior and unique solution of optimization problem (7).<sup>6</sup> Functions  $b(I)$ ,  $V(I)$  and  $e(b)$  have the following properties. (All results are proven in appendix.)

**Lemma 1** (Characterization of  $b(I)$ ,  $V(I)$ ,  $e(b)$ ).

- (i) *There exists an income level  $\underline{I} \geq 0$  such that  $b(I) > 0$  and  $b'(I) > 0$  for all  $I > \underline{I}$ .*

<sup>6</sup> Moreover, it is implicitly assumed throughout the paper that a young individual with zero wealth has positive income even for the worst realization of random variable  $\tilde{a}$ . That is, she is able to pay back the required loan  $Re(0)$  using her labor income. Otherwise, the basic model would be inconsistent with the assumption that there are no borrowing constraints to finance education.

- (ii)  $V'(I) > 0$ ,  $V''(I) < 0$  (individuals are risk-averse) and the coefficient of absolute risk aversion,  $A(I) \equiv -V''(I)/V'(I)$ , is strictly decreasing in  $I$ .
- (iii)  $e'(b) > 0$ , i.e., human capital investment is strictly increasing in family wealth.

Part (i) of Lemma 1 shows that intergenerational transfers, which equal savings of an adult, are increasing in income above a threshold income level. How the marginal propensity to save in the second period of life (MPS),  $b'(I)$ , is affected by income is generally ambiguous. As will become apparent, the relationship between inequality and the state of economic development (as measured by the level of GDP,  $Y$ ) will critically depend on the sign of  $b''(\cdot)$ , i.e., whether MPS is increasing or decreasing in income.<sup>7</sup> Decreasing absolute risk aversion (part (ii) of Lemma 1) implies that richer dynasties will spend more on education (part (iii) of Lemma 1). This parallels the result derived in the pioneering work on risky education by Levhari and Weiss (1974), who consider a two-period model with exogenous wealth (see also Eaton and Rosen, 1980). Intuitively, whereas under A1 the variance of earnings is increasing with the level of investment in human capital, investing in physical capital (i.e., financial assets) is risk-free. If  $A'(I) < 0$ , wealthier individuals are willing to bear more risk and thus invest more in education. Note that, if there were no uncertainty, then (contrary to empirical evidence) educational investment would be independent of  $b$ . To see this, consider the first-order condition implied by (7)

$$E[V'(wh(e_t^i, \tilde{a}) + R(b_t^i - e_t^i))(wh_e(e, \tilde{a}) - R)] = 0. \quad (8)$$

If there is no uncertainty and the marginal return to education is denoted by  $h'(e)$ , an interior and unique solution requires  $wh'(e) = R$ . Also note that for this solution,  $h'' < 0$  must hold. In contrast, under uncertainty, there can be an interior and unique solution even if  $h_{ee} \geq 0$  for all  $(e, a)$ . Under assumption  $h_{ee} \leq 0$  in A1, an interior solution is ensured if  $\lim_{e \rightarrow 0} h_e(0, a)$  is high enough for some  $a$ , as  $e(b) > 0$  implies  $wE[h_e(e, \tilde{a})] > R$  (expected return to education is larger than the financial return  $R = 1 + \bar{r}$ ).

#### 4.2. Long-run behavior

By not imposing functional forms on utility or the education technology, it will be possible to characterize the impact of higher initial inequality on economic development in terms of observable micro-relations in the model: education technology  $h(e, \tilde{a})$  which determines the marginal return to education, intergenerational wealth transmission as captured by function  $b(I)$ , and the educational investment function  $e(b)$ . Whether or not a stable and stationary equilibrium exists, this can be done for any period  $t$ . Nevertheless, we shall first discuss possible existence and stability of stationary equilibria.

If investing  $e_t^i = e(b_t^i)$  in period  $t$ , during adulthood individual  $i$  supplies

$$h_{t+1}^i = h(e(b_t^i), \tilde{a}) \equiv \hat{h}(b_t^i, \tilde{a}) \quad (9)$$

efficiency units of human capital. According to (4), her income in  $t + 1$  reads

$$I_{t+1}^i = w\hat{h}(b_t^i, \tilde{a}) + R(b_t^i - e(b_t^i)) \equiv \hat{I}(b_t^i, \tilde{a}). \quad (10)$$

<sup>7</sup> In the case where  $b(I)$  is a kinked function, with kink at  $I = \underline{I}$  (i.e.,  $b(I) = 0$  for  $I \leq \underline{I}$  and  $b(I) > 0$  for  $I > \underline{I}$ ),  $b(I)$  is not differentiable at  $\underline{I}$ . The subsequent analysis neglects the knife-edge case where an individual has income  $\underline{I}$  for simplicity.

Using  $b_{t+1}^i = b(I_{t+1}^i)$ , this in turn determines intergenerational transfers

$$b_{t+1}^i = b(\widehat{I}(b_t^i, \tilde{a})) \equiv \hat{b}(b_t^i, \tilde{a}). \quad (11)$$

Thus, the evolution of bequests within each dynasty  $i$  follows a discrete time Markov process defined by the first-order difference equation  $b_{t+1}^i = \hat{b}(b_t^i, \tilde{a})$ . (Recall that initial levels  $b_0^i$ ,  $i = R, P$ , are given.) According to (9) and (10), it also implies the evolution of the distribution of both human capital and income among individuals, respectively. In turn, it determines aggregates. To see this, let the economy's c.d.f. of family wealth in period  $t$  be denoted by  $\Psi_t(b)$ , with support  $\mathcal{B}_t \subset \mathbb{R}_+$ , such that we can write aggregate human capital as

$$H_{t+1} = \int_{\mathcal{A}} \int_{\mathcal{B}_t} \hat{h}(b, \tilde{a}) d\Psi_t(b) d\Phi(\tilde{a}), \quad (12)$$

which determines aggregate income  $Y_{t+1} = H_{t+1}f(\bar{k})$ .

A stationary equilibrium is reached if the distribution of  $b_t^i$  as  $t \rightarrow \infty$  is time-invariant.<sup>8</sup> Let  $\underline{a} = \inf \mathcal{A}$  and  $\bar{a} = \sup \mathcal{A}$ . There is a unique stationary equilibrium on a stable set  $[x, y] \subset \mathbb{R}_+$  of the stochastic process  $\hat{b}$  (see Wang, 1993). By definition, such a stable set  $[x, y]$  exists if (i)  $\hat{b}(x, \underline{a}) = x$ ,  $\hat{b}(y, \bar{a}) = y$ , and (ii)  $\hat{b}(b^i, \underline{a}) < b^i$ ,  $\hat{b}(b^i, \bar{a}) > b^i$  for all  $b^i \in (x, y)$ . To investigate conditions for existence of a stable set, note that  $\hat{b}(b, a) = b(\widehat{I}(b, a))$  can be characterized as follows: under differentiability, we have

$$\hat{b}_b(b, a) = b'(\widehat{I}(b, a))\widehat{I}_b(b, a), \quad (13)$$

$$\hat{b}_{bb}(b, a) = b''(\widehat{I}(b, a))\widehat{I}_b(b, a)^2 + b'(\widehat{I}(b, a))\widehat{I}_{bb}(b, a), \quad (14)$$

where, according to (10),

$$\widehat{I}_b(b, a) = wh_e(e(b), a)e'(b) + R(1 - e'(b)), \quad (15)$$

$$\widehat{I}_{bb}(b, a) = wh_{ee}(e(b), a)e'(b)^2 + (wh_e(e(b), a) - R)e''(b). \quad (16)$$

Focussing on the plausible case where  $e'(b) \leq 1$  (a marginal increase in  $b$  does not lead to a decline of investment in the financial market), part (i) of Lemma 1, (13) and (15) imply that for all  $a \in \mathcal{A}$  there exists threshold level  $\underline{b}_a \geq 0$  such that  $\hat{b}(b, a) > 0$  and  $\hat{b}_b(b, a) > 0$  when  $b > \underline{b}_a$ . Possibly,  $\hat{b}(b, a) = 0$  for  $b \leq \underline{b}_a$ . Also note that  $\hat{b}_a(b, a) > 0$  such that the  $\hat{b}$ -curves for  $\underline{a}$  and  $\bar{a}$  in  $b_t^i - b_{t+1}^i$ -space do not intersect and  $\underline{b}_{\underline{a}} \geq \underline{b}_{\bar{a}}$ .

Panels (a) and (b) of Fig. 1 show situations in which  $\hat{b}(b, \bar{a}) = 0$  for  $b \leq \underline{b}_{\bar{a}}$  and  $\underline{b}_{\bar{a}} > 0$ . This means that for low wealth levels  $b$  (hence, educational investment  $e(b)$  and income is low) also in the best states income is too low to induce positive bequests ( $I \leq \underline{I}$ ). This is consistent with the empirical observation that low-income individuals neither save nor bequeath (see Example 1 below for a microfoundation). Then, as apparent from Fig. 1, there is a stationary and stable equilibrium in which for any dynasty  $i$ ,  $b_t^i = 0$  as  $t \rightarrow \infty$  with probability one. Panel (a) shows a situation in which this is a unique and globally stable equilibrium, i.e., irrespective of initial wealth holdings, all dynasties end up with

<sup>8</sup> Formally, let  $P(b^i, \mathcal{Z}) \equiv \Pr\{\tilde{a} : \hat{b}(b^i, \tilde{a}) \in \mathcal{Z}\}$  be the probability that  $b^i$  is in the set  $\mathcal{Z}$  one period after it started in  $b^i$ , where  $\mathcal{Z}$  is a Borel set in  $\mathbb{R}_+$ . Moreover, let  $\mu_t^i(\mathcal{Z}) \equiv \Pr\{b_t^i \in \mathcal{Z}\}$  for all  $\mathcal{Z} \subset \mathbb{R}_+$ ,  $t = 0, 1, 2, \dots$ , be the probability measure associated with  $b_t^i$ . Thus, the distribution of family wealth evolves according to  $\mu_{t+1}^i(\mathcal{Z}) = \int P(b^i, \mathcal{Z})\mu_t^i(db^i)$  for all  $\mathcal{Z} \subset \mathbb{R}_+$ . A stationary equilibrium for  $b^i$  is a probability measure  $\mu^i$  such that  $\mu^i(\mathcal{Z}) = \int P(b^i, \mathcal{Z})\mu^i(db^i)$  for all  $\mathcal{Z} \subset \mathbb{R}_+$  (see Wang, 1993).

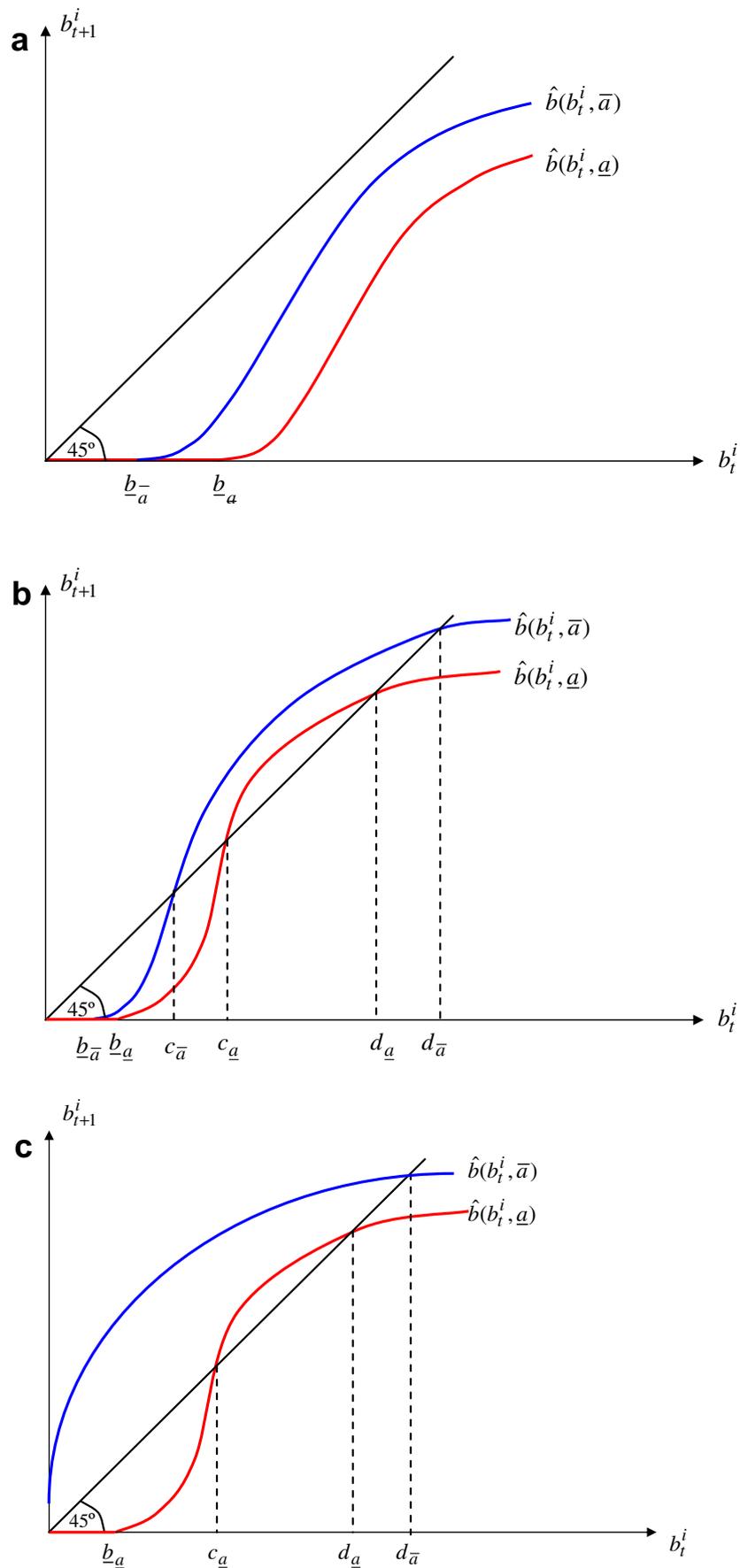


Fig. 1. Stationary equilibrium and convergence.

zero wealth in the long run. In panel (b), there is a second stationary equilibrium with positive wealth on a stable set  $[d_a, d_{\bar{a}}]$ .

In panel (c) of Fig. 1, low-wealth individuals do bequeath in the best state,  $\bar{a}$ , but not in the worst state,  $\underline{a}$ . (As implausible, we shall not consider situations in which the latter condition is violated.) In contrast to panel (b), there is a unique stationary equilibrium on the stable set  $[d_a, d_{\bar{a}}]$ .

Clearly, existence of a stable set as in panels (b) and (c) requires that both curves cross the 45-degree line from above. For this, the following two features must hold. First, the bequest motive must be sufficiently strong in the sense that there exists a  $c_a > 0$  such that  $\hat{b}(b, \underline{a}) > b$  for  $b > c_a$ . Second, for some  $b > c_a$  the  $\hat{b}(b, \underline{a})$ -curve and, if  $\hat{b}(b, \bar{a}) = 0$  for  $b \leq \underline{b}_{\bar{a}}$  where  $\underline{b}_{\bar{a}} > 0$  (panel (b)), for some  $b > c_{\bar{a}} > 0$  also the  $\hat{b}(b, \bar{a})$ -curve must be strictly concave somewhere (as the slope must turn from above unity to below unity). For instance, if  $h_{ee} = 0$  and  $e''(b)$  is negligible, this holds if  $b'' < 0$  (decreasing MPS), according to (14) and (16). For the case where MPS is non-decreasing,  $b'' \geq 0$ , we necessarily must have that the marginal return to education is eventually diminishing,  $h_{ee} < 0$ , irrespective of the sign of  $e''(b)$ . To see this, note that in the case where  $b'' \geq 0$ , strict concavity of both curves,  $\hat{b}(b, \underline{a})$  and  $\hat{b}(b, \bar{a})$ , requires  $\hat{I}_{bb}(b, a) < 0$  for  $a \in \{\underline{a}, \bar{a}\}$  and for high  $b$  (and thus high  $e(b)$ ); moreover, we have  $wh_e(e(b), \underline{a}) < R$  and  $wh_e(e(b), \bar{a}) > R$ . The salient role of decreasing returns in human capital production is consistent with a well-developed literature which in this case suggests convergence among initially heterogeneous agents in the long-run (e.g. Tamura, 1991, 1992; Galor and Moav, 2004). In the present context, otherwise, a stable set may not exist and there may be divergence. Clearly, according to (14), an increasing MPS ( $b'' > 0$ ) is a diverging force.

## 5. Role of inequality for economic development

This section examines how the distribution of initial family wealth affects economic development for finite  $t$  and, if at least one stationary and stable equilibrium exists, for  $t \rightarrow \infty$ .

Recall that, initially, there are two groups of individuals, rich and poor, and the aggregate initial transfer is  $B_0 = \lambda b_0^R + (1 - \lambda)b_0^P$ . To study the role of inequality for the process of economic development (i.e., on aggregate human capital stock,  $H_t$ , and thus on GDP,  $Y_t = H_t f(\bar{k})$ ),<sup>9</sup> suppose the distribution of initial transfers changes to

$$\check{b}_0^R \equiv b_0^R + \varepsilon, \quad \check{b}_0^P \equiv b_0^P - \varepsilon\lambda/(1 - \lambda), \quad (17)$$

i.e., aggregate family wealth at  $t = 0$ ,  $B_0$ , is held constant (mean-preserving spread). In the following, higher inequality (in the initial wealth distribution) is reflected by an increase in  $\varepsilon$ .

### 5.1. Short-run impact

To investigate the short run effect of the initial wealth distribution on the economy's state of development, reflected by  $H_1$ , note that

<sup>9</sup> Also note that the growth rate of  $Y$  between periods  $t$  and  $0$  is given by  $H_t/H_0 - 1$ .

$$\begin{aligned} H_1 &= E[\lambda \hat{h}(\check{b}_0^R, \tilde{a}) + (1 - \lambda) \hat{h}(\check{b}_0^P, \tilde{a})] \\ &= \lambda E(\hat{h}(b_0^R + \varepsilon, \tilde{a})) + (1 - \lambda) E(\hat{h}(b_0^P - \varepsilon \lambda / (1 - \lambda), \tilde{a})), \end{aligned} \quad (18)$$

according to (12) and (17). From this, we obtain the following result.

**Proposition 1** (Impact of higher inequality in the short run). *If  $h_{ee} = 0$  and  $e''(\cdot) > 0$  everywhere, then an increase in  $\varepsilon$  raises  $H_1$ . If  $h_{ee} = 0$  and  $e''(\cdot) = 0$ , then the initial wealth distribution has no effect on  $H_1$ . If  $h_{ee} \leq 0$  and  $e''(\cdot) \leq 0$ , with at least one condition holding with strict inequality, an increase in  $\varepsilon$  reduces  $H_1$ .*

The intuition of Proposition 1 is simple. For instance, suppose  $e''(\cdot) < 0$  everywhere. Then higher inequality (i.e., an increase in  $\varepsilon$ ) implies that poor individuals decrease their human capital investment more than rich individuals increase it. As we assumed that the return to education is non-increasing ( $h_{ee} \leq 0$ ), this means that the aggregate human capital stock next period decreases. This result parallels the standard view of the relationship between inequality and growth (provided there is no poverty trap), which in the literature typically relies on credit constraints and decreasing marginal returns to education (e.g. Bénabou, 1996; Moav, 2002; Galor and Moav, 2004). However, if by contrast  $e'' > 0$  and, say,  $h_{ee} = 0$ , then higher inequality in fact raises aggregate output next period. We next turn to examine the impact of a change in initial inequality on economic development for later periods.

### 5.2. Medium-run impact

For the impact of an increase in  $\varepsilon$  on the human capital stock in later periods, also the process of intergenerational wealth transmission matters. To see this, note that for family wealth  $b_t^i$  in period  $t$ , human capital of a member  $i$  of generation  $t + 1$  in period  $t + 2$ , subject to random shocks  $\tilde{a}$  and  $\tilde{a}'$ , is

$$h_{t+2}^i = \hat{h}(\hat{b}(b_t^i, \tilde{a}), \tilde{a}') \equiv \hat{\hat{h}}(b_t^i, \tilde{a}, \tilde{a}'), \quad (19)$$

according to (9) and (11), respectively. Thus, using (17), (12) and (19), the aggregate human capital stock in period 2 may be written as

$$H_2 = E \left[ \lambda E \left( \hat{\hat{h}}(b_0^R + \varepsilon, \tilde{a}, \tilde{a}') \right) + (1 - \lambda) E \left( \hat{\hat{h}}(b_0^P - \varepsilon \lambda / (1 - \lambda), \tilde{a}, \tilde{a}') \right) \right]. \quad (20)$$

Similarly, we may write

$$h_{t+3}^i = \hat{h}(\hat{b}(\hat{b}(b_t^i, \tilde{a}), \tilde{a}'), \tilde{a}'') \equiv \hat{\hat{\hat{h}}}(b_t^i, \tilde{a}, \tilde{a}', \tilde{a}''), \quad (21)$$

$$H_3 = E \left[ E \left[ \lambda E \left( \hat{\hat{\hat{h}}}(b_0^R + \varepsilon, \tilde{a}, \tilde{a}', \tilde{a}'') \right) + (1 - \lambda) E \left( \hat{\hat{\hat{h}}}(b_0^P - \varepsilon \lambda / (1 - \lambda), \tilde{a}, \tilde{a}', \tilde{a}'') \right) \right] \right], \quad (22)$$

and so on. One obtains the following result.

**Proposition 2** (Impact of higher inequality in the medium run). *Consider finite  $t \geq 2$ . (i) Suppose  $h_{ee} = 0$ . If  $b''(\cdot) \geq 0$  and  $e''(\cdot) \geq 0$  everywhere, with at least one condition holding with strict inequality, then an increase in  $\varepsilon$  raises  $H_t$ ; if  $b''(\cdot) = e''(\cdot) = 0$ , then the initial wealth distribution has no effect on  $H_t$ . (ii) Otherwise, the impact of higher inequality on  $H_t$  is generally ambiguous.*

**Proposition 2** highlights the role of intergenerational wealth transmission, hence the shape of function,  $b(I)$ , for the relationship between inequality and economic development. Even when higher inequality of the initial wealth distribution spurs development in the short-run (e.g., if  $h_{ee} = 0$  and  $e'' > 0$ , according to **Proposition 1**), it may decrease aggregate output in later periods when the MPS is decreasing in income ( $b'' < 0$ ). Also the reverse is true: if  $b'' > 0$ , then inequality may foster development in the medium run even when impeding short-run growth.

When  $\varepsilon$  increases, members of generation 0 which belong to a poor dynasty (endowed with  $\check{b}_0^P$ ) transmit less wealth to their children (in period 1), on average, whereas those from rich dynasties transmit more. For instance, suppose that  $h_{ee} = 0$  and  $e'' = 0$  everywhere (i.e.,  $H_1$  is unchanged). In such a case, average income of poor families decreases by the same amount as average income of rich families increases when  $\varepsilon$  is raised. If  $b'' > 0$ , the resulting average increase in wealth transmission of the rich to its offspring outweighs the average decrease in wealth transmission of the poor. The resulting effect on the aggregate level of human capital of members of generation 1 (in period 2,  $H_2$ ) is unambiguously positive in this case. And the same forces would prevail afterwards, so inequality fosters economic development in the medium run if  $h_{ee} = 0$ ,  $e'' = 0$  and  $b'' > 0$  everywhere.

In other scenarios, it may well be the case that the opposite holds. For instance, suppose again  $h_{ee} = 0$  but now consider the case where the impact of a higher parental transfer on educational investment is declining ( $e'' < 0$ ). In this case, an increase in  $\varepsilon$  implies that average income of poor families in period 1 decrease more than average income of rich families increase. Hence, higher inequality may depress medium run development even under an increasing MPS ( $b'' > 0$ ).

As the impact of inequality on economic development depends on observable micro-relations, which case applies is thus eventually an empirical matter and is discussed in Section 6. From a theoretical point of view, however, the shape of functions  $b(I)$  and  $e(b)$  is endogenous. In fact, to illustrate that the scenarios considered in part (i) of **Proposition 2** may occur, we have to consider specific forms of the utility function and the education technology. The first example shows that, from a theoretical point of view, inequality may indeed not matter at all, as indicative in representative agent models. The second example demonstrates that it is possible that higher inequality fosters economic development.

**Example 1.** Suppose  $u(c,b) = (1 - \gamma)\ln c + \gamma\ln(b + \xi)$ ,  $0 < \gamma < 1$ ,  $\xi > 0$ . Solving utility maximization problem (5), we obtain  $b(I) = \gamma I - (1 - \gamma)\xi$  if  $I > \underline{I} = (1 - \gamma)\xi/\gamma$  and  $b(I) = 0$  otherwise. Thus, if  $\xi$  is sufficiently high, there are  $\underline{b}_{\bar{a}}$  and  $\underline{b}_{\bar{a}}$  as in Fig. 1. Moreover, for  $I \neq \underline{I}$ ,  $b''(I) = 0$ . Indirect utility reads  $V(I) = \ln((1 - \gamma)^{1-\gamma}\gamma^\gamma) + \ln(I + \xi)$ , according to (6). (Thus, the coefficient of absolute risk aversion,  $A(I) = -V''(I)/V'(I)$ , is decreasing in  $I$ .) For the education technology, assume  $h(e, \tilde{a}) = e\tilde{a}$  (i.e.,  $h_{ee} = 0$ ), where  $\tilde{a} = \underline{a}$  with probability  $p \in (0, 1)$  and  $\tilde{a} = \bar{a}$  otherwise. According to (7), educational investment is then chosen to maximize

$$p \ln [w\underline{e}\underline{a} + R(b - e) + \xi] + (1 - p) \ln [w\bar{e}\bar{a} + R(b - e) + \xi]. \quad (23)$$

To ensure an interior solution, suppose  $w\bar{a} > R > w\underline{a}$ ,  $p w\underline{a} + (1 - p)w\bar{a} > R$  (i.e., the expected return to human capital investment exceeds  $R$ ). Consequently, (23) implies

$$e(b) = \frac{(p w\underline{a} + (1 - p)w\bar{a} - R)(Rb + \xi)}{(w\bar{a} - R)(R - w\underline{a})}, \quad (24)$$

i.e.,  $e'(b) \in (0, 1)$  and  $e''(b) = 0$ . Hence, applying **Propositions 1 and 2**, inequality is unrelated to economic development in both short-run and medium-run. Interestingly, it is easy to see from (24) that a mean-preserving spread in the distribution of random variable  $\tilde{a}$  reduces educational investment and therefore raises investment in the financial market, as in Krebs (2003).

**Example 2.** Now modify the utility function to  $u(c, b) = \alpha c - \beta c^2 + \ln b$ ,  $\alpha > 0$ ,  $\beta > 0$ . It is easy to show that this implies

$$b(I) = 0.5I - 0.25\alpha/\beta + \sqrt{0.25(I - 0.5\alpha/\beta)^2 + 0.5/\beta}. \quad (25)$$

Thus,  $b''(I) > 0$ . Moreover, by using (6) and applying the envelope theorem, we find  $V'(I) = v'(b(I)) = 1/b(I)$ . (From this, it is easy to show that, again,  $A'(I) < 0$ .) For the same education technology as in **Example 1**,  $e(b)$  is given by

$$p \frac{w\underline{a} - R}{b(w\underline{e}\underline{a} + R(b - e))} + (1 - p) \frac{w\bar{a} - R}{b(w\bar{e}\bar{a} + R(b - e))} = 0, \quad (26)$$

according to (8), where  $b(\cdot)$  is given by (25). Numerical simulations reveal that  $e(b)$  may be convex, as illustrated in Fig. 2. In Fig. 2, for the parameter values chosen, we have  $e''(b) > 0$  for small  $b$  and  $e''(b) = 0$  for larger  $b$ . According to **Propositions 1 and 2**, the implications of this example, where  $e''(b) \geq 0$ ,  $h_{ee} = 0$  and  $b''(I) > 0$ , are that initial inequality may not matter for short-run growth, but is positively related to aggregate human capital,  $H_t$ , in later periods.

### 5.3. Long-run impact

Assessing the impact of higher inequality on the aggregate human capital stock,  $H_t$ , and thus on GDP, in the long-run (as  $t \rightarrow \infty$ ) requires convergence of Markov process  $b_{t+1}^i = \hat{b}(b_t^i, \tilde{a})$  to a stationary distribution for both initial wealth levels,  $b_0^P$  and  $b_0^R$ . For reasons discussed in Section 3.2, we focus the analysis on the situations depicted in panels (a)–(c) of Fig. 1.

In panels (a) and (c), there is a unique stationary equilibrium. Thus, the income distribution between initially poor and rich dynasties (to zero wealth in panel (a) and positive wealth in (c)) converges. Obviously, in such cases initial inequality does not matter for development in the long run.

For inequality to potentially play a role as  $t \rightarrow \infty$ , multiple stationary equilibria must exist, as in panel (b) of Fig. 1. Because  $e'(b) > 0$  (part (iii) of **Lemma 1**), the long run human capital stock  $H_\infty$  is higher, the higher the fraction of dynasties whose wealth levels converge to the stable set  $[d_{\underline{a}}, d_{\bar{a}}]$  in panel (b) and therefore transmit positive wealth; see Eq. (12). In panel (b), if  $b_0^i \leq c_{\bar{a}}$ , wealth levels within dynasty  $i$  become zero with probability one in the long run (which is a locally stable stationary equilibrium). If  $b_0^i \geq c_{\underline{a}}$ , then the distribution of  $b_t^i$  converges with probability one to a locally unique stable stationary equilibrium on  $[d_{\underline{a}}, d_{\bar{a}}]$ . Thus, as long as  $b_0^i \leq c_{\bar{a}}$  or  $b_0^i \geq c_{\underline{a}}$  for all  $i = R, P$ , then a change in initial inequality again does not affect long run human capital,  $H_\infty$ . If however

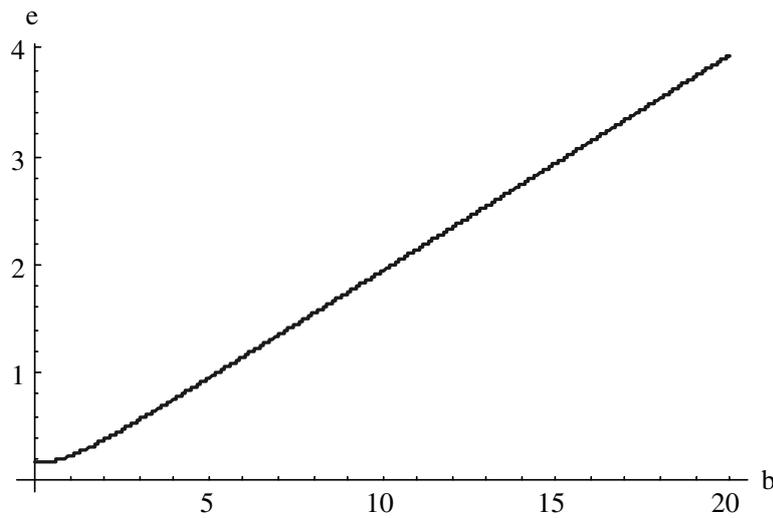


Fig. 2. Function  $e(b)$  in Example 2 for  $p = 0.5$ ,  $\underline{a} = 0$ ,  $\bar{a} = 4$ ,  $\bar{w} = 1$ ,  $\bar{R} = 1.5$ ,  $\alpha = \beta = 1$ .

$b_0^i \in (c_{\bar{a}}, c_{\underline{a}})$ , then, as  $t \rightarrow \infty$ , both  $b_t^i = 0$  or  $b_t^i \in [d_{\underline{a}}, d_{\bar{a}}]$  is possible with positive probability. Illuminating discussions of this stationary equilibrium indeterminacy in stochastic models are provided by Laitner (1981) and Wang (1993). Replicating the arguments in Laitner (1981, Section III), according to the law of large numbers, if  $b_0^P \in (c_{\bar{a}}, c_{\underline{a}})$ , some fraction of initially poor dynasties will end up with zero wealth in the long run. This fraction is increasing in the distance of  $b_0^P$  to  $c_{\underline{a}}$ . Thus, the larger  $(c_{\underline{a}} - b_0^P)$  is, the lower the fraction of initially poor dynasties which transmit positive wealth levels in the long run. Now suppose that, in addition to  $b_0^P \in (c_{\bar{a}}, c_{\underline{a}})$ , we have  $b_0^R > c_{\underline{a}}$ . This captures an economy which is sufficiently rich initially. In this case, the long run wealth distribution coincides among those initially poor dynasties who end with positive wealth and the initially rich, who all end up with positive wealth within the stable set  $[d_{\underline{a}}, d_{\bar{a}}]$ . So what matters is the fraction of individuals from the initially poor dynasties with positive wealth. Consequently, an increase in  $\varepsilon$  unambiguously leads to a decrease in  $H_\infty$ . This suggests that, if anything, when initially the economy is sufficiently rich in the above sense and there are multiple stationary equilibria as in panel (b) of Fig. 1, higher initial inequality reduces long run per capita income.

For other initial wealth levels,  $b_0^P$  and  $b_0^R$ , a positive link between initial inequality and  $H_\infty$  in the situation captured by panel (b) of Fig. 1 is possible. To see this, suppose  $b_0^i \leq c_{\bar{a}}$  for  $i = R, P$ , such that the distribution of wealth levels of all dynasties converge to zero with probability one. This reflects a poverty trap similar to the literature on inequality and growth which focusses on deterministic frameworks (e.g., Perotti, 1993; Galor and Tsiddon, 1997; Moav, 2002). In this case, sufficient redistribution to the rich may result in a situation in which wealth levels of at least some initially rich dynasties converge to the stationary equilibrium on the interval  $[d_{\underline{a}}, d_{\bar{a}}]$ , without affecting the long run wealth distribution of the initially poor (who end up with zero wealth anyway). Thus,  $H_\infty$  is raised, i.e., higher inequality may help to overcome poverty traps in initially poor economies.<sup>10</sup>

<sup>10</sup> To complete the discussion of panel (b) of Fig. 1, note that if  $b_0^i \in (c_{\bar{a}}, c_{\underline{a}})$  for all  $i$ , the impact of a change in initial inequality on  $H_\infty$  is ambiguous.

## 6. Discussion in light of empirical evidence

Propositions 1 and 2 characterize the role of inequality for the process of economic development in terms of observable micro-relations, under the assumption that there are no constraints to borrow for educational purposes. In fact, empirical evidence for advanced countries suggests that credit constraints are not binding for the bulk of individuals (see e.g. Cameron and Taber, 2004, and the references therein). Thus, the analysis applies foremost to advanced economies. The purpose of this section is to explore whether results are consistent with empirical evidence.<sup>11</sup> For instance, Barro (2000) finds a positive relationship between inequality and growth for more advanced countries and a negative one for developing countries.

According to the preceding analysis, in the absence of credit constraints as assumed, the combination of non-diminishing returns to education and a MPS which is increasing in lifetime income, captured by  $b'' > 0$ , tends to induce a positive relationship between inequality and the stock of human capital. Indeed, empirical evidence strongly suggests that the MPS is increasing in lifetime income. For instance, in a widely-received paper, Dynan et al. (2004) show that this saving pattern is prevalent for the US economy, considering various measures of savings and different time periods. According to their findings, the MPS rises from 0.08–0.09 in the lowest quintile to around 0.18–0.23 in the fourth quintile, depending on the saving measure used. Moreover, evidence by Menchik and David (1983) suggests that the marginal propensity to bequeath is increasing in lifetime earnings, which is particularly relevant in the present context of intergenerational wealth transmission. Also, it is fair to conclude that the evidence from estimating returns to schooling does not lend much support to the hypothesis that these are diminishing (e.g. Card, 1999).

Regarding the third micro-relation which drives the relationship in the model – the shape of the function  $e(b)$  – the evidence is least conclusive, unfortunately. Although the evidence strongly suggests that educational investments are increasing in family wealth ( $e'(\cdot) > 0$ ), in line with part (iii) of Lemma 1, it is unclear whether the relationship is concave or convex. There exist some estimates of the effect of parental income (which according to the model are closely related to intergenerational transfers) on children's schooling, which allow for non-linearity. For instance, Becker and Tomes (1986) suggest that the marginal impact is diminishing, whereas, if anything, Behrman and Taubman (1990) find an increasing marginal impact. Besides these contradicting results, such estimates are plagued by econometric problems associated with heterogeneity in ability of agents (see e.g. Carneiro and Heckman, 2002). Further research seems necessary to settle this debate.

Nevertheless, one may conclude that the considered overlapping generations framework with uninsurable human capital risk and intergenerational transfers is consistent with a potentially positive relationship between inequality and economic performance in advanced countries. To understand why the relationship can be negative in developing economies, one may combine the proposed model with the standard credit market imperfections approach (see e.g. the seminal paper by Galor and Zeira, 1993).

<sup>11</sup> As the long run is less interesting from an empirical point of view and because stationary equilibria may fail to exist, we focus the discussion on Propositions 1 and 2.

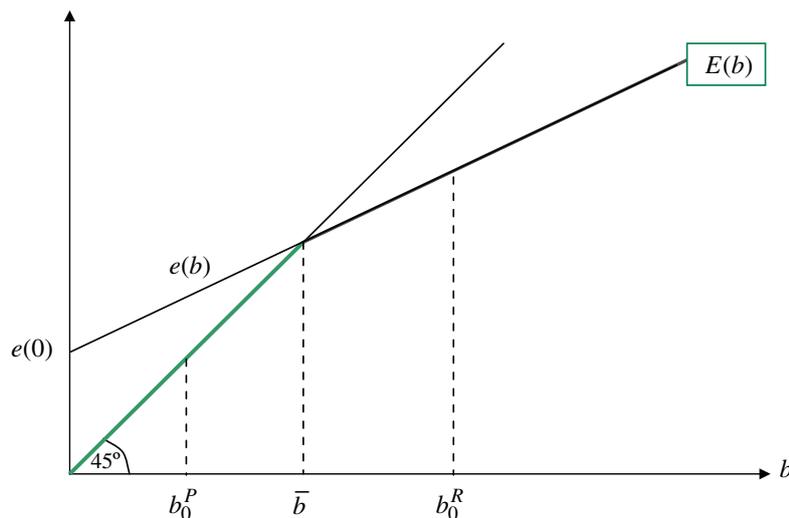


Fig. 3. Educational investment under missing credit markets.

To see how this could give further insights, suppose for simplicity that the credit market is missing completely.<sup>12</sup> Consequently, individuals cannot borrow to finance educational investment. Formally, this means that savings for old age must not be negative, i.e.,  $s_t^i = b_t^i - e_t^i \geq 0$  for all  $i$  and  $t$ . Thus, optimal investment of a young individual  $i$  in  $t$  is given by  $e_t^i = \min\{b_t^i, e(b_t^i)\} \equiv E(b_t^i)$ , where  $e(b)$  is again defined as interior solution to (7). For plausibility, we shall focus on the case where some of additional wealth is invested in the financial market, i.e.,  $e' < 1$ . In this case, missing credit markets matter only if  $e(0) > 0$  (otherwise,  $E(b) = e(b)$  and we are back to the basic model). If  $e' < 1$  and  $e(0) > 0$ , function  $E(b)$  is kinked, as shown in Fig. 3, and individuals with low wealth ( $b < \bar{b}$ ) are credit-constrained.

Now suppose that only the poor are initially credit constrained. Thus, in Fig. 3,  $b_0^P < \bar{b}$  and  $b_0^R \geq \bar{b}$ ; that is,  $E(b_0^P) = b_0^P$  and  $E(b_0^R) = e(b_0^R) \leq b_0^R$ . Now suppose inequality rises initially, such the poor increase and the rich decrease educational investment. Clearly, since  $e'(b) < 1$ , the reduction of the average investment of the poor in the initial period is higher than the increase of the rich, implying that aggregate educational investment decreases. Hence, in the short-run and possibly also in later periods (where effects also depend on intergenerational wealth transmission), higher inequality reduces the aggregate level of human capital, and therefore output,  $Y_t$ .

In sum, it is well possible that for the same education technology and the same micro-relations  $e(b)$  and  $b(I)$ , the relationship between initial inequality and economic development is positive in advanced economies, when credit constraints are negligible, and negative in developing economies, where at least the poor are severely credit-constrained. As a cautionary note, however, a rigorous empirical test of the proposed theory would require knowledge about at which level of development, all other things equal, borrowing constraints on educational investments become negligible.

<sup>12</sup> This is a standard assumption in the growth literature. For exceptions, see Galor and Zeira (1993) and Föllmi and Oechslin (2005).

## 7. Concluding remarks

The growth literature has recently emphasized the dominant role of human capital for the process of development (e.g. Glaeser et al., 2004). Human capital formation itself depends on individual education decisions, which are subject to and affected by considerable risk.

This paper has examined the implications of human capital risk for the role of inequality on the process of economic development fueled by private education investments. It has demonstrated that the initial distribution of family wealth may play an important role for development even when there are no credit constraints, due to missing insurance markets for human capital risk. The analysis suggests that – when the marginal propensity to save for intergenerational transfers is increasing in income and credit-constraints can be neglected, as supported by evidence for advanced countries – higher inequality tends to increase the aggregate human capital stock in the process of development. However, for developing economies where borrowing constraints are an important obstacle for human capital investment of poorer individuals, higher inequality tends to impede the macroeconomic performance. Overall, and consistent with recent evidence, this suggests that higher initial inequality slows down growth in earlier stages of development but has positive growth effects in mature stages of development.

It is important to note that even under a positive relationship between inequality and macroeconomic performance, the proposed theory does not suggest a rationale for egalitarian policies. This is because in a small open economy there is no “trickle-down” from growth to wages. Hence, an equity-growth trade-off may arise. To examine the role of redistributive taxation for growth by taking into account interactions between intergenerational wealth transmission and educational investments under human capital risk would thus be an interesting topic for future research. Another interesting question is how the impact of a reduction in human capital risk (possibly via public policy) on the aggregate human capital stock depends on the degree of inequality in an economy.

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## Appendix

**Proof of Lemma 1.** We start by proving part (i). Note that, according to (5),  $b_{t+1}^i$  is implicitly given by the first-order condition

$$\Omega(b_{t+1}^i, I_{t+1}^i) \equiv -u'(I_{t+1}^i - b_{t+1}^i) + v'(b_{t+1}^i) \leq 0, \quad (\text{A.1})$$

which holds with equality if  $b_{t+1}^i > 0$ . According to (A.1) and Assumption A2,  $\Omega_b = u'' + v'' < 0$  and  $\Omega_I = -u'' > 0$ . Hence, for  $b_{t+1}^i > 0$ ,  $b'(I) = -\Omega_I/\Omega_b = u''/(u'' + v'') > 0$ . Next, suppose  $b_{t+1}^i = 0$ . Since  $\Omega_I > 0$ , the left-hand side of inequality (A.1) is strictly increasing in  $I_{t+1}^i$ . Hence, using  $\lim_{I \rightarrow \infty} u'(I) < v'(0)$  from A2, eventually,  $b_{t+1}^i > 0$  if income  $I_{t+1}^i$  exceeds some level  $\underline{I} \geq 0$ . This confirms part (i).

We now turn to part (ii). First, if  $b_{t+1}^i = b(I_{t+1}^i) = 0$ , we have  $V(I_{t+1}^i) = u(I_{t+1}^i) + v(0)$ ; thus,  $V'(I) = u'(I) > 0$  and  $V''(I) = u''(I) < 0$ . Hence,  $A(I) = -V''(I)/V'(I) = -u''(I)/u'(I)$  is decreasing in  $I$  if  $u'''u' > (u'')^2$ , which holds by Assumption A2. If  $b(I) > 0$ , then  $V'(I) = u'(I - b(I)) > 0$ , according to (6) and  $u' = v'$  from (A.1), applying the envelope theorem. Thus,  $V'' = (1 - b')u'' = u''v''/(u'' + v'') < 0$  and

$$A(I) = -\frac{u''(I - b(I))v''(b(I))}{u'(I - b(I))[u''(I - b(I)) + v''(b(I))]} \tag{A.2}$$

Straightforward algebra reveals that  $A' < 0$  if  $(v'')^3[u'u''' - (u'')^2] + (u'')^3[v'v''' - (v'')^2] > 0$ , as assumed in A2. This confirms part (ii).

To prove the final part (iii) of Lemma 1, first recall that  $e_t^i = e(b_t^i)$  is given by the first-order condition

$$\Xi(b_t^i, e_t^i) \equiv E[V'(wh(e_t^i, \tilde{a}) + R(b_t^i - e_t^i))(wh_e(e_t^i, \tilde{a}) - R)] = 0, \tag{A.3}$$

according to (8).  $V'' < 0$  and  $h_{ee} \leq 0$  imply  $\Xi_e < 0$ . Thus, according to the implicit function theorem,  $e'(b_t^i) > 0$  if and only if  $\Xi_b(b_t^i, e_t^i)|_{e_t^i=e(b_t^i)} > 0$ . For notational simplicity, indices  $t$  and  $i$  are suppressed in the remainder of this proof. Then,

$$\begin{aligned} \Xi_b(b, e)|_{e=e(b)} &= E[V''(\widehat{I}(b, \tilde{a}))(wh_e(e(b), \tilde{a}) - R)]R, \\ &= E[A(\widehat{I}(b, \tilde{a}))V'(\widehat{I}(b, \tilde{a}))(R - wh_e(e(b), \tilde{a}))]R, \end{aligned} \tag{A.4}$$

according to (A.3), where  $A(I) = -V''(I)/V'(I)$  has been used for the latter equation. The result is proven for an infinite set  $\mathcal{A}$ . (The proof for a finite  $\mathcal{A}$  is then straightforward.) Define sets  $\mathcal{A}_1$  and  $\mathcal{A}_2$  such that  $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$ ,  $wh_e(e, a) < R$  for all  $a \in \mathcal{A}_1$  and  $wh_e(e, a) \geq R$  for all  $a \in \mathcal{A}_2$ . We can write

$$\begin{aligned} E[A(\widehat{I}(b, \tilde{a}))V'(\widehat{I}(b, \tilde{a}))(R - wh_e(\cdot, \tilde{a}))] &= \int_{\mathcal{A}_1} A(\widehat{I}(b, \tilde{a}))V'(\widehat{I}(b, \tilde{a}))(R - wh_e(\cdot, \tilde{a}))d\Phi(\tilde{a}) \\ &\quad + \int_{\mathcal{A}_2} A(\widehat{I}(b, \tilde{a}))V'(\widehat{I}(b, \tilde{a}))(R - wh_e(\cdot, \tilde{a}))d\Phi(\tilde{a}). \end{aligned} \tag{A.5}$$

According to the definitions of  $\mathcal{A}_1$  and  $\mathcal{A}_2$ , the first integral in (A.5) is positive, whereas the second one is negative. Moreover, since  $h_{\tilde{a}} > 0$ ,  $\widehat{I}(b, \tilde{a})$  is increasing in  $\tilde{a}$ , according to (4); thus, using  $A' < 0$  from part (ii) of Lemma 1,  $A(\widehat{I}(b, \tilde{a}))$  is strictly decreasing in  $\tilde{a}$ . Also note that  $h_{e\tilde{a}} > 0$  (Assumption A1) implies that there exists  $\check{a} \in \mathcal{A}$  such that  $\mathcal{A}_1 = \{a \in \mathcal{A} | a < \check{a}\}$  and  $\mathcal{A}_2 = \{a \in \mathcal{A} | a \geq \check{a}\}$ . Hence,

$$A(\widehat{I}(b, \tilde{a})) \int_{\mathcal{A}_1} V'(\widehat{I}(b, \tilde{a}))(R - wh_e(\cdot, \tilde{a}))d\Phi(\tilde{a}) < \int_{\mathcal{A}_1} A(\widehat{I}(b, \tilde{a}))V'(\widehat{I}(b, \tilde{a}))(R - wh_e(\cdot, \tilde{a}))d\Phi(\tilde{a}), \quad (\text{A.6})$$

$$A(\widehat{I}(b, \tilde{a})) \int_{\mathcal{A}_2} V'(\widehat{I}(b, \tilde{a}))(R - wh_e(\cdot, \tilde{a}))d\Phi(\tilde{a}) < \int_{\mathcal{A}_2} A(\widehat{I}(b, \tilde{a}))V'(\widehat{I}(b, \tilde{a}))(R - wh_e(\cdot, \tilde{a}))d\Phi(\tilde{a}). \quad (\text{A.7})$$

Adding up (A.6) and (A.7) and using  $A(I) = -V''(I)/V'(I)$  yields

$$A(\widehat{I}(b, a'))E[V'(\widehat{I}(b, \tilde{a}))(R - wh_e(\cdot, \tilde{a}))] < E[V''(\widehat{I}(b, \tilde{a}))(wh_e(\cdot, \tilde{a}) - R)]. \quad (\text{A.8})$$

Given optimal human capital investment,  $e(b)$ , the left-hand side of (A.8) is zero, according to (A.3). Thus,  $E[V''(\widehat{I}(b, \tilde{a}))(wh_e(e(b), \tilde{a}) - R)] > 0$ , implying  $\Xi_b(b, e)|_{e=e(b)} > 0$ , according to (A.4). Hence,  $e'(b) > 0$ . This concludes the proof.  $\square$

**Proof of Proposition 1.** According to (18), differentiating  $H_1$  with respect to  $\varepsilon$  yields

$$\frac{\partial H_1}{\partial \varepsilon} = \lambda E[\hat{h}_b(\check{b}_0^R, \tilde{a}) - \hat{h}_b(\check{b}_0^P, \tilde{a})], \text{ where} \quad (\text{A.9})$$

$$\hat{h}_b(b, a) = h_e(e(b), a)e'(b) \quad (\text{A.10})$$

according to (9). Since  $\check{b}_0^R > \check{b}_0^P$ , we have  $\partial H_1/\partial \varepsilon > (=, <)0$  if, for all  $a \in \mathcal{A}$  and for all  $b \in \mathbb{R}_{++}$ ,

$$\hat{h}_{bb}(b, a) = h_{ee}(e(b), a)e'(b)^2 + h_e(e(b), a)e''(b) > (=, <)0. \quad (\text{A.11})$$

This confirms the result.  $\square$

**Proof of Proposition 2.** First, note that (20) implies

$$\frac{\partial H_2}{\partial \varepsilon} = \lambda E[E[\hat{h}_b(\check{b}_0^R, \tilde{a}, \tilde{a}') - \hat{h}_b(\check{b}_0^P, \tilde{a}, \tilde{a}')]]. \quad (\text{A.12})$$

Since  $\check{b}_0^R > \check{b}_0^P$ , we have  $\partial H_2/\partial \varepsilon > (=)0$  if for all  $b \in \mathbb{R}_{++}$ ,  $E[E[\hat{h}_{bb}(b, \tilde{a}, \tilde{a}')] > (=)0$ . According to (9) and (19), we have  $\hat{h}(b, a, a') = h(e(\hat{b}(b, a)), a')$ . Thus,

$$\hat{h}_b(b, a, a') = h_e(e(\hat{b}(b, a)), a')e'(\hat{b}(b, a))\hat{b}_b(b, a), \quad (\text{A.13})$$

$$\begin{aligned} \hat{h}_{bb}(b, \tilde{a}, \tilde{a}') &= h_{ee}(e(\hat{b}(b, a)), a')e'(\hat{b}(b, a))^2\hat{b}_b(b, a)^2 \\ &\quad + h_e(e(\hat{b}(b, a)), a')[e''(\hat{b}(b, a))\hat{b}_b(b, a)^2 + e'(\hat{b}(b, a))\hat{b}_{bb}(b, a)]. \end{aligned} \quad (\text{A.14})$$

Using (14) and (16), we can rewrite (A.14) to

$$\begin{aligned} \hat{h}_{bb}(b, \tilde{a}, \tilde{a}') &= h_{ee}(e(\hat{b}(b, a)), a')e'(\hat{b}(b, a))^2\hat{b}_b(b, a)^2 + h_e(e(\hat{b}(b, a)), a') \\ &\quad \times \{e''(\hat{b}(b, a))\hat{b}_b(b, a)^2 + e'(\hat{b}(b, a))[b''(\widehat{I}(b, a))\widehat{I}_b(b, a)^2 \\ &\quad + b'(\widehat{I}(b, a))(wh_{ee}(e(b), a)e'(b)^2 + (wh_e(e(b), a) - R)e''(b))]\}. \end{aligned} \quad (\text{A.15})$$

Now define

$$G(b, a, a') \equiv h_e(e(\hat{b}(b, a)), a')e'(\hat{b}(b, a))b'(\widehat{I}(b, a)). \quad (\text{A.16})$$

To confirm part (i), suppose  $h_{ee} = 0$  first. Then we have  $\partial G(b, a, a') / \partial a > 0$  if  $b''(I) \geq 0$  and  $e''(b) \geq 0$ , with at least one strict inequality and  $\partial G(b, a, a') / \partial a = 0$  if  $b''(I) = e''(b) = 0$  (use the facts  $\hat{I}_a(b, a) > 0$  and  $\hat{b}_a(b, a) > 0$ ). Moreover, recall that  $e(b) > 0$  requires  $E[wh_e(e(b), \tilde{a}) - R] > 0$ . Thus, for all  $a' \in \mathcal{A}$ , we have  $E[G(b, \tilde{a}, a')(wh_e(e(b), \tilde{a}) - R)] > 0$  if  $b''(I) \geq 0$  and  $e''(b) \geq 0$ . Inspection of (A.15) then reveals that  $E[E[\hat{h}_{bb}(b, \tilde{a}, \tilde{a}')] > 0$  if  $b''(I) \geq 0$  and  $e''(b) \geq 0$ , with at least one strict inequality, and  $E[E[\hat{h}_{bb}(b, \tilde{a}, \tilde{a}')] = 0$  if  $b''(I) = e''(b) = 0$ . This proves the result for  $t = 2$ . For  $t = 3$ , note that (22) implies

$$\frac{\partial H_3}{\partial \varepsilon} = \lambda E \left[ E \left[ E \left[ \hat{h}_b(\check{b}_0^R, \tilde{a}, \tilde{a}', \tilde{a}'') - \hat{h}_b(\check{b}_0^P, \tilde{a}, \tilde{a}', \tilde{a}'') \right] \right] \right], \text{ where} \quad (\text{A.17})$$

$$\hat{h}_b(b, a, a', a'') = \hat{h}_b(\hat{b}(b, a), a', a'') \hat{b}_b(b, a), \quad (\text{A.18})$$

according to (21). Hence,  $\partial H_3 / \partial \varepsilon > (=) 0$  if for all  $b \in \mathbb{R}_{++}$ ,  $E \left[ E \left[ E \left[ \hat{h}_{bb}(b, \tilde{a}, \tilde{a}') \right] \right] \right] > (=) 0$ , where

$$\hat{h}_{bb}(b, a, a', a'') = \hat{h}_{bb}(\hat{b}(b, a), a', a'') \hat{b}_b(b, a) + \hat{h}_b(\hat{b}(b, a), a', a'') \hat{b}_{bb}(b, a). \quad (\text{A.19})$$

Using (A.13) for  $\hat{h}_b$ , (13) for  $\hat{b}_b$ , (14) for  $\hat{b}_{bb}$  (after substituting (16)), and (A.15) for  $\hat{h}_{bb}$ , we can proceed in an analogous way as in the case  $t = 2$ . Applying the same reasoning to  $t \geq 4$  shows that the result holds for all finite  $t$ . This confirms part (i).

For part (ii), note that if  $h_{ee} < 0$  or  $b'' < 0$  it is possible that  $\partial G(b, a, a') / \partial a < 0$  and therefore we may have any sign of  $E[G(b, \tilde{a}, a')(wh_e(e(b), \tilde{a}) - R)]$ . According to (A.15), if  $e'' \neq 0$ , the sign of  $E[E[\hat{h}_{bb}(b, \tilde{a}, \tilde{a}')] > (=) 0$  therefore cannot be determined. This concludes the proof.  $\square$

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