

# Quantifying Optimal Growth Policy\*

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## Abstract

We determine the optimal growth policy within a comprehensive endogenous growth model. The model accounts for important elements of the tax-transfer system and for transitional dynamics. It captures the three main growth engines based on standard ingredients in order to understand the quantitative policy and welfare implications of the existing theory. Our calibrated model indicates that the current policy leads to severe underinvestment in both R&D and physical capital, implying that both R&D and capital investment subsidies should be increased substantially. We argue that previous research has overlooked a strong evidence for the welfare significance of the quest for the optimal growth policy by abstaining to calibrate the distortionary tax system.

**Key words:** Economic growth; Endogenous technical change; Optimal growth policy; Tax-transfer system; Transitional dynamics.

**JEL classification:** H20, O30, O40.

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# 1 Introduction

How does the optimal growth policy in advanced economies look like and what are the quantitative effects of implementing it? One must acknowledge that, after three decades of Endogenous Growth Theory, we still do not have a sound answer to this exceptionally important research question. In this sense, the topic on optimal growth policy in advanced economies appears heavily underresearched.

This paper employs a comprehensive endogenous growth model to quantitatively derive the optimal policy mix and to determine the potential welfare gain from an optimal policy reform. Our model accounts for important elements of the tax-transfer system and for transitional dynamics. It captures the three main growth engines, allowing for investment in physical capital, human capital, and R&D. The modelling of the growth engines is consciously based on well-understood and widely-used ingredients in order to understand the quantitative policy and welfare implications of the existing theory.

Our results strongly suggest that the current policy mix is suboptimal and there is potential to realize substantial welfare gains. As will become apparent below, the quantitative implications are large and hence the results seem provocative. We conclude that there is strong indication for the welfare significance of the quest for the optimal growth policy.

Endogenous growth theory provides a natural analytical framework for studies that aim at advising policy makers about the design of welfare-maximizing growth policy, by taking into account the general equilibrium dimension and the intertemporal dimension associated with R&D. However, any such analysis faces the problem to meet a balance between maintaining analytical tractability and avoiding that the model is too stylized to base policy recommendations upon it. It is true that any specific policy advise (like the calculation of the optimal R&D subsidy rate) requires numerical evaluations at some stage of the analysis. Nevertheless, we want to limit ourselves to models where the steady state can be derived analytically for at least two reasons. First, analytical solutions are generally useful in understanding the mechanics of a model such that numerical results can then be used mainly for quantification purposes. Second, analytical steady state results are salient to match endogenous variables to observables when calibrating the model. Using the steady state as an anchor for calibration appears as a reasonable strategy in the case of the US economy. This allows us to limit the degree of freedom in the numerical analysis substantially.

We think that any serious and careful study on optimal growth policy in advanced economies should at least meet the following two requirements. First, it should capture important elements of the income tax system. Taxes on labor income, bond yields, capital gains and corporate income may be levied for other (e.g. redistributive) purposes than stimulating economic growth. However, like externalities and market power, they may directly distort investment decisions. Failing to account for income taxation thus potentially gives rise to misleading growth policy recommendations. Another reason to take the public finance side seriously when calculating quantitative policy recommendations results from the requirement of a careful calibration strategy. Setting model

parameters such that endogenous variables match observables requires to take public policy into account since endogenous variables may depend on public policy. Accounting for this fact turns out to have important consequences for our results compared to the existing literature.

The second requirement to study our research question is to take transitional dynamics into account in the numerical evaluation of growth policy reforms. This requires to calculate the entire transition path in response to policy shocks. It is well-known that, in growth models with decreasing marginal productivity of capital, it may take a long time after some shock until per capita income adjusts to anywhere near the new steady state. It is thus salient to compute the policy mix which maximizes the intertemporal welfare gain from a policy reform and not just focus on maximization of steady state welfare. Moreover, the underlying R&D-based growth model represents a non-linear, highly dimensional, saddle-point stable, differential-algebraic system. For plausible calibrations, the stable eigenvalues differ substantially in magnitude; hence, the dynamic system belongs to the class of stiff differential equations. Simulating such a dynamic model is all but trivial.<sup>1</sup> We employ a recent procedure, called relaxation algorithm (Trimborn, Koch and Steger, 2008), which can deal with these conceptual difficulties.

Our analysis suggests that the current R&D subsidization in the US leads to more dramatic underinvestment in R&D than has been previously found in the literature. The main reason is that by not accounting for capital income taxation, which distorts incentives to invest in physical capital, households have to be calibrated to be less patient to match observable income growth than they may actually be. Thus, socially optimal investment levels are found to be closer to the market equilibrium than it may be the case. According to our preferred calibration, innovating firms should be allowed to deduct more than twice their R&D costs from sales revenue for calculating taxable corporate income, rather than just 1.1 times their R&D costs under the current policy. The US stimulus for investment in physical capital is also suboptimally low. In addition to capital income taxation, the investment rate is biased downwards also because of price setting power of firms. Our calibrated model implies that firms should be allowed to deduct about 1.5 times their capital costs from sales revenue, rather than full deduction of their capital costs under the current policy. Investment in human capital should also be subsidized, roughly to the extent labor income is taxed. A policy reform targeted simultaneously to all three growth engines may entail huge welfare gains. An appropriate policy reform could achieve an intertemporal welfare gain which is equivalent to a permanent doubling of per capita consumption. The welfare gain in response to the implementation of the optimal growth policy program is only slightly smaller if the government adjusts a distortionary tax instead of adjusting a lump-sum tax to achieve a balanced budget. Although the potential welfare gain varies with the calibration, the optimal policy mix is remarkably robust to parameter changes.

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<sup>1</sup>The growth literature has used the techniques of linearization, time elimination, or backward integration. Linearization delivers bad approximations if the deviation from the steady state is large, time elimination does not work if there are non-monotonic adjustments, and backward integration fails in case of stiff differential equations.

There is common sense among economists that private firms in advanced economies conduct too little R&D. This conviction can be substantiated by noting that the social rate of return to business enterprise R&D is far above the private rate of return. The empirical literature has identified social rates of return to R&D between 70 percent and more than 100 percent (e.g. Scherer, 1982; Griliches and Lichtenberg, 1984). Jones and Williams (1998) argue that, due to methodical shortcomings, these estimates should indeed be viewed as lower bounds. Hall (1996) reports that estimates of the private rate of return to R&D cluster around 10 percent to 15 percent. It is also widely believed that this R&D underinvestment bias is likely to cause a substantial welfare loss. Moreover, there is strong evidence showing that fiscal incentives are effective in increasing the economy-wide R&D intensity (e.g. Bloom, Griffith and van Reenen, 2002). This raises the important question addressed in our paper about the level of fiscal intervention which is required to remove the R&D underinvestment gap.

Our paper is closely related to the theoretical literature on underinvestment gaps which, however, has focussed on a steady state analysis. Our main point of reference is the innovative study by Jones and Williams (2000). Like we do, they employ a horizontal innovation model without strong scale effects à la Jones (1995). However, they neither consider transitional dynamics nor do they calculate the optimal policy mix. We identify much larger underinvestment gaps and show that the difference to their results can be attributed to their neglect of calibrating the distortionary tax system. Other contributions in this direction are Steger (2005) and Strulik (2007) who find an even smaller degree of R&D underinvestment than Jones and Williams (2000). Similar to our steady state analysis, however, Steger (2005) finds that the market economy quite heavily underinvests in physical capital accumulation.<sup>2</sup> A limited number of studies explicitly derive the optimal R&D subsidy. In an important contribution, Sener (2008) numerically determines the optimal R&D subsidy in an endogenous growth model without scale effects but policy-dependent long run growth. He focusses on a steady state and abstracts from tax distortions. By contrast, we compute the welfare-maximizing policy reform by taking into account the entire transition path and the public finance side. Finally, Grossmann, Steger and Trimborn (2013a) employ a stylized R&D-based growth model to investigate the welfare implications of time-invariant R&D subsidies as compared to time-varying R&D subsidies. Their stylized model is less informative, however, on optimal levels of policy instruments.<sup>3</sup>

The plan of the paper is as follows. Section 2 describes the underlying model. Section 3 analyzes the decentralized market equilibrium and analytically derives the balanced growth equilibrium. Section 4 derives the social planning optimum and the

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<sup>2</sup>Chu (2009) also finds underinvestment in R&D and physical capital. He proposes to increase patent breadth and optimize the profit sharing rule between current and former inventors to increase welfare. In his framework, these policy measures do not lead to the first best allocation.

<sup>3</sup>Our paper is more loosely related to the literature on optimal capital income taxation in the tradition of Chamley (1986) and Judd (1985); for recent developments, see e.g. Krueger, Conesa and Kitao (2008) and Krueger and Ludwig (2013), and the references therein. Whereas our paper focusses on underinvestment and (dynamically) optimal subsidies by taking as given the tax rates on income and capital gains, the standard optimal taxation (“Ramsey”) problem is to find the optimal tax schedule under the constraint that an exogenous government tax revenue is raised.

optimal long run policy mix. The calibration strategy is outlined in Section 5. Section 6 numerically examines the optimal long run policy mix, whereas Section 7 derives the dynamically optimal policy program. Section 8 summarizes the main conclusions and discusses the route for further research. Technical details have been relegated to an appendix.

## 2 The Model

Consider the following continuous-time model with three engines of economic growth: horizontal innovations performed by a competitive R&D sector, accumulation of physical capital which is provided by a monopolistically competitive producer goods for the production of a homogenous consumption good and human capital formation by households.<sup>4</sup> A representative household owns the intermediate good firms by holding equity and lend to the intermediate good firms in the form of bonds.

### 2.1 Government

The government possesses a variety of policy instruments which potentially affect the three engines of growth. At the household level it may subsidize education at rate  $s_H$  per unit of educational input. At the firm level, we assume that there is corporate income taxation. The corporate tax rate is identical across sectors and denoted by  $\tau_c$ . Intermediate good firms may deduct a fraction  $s_d$  of their capital costs (for instance, via depreciation allowances or an investment tax credit) from pre-tax profits to obtain the corporate tax base. If  $s_d = 0$ , capital costs are fully deductible from sales revenue; if  $s_d < (>)0$ , they are less than (more than) fully deductible. Similarly, the R&D sector may deduct a fraction  $s_R$  of their R&D spending from pre-tax profits to obtain the corporate tax base. Households are taxed in various ways. There is a tax on wage income at rate  $\tau_w$ , a tax on income from bond holdings at rate  $\tau_r$ , and a capital gains tax paid on increases in share prices. To be able to calibrate all the tax instruments at observed levels, we also allow for redistribution via a lump-sum transfer to households.<sup>5</sup> The government balances the budget in each point of time. The policy instruments are central to our analysis; its notation is restated for convenience in Table 1.

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<sup>4</sup>We abstract from infrastructure provided by the government. For a recent study investigating growth and welfare maximizing policies which focuses on public infrastructure, see Mischi, Gemmill and Kneller (2013).

<sup>5</sup>We assume homothetic preferences (see below), such that there exists a representative consumer in the economy; see e.g. Mas-Colell, Whinston and Green (1995). Thus, households could well be heterogeneous in individual endowments. In that case, the considered tax scheme is redistributive, if the lump-sum transfer is positive.

Notation	Policy instrument
$\tau_r$	Tax rate on bond holdings (physical capital)
$\tau_c$	Corporate income tax rate
$\tau_w$	Labor income tax rate
$\tau_g$	Tax rate on capital gains (equity holdings)
$s_d$	Deduction rate of capital costs from pre-tax profits
$s_R$	Deduction rate of R&D costs from pre-tax profits
$s_H$	Subsidy rate on education costs

**Table 1:** Policy instruments

## 2.2 Firms

There is a homogenous final output good with price normalized to unity. Final output is produced under perfect competition according to

$$Y = \left( \int_0^A (x_i)^{\frac{\beta-1}{\beta}} di \right)^{\frac{\alpha\beta}{\beta-1}} (H^Y)^{1-\alpha}, \quad (1)$$

$0 < \alpha < 1$ ,  $\beta > 1$ , where  $H^Y$  is human capital (efficiency units of labor) in the manufacturing sector,  $A$  is the mass (“number”) of intermediate goods and  $x_i$  denotes the quantity of intermediate good  $i$ . (Time index  $t$  is omitted whenever this does not lead to confusion.) The number of varieties,  $A$ , expands through horizontal innovations, protected with (potentially) infinite patent length. As usual,  $A$  is interpreted as the economy’s stock of knowledge.  $A_0 > 0$  is given. The labor market is perfect.

In each sector  $i$  there is one firm – the innovator or the buyer of a blueprint for an intermediate good – which has access to a one-to-one technology: one unit of foregone consumption (capital) can be transformed into one unit of output. Capital depreciates at rate  $\delta_K \geq 0$ . Capital supply in the initial period,  $K_0 > 0$ , is given. The capital market is perfect.

Moreover, in each sector  $i$  there is a competitive fringe which can produce a perfect substitute for good  $i$  (without violating patent rights) but is less productive in manufacturing the good (see, e.g., Aghion and Howitt, 2005): one unit of output requires  $\kappa$  units of capital;  $1 < \kappa \leq \frac{\beta}{\beta-1}$ .

There is free entry into the R&D sector. Suppose that in each point of time,  $(1 + \psi)\dot{A}$  patents are generated. As in Jones and Williams (2000),  $\psi\dot{A}$  of these patents replace existing patents, such that there will be “business stealing”. Thus, in each point of time, the probability of an existing innovator to be replaced is equal to the fraction of firms driven out of business,  $\psi\dot{A}/A$ ; the expected effective patent life is therefore limited to the inverse of this probability. Ideas for new intermediate goods are generated according to

$$(1 + \psi)\dot{A} = \tilde{\nu} A^\phi H^A, \quad \text{with } \tilde{\nu} \equiv \nu (H^A)^{-\theta}, \quad (2)$$

where  $H^A$  is the human capital level in the R&D sector,  $\nu > 0$ ,  $\phi < 1$ ,  $0 \leq \theta < 1$ ,  $\psi \geq 0$ .  $\tilde{\nu}$  is taken as given in the decision of R&D firms; that is, R&D firms perceive a constant returns to scale R&D technology, although the social return to higher R&D input is decreasing whenever  $\theta > 0$ . The wedge between private and social return may arise because firms do not take into account that rivals may work on the same idea such that, from a social point of view, some of the R&D input is duplicated (“duplication externality”). Parameter  $\theta$  captures the extent of this externality. For  $\theta \rightarrow 1$ , social returns to R&D investment approach zero.  $\phi > 0$  gives the strength of the standard intertemporal knowledge spill-over (or standing on shoulders effect).

### 2.3 Households

There is an infinitely-living, representative dynasty. Household size,  $N$ , grows with constant exponential rate,  $n \geq 0$ .  $N_0$  is given and normalized to unity. Preferences are represented by the standard utility function

$$U = \int_0^{\infty} \frac{c^{1-\sigma} - 1}{1-\sigma} e^{-(\rho-n)t} dt, \quad (3)$$

$\sigma > 0$ , where  $c$  is consumption per capita. Households take factor prices as given.

The process of skill accumulation depends on the amount of human capital an individual invests (e.g., paying teachers) in education,  $h^H$ . Moreover, it is characterized by human capital transmission within the representative dynasty.<sup>6</sup> We also assume that human capital depreciates over time at rate  $\delta_H > 0$ . Formally, suppose that the human capital level per capita,  $h$ , evolves according to<sup>7</sup>

$$\dot{h} = \xi (h^H)^\gamma h^\eta - \delta_H h, \quad (4)$$

$\gamma, \eta, \xi > 0$ ,  $\gamma + \eta < 1$ ;  $h_0 > 0$ .  $\gamma < 1$  captures decreasing returns to teaching input. Parameter  $\eta$  is associated with human capital transmission within the dynasty over time.  $\gamma + \eta < 1$  (thus,  $\eta < 1$ ) implies that, on a balanced growth path,  $h$  assumes a stationary long-run value.

Let  $w$  and  $r$  denote the wage rate per unit of human capital and the interest rate, respectively. Moreover, denote by  $T$  the transfer per capita, which equals the sum of tax revenue minus subsidies, both divided by  $N$ . Financial wealth per individual,  $a$ , accumulates according to

$$\dot{a} = [(1 - \tau_r)r - n]a + (1 - \tau_w)wh - (1 - s_H)wh^H - c + T. \quad (5)$$

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<sup>6</sup>There is overwhelming evidence for the hypothesis that the education of parents affects the human capital level of children, even when controlling for family income. For recent studies, also providing an overview of the previous literature, see Plug and Vijverberg (2003) as well as Black, Devereux and Salvanes (2005).

<sup>7</sup>Grossmann and Steger (2013) introduce heterogeneity of R&D skills in the standard framework by Jones (1995). They show that the analytical solution for the optimal long run subsidy on R&D and capital costs does not depend on the distribution of R&D skills.

It turns out that, for the transversality conditions of both the household optimization problem and the social planner problem to hold and the value of the utility stream,  $U$ , to be finite, we have to restrict the parameter space such that

$$\rho - n + (\sigma - 1)g > 0 \text{ with } g \equiv \frac{\alpha(1 - \theta)n}{(1 - \alpha)(\beta - 1)(1 - \phi)}. \quad (\text{A1})$$

As will become apparent,  $g$  is the economy's long run growth rate both in decentralized equilibrium and in social planning optimum. We maintain assumption A1 throughout.

In most of our numerical analysis, we focus on the case where  $\tau_w = s_H$ , i.e. the costs of education can be fully deducted from the labor income tax base. Notably, when  $\tau_w = s_H$ , the human capital accumulation process (4) is similar to that in the seminal work of Lucas (1988).<sup>8</sup> In Lucas (1988), individuals devote a fraction of their time to acquire education rather than purchasing teaching input. Let us define the fraction of human capital devoted to education as  $\mathfrak{h}^H \equiv h^H/h$ , i.e.  $\dot{h} = \xi (\mathfrak{h}^H)^\gamma h^{\gamma+\eta} - \delta_H h$  and  $h - h^H = (1 - \mathfrak{h}^H)h$ . Thus, using  $\tau_w = s_H$  in (5), education involves costs in terms of foregone labor income and  $\mathfrak{h}^H$  could be interpreted as the fraction of time devoted to education. In this sense, the human capital accumulation process in Lucas (1988) is a special case of our formulation. However, in contrast to Lucas (1988), who assumes  $\gamma = 1$  and  $\eta = 0$ , we do not allow the stock of human capital per capita to grow infinitely. Moreover, allowing for  $\tau_w \neq s_H$  and  $\tau_w \neq 0$ , therefore viewing education costs as real expenses, is important in our model for the labor income tax to be potentially distortionary. In Section 7.2, where we study the dynamically optimal policy reform and the associated welfare gain, we contrast the case where the government achieves a balanced budget by adjusting labor income tax rate instead of adjusting the non-distortionary lump-sum tax.

### 3 Equilibrium Analysis

We first analyze the decentralized equilibrium, focussing on a comparative-static analysis of the impact of changes of tax parameters on the long run equilibrium allocation of human capital.

We start with intermediate goods producers. Denote by  $R \equiv r + \delta_K$  the user cost of capital for an intermediate good firm (before taxation). It turns out that

$$s_K \equiv \frac{\tau_c s_d}{1 - \tau_c} \quad (6)$$

is the behaviorally relevant subsidy rate on capital costs. To see this, use the definition of policy instruments in Table 1 to find that producer  $i$  has profits

$$\pi_i = p_i x_i - R x_i - \tau_c [p_i x_i - (1 + s_d) R x_i] \quad (7)$$

$$\equiv (1 - \tau_c) [p_i - (1 - s_K) R] x_i, \quad (8)$$

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<sup>8</sup>Lucas (1988) implicitly assumes  $\tau_w = s_H = 0$ .

where  $p_i$  is the price of good  $i$ . According to (1), the demand function for intermediate good  $i$  reads as

$$x_i = \frac{\alpha Y (p_i)^{-\beta}}{P^{1-\beta}}, \quad (9)$$

where

$$P \equiv \left( \int_0^A (p_i)^{1-\beta} di \right)^{\frac{1}{1-\beta}} \quad (10)$$

is a price index. Profit maximization implies that the optimal price of each firm  $i$  is given by

$$p_i = p = \kappa(1 - s_K)R. \quad (11)$$

To see this, note that a firm which owns a blueprint perceives the price elasticity of demand as being  $-\beta$  (taking aggregates  $Y$  and  $P$  as given). Thus, it would choose a mark-up factor which is equal to  $\frac{\beta}{\beta-1}$  if it were not facing a competitive fringe. Moreover, the competitive fringe would make losses at a price lower than  $\kappa(1 - s_K)R$ . Thus, as  $\kappa \leq \frac{\beta}{\beta-1}$ , each firm  $i$  sets the maximal price allowing it to remain monopolist. According to (9) - (11), resulting output is given by

$$x_i = x = \frac{\alpha Y}{A\kappa(1 - s_K)R}. \quad (12)$$

Substituting (12) into (1) and solving for  $Y$  implies

$$y \equiv \frac{Y}{N} = A^{\frac{\alpha}{(1-\alpha)(\beta-1)}} \left( \frac{\alpha}{\kappa(1 - s_K)R} \right)^{\frac{\alpha}{1-\alpha}} h^Y \quad (13)$$

for per capita income, where  $h^Y \equiv H^Y/N$ . Thus, the total amount of physical capital,  $K = \int_0^A x_i di = Ax$ , divided by population size, is given by

$$k \equiv \frac{K}{N} = A^{\frac{\alpha}{(1-\alpha)(\beta-1)}} \left( \frac{\alpha}{\kappa(1 - s_K)R} \right)^{\frac{1}{1-\alpha}} h^Y. \quad (14)$$

Expressions (13) and (14) suggest that, if the interest rate  $r$  is stationary in the long run, the capital stock per capita and per capita income grow at the same rate along a balanced growth path.

Let  $P^A$  denote the present discounted value of the (after-tax) profit stream generated by an innovation. Thus,  $P^A$  is the price an intermediate good producer pays to the R&D sector for a new blueprint as well as the stock market evaluation of an intermediate good firm. In equilibrium, arbitrage possibilities in the capital market are absent. The dividends paid out by an intermediate good firm (being identical for all  $i$  due to symmetry, i.e.,  $\pi_i = \pi$ ),  $\pi/P^A$ , plus the growth rate of  $P^A$  after capital gains are taxed,  $(1 - \tau_g)\dot{P}^A/P^A$ , must be equal to the sum of the after-tax interest

rate,  $(1 - \tau_r)r$ , and the probability that an existing innovator is driven out of business,  $\psi\dot{A}/A$ .<sup>9</sup> The no arbitrage condition for the capital market therefore reads as

$$(1 - \tau_g)\frac{\dot{P}^A}{P^A} + \frac{\pi}{P^A} = (1 - \tau_r)r + \frac{\psi\dot{A}}{A}. \quad (15)$$

It turns out that

$$s_A \equiv \frac{s_R\tau_c}{1 - \tau_c} \quad (16)$$

is the behaviorally relevant subsidy rate of R&D costs. To see this, use the definition of policy instruments in Table 1 to find that profits in the R&D sector read as

$$\Pi = P^A(1 + \psi)\dot{A} - wH^A - \tau_c \left[ P^A(1 + \psi)\dot{A} - (1 + s_R)wH^A \right] \quad (17)$$

$$\equiv (1 - \tau_c) \left[ P^A\tilde{\nu}A^\phi H^A - (1 - s_A)wH^A \right], \quad (18)$$

taking  $A$  and  $\tilde{\nu}$  as given, where we used (2) for the latter equation.

The household chooses the optimal consumption path, where savings are supplied to the financial market, and the optimal (path of) education investment. Indexing time by subscript  $t$ , formally, the household's problem is to solve

$$\max_{\{c_t, h_t^H\}_{t=0}^\infty} U \quad \text{s.t. (4), (5), } h_t \geq 0, \quad \lim_{t \rightarrow \infty} a_t \exp \left( - \int_0^t [(1 - \tau_r)r_s - n] ds \right) \geq 0. \quad (19)$$

**Definition.** A market equilibrium consists of time paths for the quantities  $\{h_t^A, h_t^Y, h_t^H, c_t, \{x_{it}\}_{i=0}^A, a_t, Y_t, K_t, A_t, T_t\}_{t=0}^\infty$  and prices  $\{P_t^A, \{p_{it}\}_{i=0}^A, w_t, r_t\}_{t=0}^\infty$  such that

1. final goods producers, intermediate goods producers and R&D firms maximize profits,
2. households maximize intertemporal welfare,
3. the capital resource constraint  $\int_0^A x_{it} di = K$  holds,
4. the capital market equilibrium condition, equ. (15), holds,
5. the labor market clears (i.e.  $h^A + h^Y + h^H = h$ ), the intermediate goods markets clear, and the financial market clears (i.e.  $aN = K + P^AA$ ),<sup>10</sup>
6. the government runs a balanced budget, i.e. given the policy instruments in Table 1, the transfer per capita  $T$  equals total tax revenues minus total subsidies, both divided by  $N$ .

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<sup>9</sup>Note that the after-tax income from asset holding of a household is  $(1 - \tau_r)rK/N + (1 - \tau_g)\dot{P}^AA/N + \pi A/N - P^A\psi\dot{A}/N$ . Under (15) and since  $Na = K + P^AA$ , this equals  $(1 - \tau_r)ra$ , as reflected in the budget constraint (5) of a household.

<sup>10</sup>According to Walras' law, the final goods market then clears as well.

In the proof of the first proposition we derive the full dynamical system (employed in the numerical analysis of Section 6), which describes, given initial conditions  $A_0$ ,  $h_0$ ,  $N_0$  and  $K_0$ , the dynamic evolution of the economy as well as the steady state equilibrium. Let  $h^A \equiv H^A/N$ . The long run equilibrium allocation can be described by the fraction of human capital devoted to education and R&D,  $\mathfrak{h}^H = h^H/h$  and  $\mathfrak{h}^A \equiv h^A/h$ , respectively, as well as the capital investment rate (which is also the economy's savings rate).

$$sav \equiv \frac{\dot{K} + \delta_K K}{Y} = \left( \frac{\dot{K}}{K} + \delta_K \right) \frac{k}{y} \left[ = 1 - \frac{c}{y} \right] \quad (20)$$

The following holds in a steady state:

**Proposition 1.** (Long run market equilibrium) *There exists a unique balanced growth equilibrium, which is characterized as follows.*

(i) *The number of ideas grows with rate*

$$\frac{\dot{A}}{A} = \frac{(1-\theta)n}{1-\phi} \equiv g_A. \quad (21)$$

(ii) *Equity wealth per capita ( $q \equiv P^A A/N$ ), the wage rate ( $w$ ), income per capita ( $y$ ), consumption per capita ( $c$ ), financial wealth per capita ( $a$ ), and the physical capital stock per capita ( $k$ ) grow with rate*

$$g = \frac{\alpha g_A}{(1-\alpha)(\beta-1)}. \quad (22)$$

(iii) *The human capital level per capita ( $h$ ) is stationary and we have*

$$\frac{h^H}{h} = \frac{1-\tau_w}{1-s_H} \frac{\gamma \delta_H}{\rho - n + g(\sigma-1) + \delta_H(1-\eta)} \equiv \mathfrak{h}^{H*}, \quad (23)$$

$$\frac{h^A}{h} = \frac{1-\mathfrak{h}^{H*}}{\frac{1-s_A}{1-\tau_c} \Lambda(\tau_g) + 1} \equiv \mathfrak{h}^{A*} \text{ with} \quad (24)$$

$$\Lambda(\tau_g) \equiv \frac{\sigma g + \rho + \psi g_A - (n+g-g_A)(1-\tau_g)}{(1-1/\kappa)(\beta-1)(1+\psi)g}. \quad (25)$$

(iv) *The savings and investment rate is given by*

$$sav = \frac{\alpha(n+g+\delta_K)}{(1-s_K)\kappa \left( \frac{\sigma g + \rho}{1-\tau_r} + \delta_K \right)} \equiv sav^*. \quad (26)$$

**Proof.** See Appendix. ■

Like in Jones (1995), the growth rate of per capita income along a balanced growth path is independent of economic policy (in contrast to the level of income). This is an

attractive feature for our numerical analysis. It allows us to attribute growth effects of policy shocks (starting from balanced growth equilibrium) entirely to the transitional dynamics. Proposition 1 also implies that life-time utility (3) is finite if and only if assumption (A1) holds.

Moreover, Proposition 1 shows that an increase in the behaviorally relevant capital cost subsidy,  $s_K$ , raises the long run savings rate and investment share,  $sav^*$ . It does neither affect the level nor the allocation of human capital in long run equilibrium. An increase in the education subsidy rate,  $s_H$ , raises the long run fraction of human capital devoted to education,  $\mathfrak{h}^{H*}$ , and therefore also raises the long run level of human capital per capita. It does not affect the investment rate,  $sav^*$ . An increase in the behaviorally relevant R&D subsidy,  $s_A$ , stimulates R&D activity of firms (i.e.,  $\mathfrak{h}^{A*}$  increases) but does not affect incentives to invest in education or physical capital in long run equilibrium.

How can we understand the intuition behind the neutrality implications? Let us consider properties  $\frac{\partial \mathfrak{h}^{H*}}{\partial s_A} = 0$  and  $\frac{\partial \mathfrak{h}^{H*}}{\partial s_K} = 0$  first. One may be led to think that an increase in  $s_A$  or  $s_K$ , by fostering R&D and capital accumulation, raises the wage rate per unit of human capital,  $w$ , and therefore induces more education. However, not only is the the marginal cost (foregone consumption) of increasing teaching input,  $h^H$ , proportional to  $w$ , but also the long run marginal benefit (i.e. the value of human capital). Consequently, any policy that increases the wage rate does not impact on education decisions in the steady state because it equally affects costs and benefits. Consider next the properties  $\frac{\partial sav^*}{\partial s_A} = 0$  and  $\frac{\partial sav^*}{\partial s_H} = 0$ . Again one may be led to think that an increase in  $s_A$  or  $s_H$  should raise the marginal product of machines and induce a higher rate of capital accumulation. However, note that the capital-output ratio reads as  $\frac{k}{y} = \frac{\alpha}{\kappa(1-s_K)R}$ , according to (13) and (14); thus, according to (20), the capital investment rate,  $sav$ , is decreasing in the user cost of capital,  $R = r + \delta_K$ . According to the Keynes-Ramsey rule, individual consumption growth reads as

$$\frac{\dot{c}}{c} = \frac{(1 - \tau_r)r - \rho}{\sigma} \quad (27)$$

(see the proof of Proposition 1), where  $\dot{c}/c = g$  in steady state, according to Proposition 1. Thus,<sup>11</sup>

$$\sigma g + \rho = (1 - \tau_r)r, \quad (28)$$

implying that the interest rate and therefore the capital investment rate is constant in the steady state. Therefore,  $sav^*$  does neither depend on  $s_A$  nor on  $s_H$ . An increase in the marginal product of machines does indeed lead to more machines. Due to diminishing returns, however, the marginal product of machines declines such that the interest rate and hence the capital investment rate remain unaffected.

Furthermore, we find that taxing wages gives a disincentive to invest in education, i.e., an increase in  $\tau_w$  lowers  $\mathfrak{h}^{H*}$ . Similarly, an increase in the corporate tax rate (entering arbitrage condition (15) via instantaneous profits of intermediate good firms,  $\pi$ ) gives a disincentive to invest in R&D; consequently,  $\mathfrak{h}^{A*}$  is decreasing in  $\tau_c$ , all other

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<sup>11</sup>Rewriting assumption (A1) by using (28) implies that  $(1 - \tau_r)r > n + g$ , i.e., the after-tax interest rate must exceed the long-run growth rate of aggregate income.

things equal. Moreover, an increase in the rate at which capital gains are taxed,  $\tau_g$ , lowers R&D incentives, leading to a decline in  $\mathfrak{h}^{A*}$ , if  $n + g > g_A$  (which turns out to hold with our calibration outlined in Section 5). Finally, the long run savings rate,  $sav^*$ , is decreasing in the capital income tax rate,  $\tau_r$ .

## 4 Social Planning Optimum

We next derive the social planning optimum and the optimal policy mix which implements it. A social planner chooses a symmetric capital allocation across intermediate firms, i.e.,  $x_i = K/A$  for all  $i$ . Noting the output technology (1), per capita output ( $y = Y/N$ ) may be expressed as:

$$y = A^{\frac{\alpha}{\beta-1}} k^\alpha (h^Y)^{1-\alpha}. \quad (29)$$

Thus, the capital stock per capita ( $k = K/N$ ) evolves according to

$$\dot{k} = A^{\frac{\alpha}{\beta-1}} k^\alpha (h^Y)^{1-\alpha} - (\delta_K + n)k - c. \quad (30)$$

Also note that the social planner takes R&D externalities into account. Using (2), he observes the knowledge accumulation condition

$$\dot{A} = \frac{\nu}{1+\psi} A^\phi (Nh^A)^{1-\theta}. \quad (31)$$

The social planner's problem thus is to solve

$$\max U \quad \text{s.t. (4), (30), (31), } h^A = h - h^Y - h^H, \quad (32)$$

and non-negativity constraints, where  $c$ ,  $h^A$ ,  $h^H$ ,  $h^Y$  are control variables and  $h$ ,  $k$ ,  $A$  are state variables.

**Proposition 2.** (Long run social optimum) *There exists an interior, unique long-run solution of the social planner problem (32) which is characterized as follows:*

(i) *As in decentralized long run equilibrium, the growth rate of  $A$  is given by  $g_A$  (see (21)) and the growth rates of  $c$ ,  $k$ ,  $y$  are given by  $g$  (see (22)).*

(ii) *The fraction of human capital devoted to education and R&D are given by*

$$\frac{h^H}{h} = \frac{\gamma\delta_H}{\rho - n + g(\sigma - 1) + \delta_H(1 - \eta)} \equiv \mathfrak{h}_{opt}^H, \quad (33)$$

$$\frac{h^A}{h} = \frac{1 - \mathfrak{h}_{opt}^H}{\Gamma + 1} \equiv \mathfrak{h}_{opt}^A \quad \text{with } \Gamma \equiv \frac{\rho + g\sigma - \theta n - g}{(1 - \theta)g}. \quad (34)$$

(ii) *The capital investment rate reads as follows*

$$sav = \frac{\alpha(n + g + \delta_K)}{\sigma g + \rho + \delta_K} \equiv sav_{opt}. \quad (35)$$

**Proof.** See Appendix. ■

As for the decentralized equilibrium, the productivity of R&D and education, parameterized by  $\nu$  and  $\xi$ , respectively, do neither affect the allocation and level of human capital nor the investment rate in the long run social optimum. Unlike in steady state market equilibrium, also parameter  $\psi$ , which captures the strength of the business stealing effect, and the mark-up factor  $\kappa$  do not affect the optimal resource allocation.

Like in Jones and Williams (2000), there are four R&D externalities. The duplication externality ( $\theta > 0$ ) promotes overinvestment in R&D, whereas a standing on shoulders effect ( $\phi > 0$ ) promotes underinvestment. The possibility of business stealing ( $\psi > 0$ ) gives rise to two counteracting effects on the human capital allocation in the market economy, relative to the unaffected social optimum. On the one hand, existing intermediate good firms are at risk of being replaced by future innovators. An increase in  $\psi$  thus lowers the value of patents ( $P^A$ ) by raising the effective discount rate (right-hand side of (15)) and therefore depresses the long run equilibrium fraction of human capital devoted to R&D,  $h^A/h$ . On the other hand, an innovator obtains a rent from an innovation even when he does not contribute to the knowledge stock of the economy,  $A$ . To achieve the same increase in  $A$ , more R&D labor is required if  $\psi$  increases, which tends to raise the equilibrium value of  $h^A/h$ . If and only if the latter effect dominates, the fraction of human capital devoted to R&D in decentralized equilibrium increases in  $\psi$ . In this case,  $\psi > 0$  promotes overinvestment. According to (24),  $\mathfrak{h}^{A*}$  is increasing in  $\psi$ , for instance, if the capital gains tax rate ( $\tau_g$ ) is zero or small. Finally, innovators may not be able to appropriate the full economic surplus from raising the knowledge stock of the economy. To see this, note from (8), (11) and (12) that instantaneous profit of an intermediate goods firm  $i$  reads as  $\pi_i = \alpha(1 - \tau_c)\frac{\kappa-1}{\kappa}\frac{Y}{A}$ , whereas  $\frac{\partial Y}{\partial A} = \frac{\alpha}{\beta-1}\frac{Y}{A}$  holds, according to (29). If and only if  $(1 - \tau_c)(1 - \frac{1}{\kappa}) < \frac{1}{\beta-1}$ , there is a “surplus appropriability problem” which promotes underinvestment. (If  $(1 - \tau_c)(1 - \frac{1}{\kappa}) > \frac{1}{\beta-1}$ , there is a force towards overinvestment.) Thus, depending on parameter values, there may be over- or underinvestment in R&D. This leaves a critical role for the calibration strategy to obtain useful numerical results on the optimal resource allocation and policy mix.

Comparing (23) and (33), we find that in the case where the tax rate on wage income equals the effective education subsidy rate ( $\tau_w = s_H$ ), both the long run fraction of human capital devoted to education and the long run level of human capital are socially optimal. That is, the distortion stemming from wage taxation can be exactly offset by an education subsidy. Generally, we find that  $\mathfrak{h}^{H*} < (=, >) \mathfrak{h}_{opt}^H$  if  $s_H < (=, >) \tau_w$ . Finally, in absence of a capital cost subsidy ( $s_d = s_K = 0$ ), the savings rate will be too low whenever  $\tau_r \geq 0$ , i.e.,  $sav^* < sav_{opt}$ , according to (26) and (35).

We next characterize the optimal policy mix in the long run.

**Proposition 3.** (Optimal long run policy mix) *There exists a policy mix  $(s_H^{opt}, s_A^{opt}, s_K^{opt})$  which for any feasible values of tax parameters  $(\tau_w, \tau_c, \tau_g, \tau_r)$  implements the long-run*

*social planning optimum. It is characterized as follows:*

$$s_H = \tau_w, \quad (36)$$

$$s_A = 1 - \frac{(1 - \tau_c)\Gamma}{\Lambda(\tau_g)} \equiv s_A^{opt}, \text{ i.e. } s_R = \frac{1 - \tau_c}{\tau_c} s_A^{opt} \equiv s_R^{opt}, \quad (37)$$

$$s_K = 1 - \frac{\sigma g + \rho + \delta_K}{\kappa \left( \frac{\sigma g + \rho}{1 - \tau_r} + \delta_K \right)} \equiv s_K^{opt}, \text{ i.e. } s_d = \frac{1 - \tau_c}{\tau_c} s_K^{opt} \equiv s_d^{opt}. \quad (38)$$

**Proof.** Set  $\mathfrak{h}^{H*} = \mathfrak{h}_{opt}^H$ ,  $\mathfrak{h}^{A*} = \mathfrak{h}_{opt}^A$  and  $sav^* = sav_{opt}$  to derive (36), (37) and (38), respectively, by using the expressions in Proposition 1 and 2. ■

How optimal long run subsidies on R&D and capital costs depend on tax parameters follows from the tax distortions discussed after Proposition 1. Moreover, note that a higher mark up factor  $\kappa$  drives a bigger wedge between the equilibrium investment rate and the socially optimal investment rate, provided that capital income is not subsidized ( $\tau_r \geq 0$ ). Thus, an increase in price setting power calls for a higher subsidy on capital costs.

According to Proposition 3, the first-best allocation in the steady state can be restored, despite numerous distortions from goods market imperfection, externalities and income taxation, with a very limited number of tax/subsidy instruments (one targeted to each engine of growth).<sup>12</sup> This remarkable result follows from the fact that the dynamic system is governed by the three allocation variables,  $\mathfrak{h}^H$  (fraction of human capital devoted to education)  $\mathfrak{h}^A$  (fraction of human capital devoted to R&D) and  $sav$  (capital investment rate). Thus, we need exactly three policy instruments to implement the first best.

## 5 Calibration

A calibration strategy is proposed which attempts to match observables for the US in the 2000s before the financial crisis unfolded. We assume that observable endogenous variables correspond to steady state values in the model under the status quo policy. Table 2 provides an overview on the calibration strategy.

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<sup>12</sup>Independent research by Nuño (2011) has led to a similar result. He shows that the first-best, long-run allocation can be supported by an appropriate investment subsidy and R&D subsidy.

Endogenous observables	Value	Source
$sav$	0.21	Heston et al. (2006)
$k/y$	3	US Bureau of Economic Analysis
$r$	0.07	Mehra and Prescott (1985)
$g$	0.021	Heston et al. (2006)
$\omega^A \equiv \frac{wh^A}{y}$	0.02 (0.07)	OECD (2008a); cf. discussion in text
$\omega^H \equiv \frac{wh^H}{y}$	0.05	OECD (2007b); cf. discussion in text
Parameters set by authors	Value	Source
$\tau_r = \tau_c$	0.395	OECD Tax Database (2008)
$\tau_w$	0.30	OECD Tax Database (2008)
$\tau_g$	0.12	cf. discussion in text
$s_d$	0	Devereux et al. (2002); cf. discussion in text
$s_A$	0.066	OECD (2007a); cf. discussion in text
$s_H$	0.3	cf. discussion in text
$\rho$	0.02	cf. discussion in text
$\varphi$	0.23	Coe and Helpman (1995)
$\delta_H$	0.03	cf. discussion in the text
$n$	0.01	Heston et al. (2006)
$EPL$	10	Jones and Williams (2000)
$\eta$	0.15	cf. discussion in text
$\theta$	0.5	cf. discussion in text
Implied parameters	At $\omega^A = 0.02$ (at $\omega^A = 0.07$ )	
$s_K$	0 (0)	implied by equ. (6)
$s_R$	0.1 (0.1)	implied by equ. (16)
$\delta_K$	0.04 (0.04)	implied by equ. (39)
$\sigma$	1.08 (1.08)	implied by equ. (28)
$\alpha$	0.36 (0.44)	implied by equ. (47)
$\beta$	2.58 (2.93)	implied by equ. (41)
$\psi$	1.74 (2.00)	implied by equ. (42)
$\kappa$	1.1 (1.35)	implied by equ. (45)
$\phi$	0.91 (0.89)	implied by equ. (44)
$\gamma$	0.087 (0.090)	implied by equ. (48)

**Table 2:** Baseline calibration: endogenous observables, observable parameters and implied parameters.

## 5.1 Observable Parameters

### 5.1.1 Policy instruments

Let us start with the calibration of policy parameters in Table 1. In the US, the statutory tax rate on dividend income and corporate income coincide. We thus set  $\tau_r = \tau_c = 0.395$ , as published by the OECD tax database (federal and sub-central government taxes combined). Using the same source, the labor income tax,  $\tau_w$ , is set equal to the total tax wedge (wage income tax rate including all social security contributions and from all levels of governments combined) which applies to average wage income. It is given by  $\tau_w = 0.3$ . The behaviorally relevant R&D subsidy rate,  $s_A$ , is (for the year 2007) taken from OECD (2007a, p.73),  $s_A = 0.066$ .<sup>13</sup> Using  $s_R = \frac{1-\tau_c}{\tau_c} s_A$ , according to (16), we have  $s_R = 0.1$ .

Devereux, Griffith and Klemm (2002, p. 459) report for the US a rate of depreciation allowances for capital investments of almost 80 percent. This would suggest that  $s_d$

<sup>13</sup>The OECD reports a R&D subsidy rate  $R D T S = 1 - B i n d e x$ , where the so-called B-index is given by  $B i n d e x = \frac{1-\Xi}{1-\tau_c}$ , with  $\tau_c$  being the statutory corporate income tax rate and  $\Xi$  the net present discounted value of depreciation allowances, tax credits and special allowances on R&D assets. In the context of our model,  $\Xi = \tau_c(1 + s_R)$ . Thus,  $R D T S = \frac{\tau_c s_R}{1-\tau_c} = s_A$ .

is somewhat above  $-0.2$  and thus  $s_K < 0$ . However, as the authors point out, the definition of corporate income tax base is very complex and there are other possibilities than depreciation allowances to deduct capital costs, which they cannot provide data on. We take into account further allowances by assuming that, initially,  $s_d = s_K = 0$  (i.e. full deduction of capital costs).

As a result of the ‘Jobs and Growth Tax Relief Reconciliation Act’ of 2003, long-term capital gains are taxed at 15 percent if income is above some threshold. Otherwise, until 2008 it was 5 percent and until 2010 it was 0 percent. Before 2003 it was 20 percent. We calibrate  $\tau_g$  to 12 percent throughout. Fortunately, our results are strongly robust with respect to changes in  $\tau_g$  (to save space the sensitivity analysis is not displayed).

Finally, we need to calibrate the education subsidy rate ( $s_H$ ), which is most difficult. For instance, we observe the fraction of public education expenditure in total expenditure. In the year 2004, the average was 68.4 percent in the US (OECD, 2007b, Table B3.1, p. 219); among the public spending, 20.7 percent was on student loans, scholarships and other household grants (rather than direct public spending on institutions). To complicate things further, a substantial fraction of total household spending on education is unobservable, like private teachers at home, time costs of parents etc. (neither counted as education expenditure in databases nor subsidized). It is thus difficult to come up with a well-founded estimate. We assume that the education subsidy is set such that the long run fraction of human capital devoted to education,  $h^{H*}$ , and the long run level of human capital are socially optimal, given the distortion introduced by wage taxation,  $s_H = \tau_w (= 0.3)$ . That is, we focus on distortions of R&D investment and physical capital investment in our numerical analysis.

### 5.1.2 The Growth Rate of Population Size and Per Capita Income

Other parameters are calibrated as follows. First,  $n$  is set to the average population growth rate for the period 1990-2004. Taking data from the Penn World Tables (PWT) 6.2 (Heston, Summers and Aten, 2006), we find  $n = 0.01$ . For the same period, and again from PWT 6.2, the average growth rate of per capita income is 2.1 percent. We calibrate  $g$  to match this growth rate (thereby averaging out business cycle phenomena).

### 5.1.3 Scale Parameters

Scale parameters  $\nu$  and  $\xi$  in the technology of accumulating knowledge and human capital, respectively, do not enter the long run values for the allocation variables derived in Proposition 1 (decentralized equilibrium) and Proposition 2 (social optimum). They also do not affect the allocation variables of interest in the transitional dynamics and can thus be set arbitrarily.<sup>14</sup>

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<sup>14</sup>We can show numerically that  $\nu$  and  $\xi$  do not affect the eigenvalues of the dynamical system.

## 5.2 Relationships of Observables and Unobservables

### 5.2.1 Depreciation Rates

We use measures for the investment rate ( $sav$ ) and the capital-output ratio to calibrate the depreciation rate of physical capital,  $\delta_K$ , as follows. Using  $\dot{K}/K = n + g$  in (20) and solving for  $\delta_K$  yields

$$\delta_K = \frac{sav}{k/y} - n - g. \quad (39)$$

Averaging over the period 1990-2004,  $sav$  is equal to about 21 percent, according to PWT 6.2. For the capital-output ratio, we take averages over the period 2002-2007 calculated from data of the US Bureau of Economic Analysis. The capital stock is taken to be total fixed assets (private and public structures, equipment and software). At current prices, this gives us  $k/y = 3$ . From (39), the evidence then suggests that  $\delta_K$  is about 4 percent, which is a standard value in the literature. In the literature, the depreciation rate of human capital is typically set slightly lower than  $\delta_K$ . We choose  $\delta_H = 0.03$ . This is in the range of the estimated value in Heckman (1976), who finds that  $\delta_H$  is between 0.7 and 4.7 percent. For the steady state analysis in Section 6, we do not need to know  $\delta_H$ , as will become apparent.

### 5.2.2 Patience

The steady state interest rate is set to the real long-run stock market return estimated by Mehra and Prescott (1985), suggesting  $r = 0.07$ .<sup>15</sup> Recall from (28), that preference parameters  $(\sigma, \rho)$  fulfill  $\sigma g + \rho = (1 - \tau_r)r$ . Thus, in an numerical analysis, accounting for tax distortions plays a potentially important role for calibrated parameters. In particular, matching the observed growth rate of the economy,  $g$ , requires to assume that households are more patient if accounting for capital income taxation ( $\tau_r > 0$ ) than if setting  $\tau_r = 0$ . Thus, the gap between socially optimal investment levels and equilibrium investment levels are potentially larger when accounting for capital income taxation. For  $g = 0.021$ ,  $r = 0.07$ ,  $\tau_r = 0.395$  and a typical value for the time preference rate of  $\rho = 0.02$ , we find  $\sigma = 1.08$  (whereas  $\sigma = 1.5$  when setting  $\tau_r = 0$ ). Note that a value of  $\sigma$  around unity is also more in line with a large body of evidence from the public finance literature (e.g. Chetty, 2006).

### 5.2.3 Knowledge Production

Production technology parameters  $\alpha$  and  $\beta$  are potentially critical since they determine the elasticity of output with respect to the state of knowledge,  $A$ . To see this, use  $x_i = K/A$  for all  $i$  and  $H^Y = Nh^Y$  in (1) to find

$$Y = BK^\alpha N^{1-\alpha} \text{ with } B \equiv A^{\frac{\alpha}{\beta-1}} (h^Y)^{1-\alpha}. \quad (40)$$

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<sup>15</sup>Jones and Williams (2000) argue that this rate of return is more appropriate for calibration of growth models than the risk-free rate of government bonds.

We employ a relationship between  $\alpha$  and  $\beta$  which can be recovered from estimates of the output elasticity with respect to the R&D capital stock. Using (40), this elasticity is equal to  $\frac{\partial Y}{\partial A} \frac{A}{Y} = \frac{\alpha}{\beta-1} \equiv \varphi$ . Thus,

$$\beta = 1 + \frac{\alpha}{\varphi}. \quad (41)$$

We can write  $\log B = \Upsilon + \varphi \log A$ , where  $\Upsilon \equiv (1 - \alpha) \log h^Y$ , according to (40). Regressing  $\log B$  (by using that the total factor productivity is given by  $B = YK^{-\alpha}N^{\alpha-1}$ ) on a measure of knowledge capital ( $\log A$ ), Coe and Helpman (1995) obtain  $\varphi = 0.23$ , which is the value we use to fix the relationship between  $\alpha$  and  $\beta$  given by (41).

The steady state fraction of intermediate good firms driven out of the market each instant is  $\psi g_A$ . Its inverse is equal to the effective patent life,  $EPL$ . Thus, we have

$$\psi = \frac{1}{EPL} \frac{1}{g_A}, \quad (42)$$

where

$$g_A = \frac{(1 - \alpha)(\beta - 1)g}{\alpha}, \quad (43)$$

according to (22). In our baseline calibration we follow Jones and Williams (2000) in assuming an effective patent life of 10 years ( $EPL = 10$ ) and investigate the sensitivity of our results to alternative values.

The duplication externality parameter  $\theta$  and the standing on shoulders parameter  $\phi$  play an important role for the extent of R&D underinvestment. They cannot be set independently. Given  $g$ ,  $n$ ,  $\alpha$ ,  $\beta$  and  $\theta$ , we obtain  $\phi$  from (21) and (22):

$$\phi = 1 - \frac{\alpha n(1 - \theta)}{(1 - \alpha)(\beta - 1)g}. \quad (44)$$

We consider variations of  $\theta$  and  $\phi$  which fulfill (44), taking the intermediate value  $\theta = 0.5$  as baseline. We consider a wide range for  $\theta$  as sensitivity analysis.

#### 5.2.4 Price Setting Power

The literature has tried to come up with estimates for the price setting power of firms. For instance, Norrbin (1993) estimates mark-up factor  $\kappa$  to be in the range (between 1.05 and 1.4). Because this range is rather large, we attempt to pin down  $\kappa$  indirectly, by using (13) and (14) together with  $R = r + \delta_K$ , to find

$$\kappa = \frac{\alpha}{(1 - s_K)(r + \delta_K)^{\frac{\kappa}{y}}}. \quad (45)$$

Thus, if we knew  $\alpha$ , then  $\kappa$  would be implied by the already discussed values in the denominator of (45).

### 5.2.5 Human Capital Shares

According to (1), the human capital share in manufacturing is  $1 - \alpha$ . According to (41)-(45),  $\alpha$  is a key parameter which determines  $\beta$ ,  $\psi$ ,  $g_A$  and  $\kappa$ , for given calibrated values of  $\varphi$ ,  $EPL$ ,  $g$ ,  $s_K$ ,  $r$ ,  $\delta_K$ ,  $k/y$ . In the literature, the value of  $\alpha$  is typically motivated by using the labor share in total income. However, due to the existence of R&D workers and teachers in the model,  $\alpha$  is related to the fraction of income which accrues to production workers only (rather than to the entire labor share): we have  $wh^Y/y = 1 - \alpha$ . Moreover, as pointed out by Krueger (1999), among others, there is little consensus on how to measure the total labor share as fraction of GDP. In our context, the labor share is  $wh/y$ . When two thirds of business proprietor's income is added to labor income, Krueger (1999) shows that the US labor share fluctuates over time between 75 and 80 percent. Otherwise the labor share would be significantly lower. For instance, the OECD reports a labor share around 65 percent for the US. Due to the uncertainty about the labor share, we propose a different route than typically taken in the literature. Our calibration strategy is to determine the human capital income share endogenously, together with the salient parameter  $\alpha$ . This is done as follows. Defining  $\omega \equiv wh/y$ ,  $\omega^H \equiv wh^H/y$  and  $\omega^A \equiv wh^A/y$ , we obtain from  $h^Y + h^A + h^H = h$  and  $wh^Y/y = 1 - \alpha$  that

$$\omega = 1 + \omega^H + \omega^A - \alpha. \quad (46)$$

By definition, we have  $h^A/h = \omega^A/\omega$  and  $h^H/h = \omega^H/\omega$ . Substituting both  $\mathfrak{h}^{A*} = \omega^A/\omega$  and  $\mathfrak{h}^{H*} = \omega^H/\omega$  into expression (24) for the long run equilibrium fraction of human capital devoted to R&D, and then using (46), we find

$$\frac{1 - s_A}{1 - \tau_c} \Lambda(\tau_g) \omega^A = 1 - \alpha. \quad (47)$$

Given  $\omega^A$  and taking into account relationships (28), (41), (42), (43) and (45) to find  $\Lambda(\tau_g)$  as defined in (25),  $\alpha$  is implied by (47). Note that  $\omega^A$  is the R&D intensity in the model. For the period 1990-2006 we find that the average R&D costs of business enterprises (BERD) as a fraction of GDP is 1.9 percent (OECD, 2008a). When we use gross R&D investment intensity (GERD), the figure would be higher (about 2.6 percent). As most but not all R&D costs are labor costs, this suggests to calibrate  $\omega^A = 0.02$ . However, one may argue that not all R&D activity in the sense of the model is captured by typical R&D intensity measures. According to OECD (2008b, Table 1.1), total investment in intangible assets in the US as a fraction of GDP was almost 12 percent for the period 1998-2000. However, 5 percent of GDP was spent to develop intangible assets like brand equity, firm-specific human capital and the organizational firm structure, which are not R&D activities in the sense of our model. We therefore consider  $\omega^A = 0.07$  as an alternative scenario to the case of  $\omega^A = 0.02$  in our numerical analysis.

Note from (47) that we do not need to know the fraction of human capital used in education,  $\omega^H$ , to calibrate  $\alpha$ . Moreover, all parameters which are needed to find  $\alpha$  can be led back to observables. With  $n = 0.01$ ,  $g = 0.021$ ,  $r = 0.07$ ,  $k/y = 3$ ,  $sav = 0.213$  (thus,  $\delta_K = 0.04$ ),  $EPL = 10$ ,  $\tau_r = \tau_c = 0.395$ ,  $\tau_w = 0.3$ ,  $\tau_g = 0.12$ , we find for the

case where the R&D intensity is  $\omega^A = 0.02$  that  $\alpha = 0.36$ . In turn, this value of  $\alpha$  implies  $\beta = 2.58$ ,  $\psi = 1.74$ ,  $g_A = 0.06$  and  $\kappa = 1.1$ . If the R&D intensity is set to  $\omega^A = 0.07$ , we obtain  $\alpha = 0.44$ ,  $\beta = 2.93$ ,  $\psi = 2$ ,  $g_A = 0.05$  and  $\kappa = 1.35$ .

To calculate the long run equilibrium allocation of human capital, characterized by  $\mathfrak{h}^{A*} = \omega^A/\omega$  and  $\mathfrak{h}^{H*} = \omega^H/\omega$ , we next need to find the human capital income share,  $\omega$ , by calibrating  $\omega^H$  and using (46). To calibrate  $\omega^H$ , we add expenditure from public and private sources over all education levels. This gives us an average value of 7 percent for the time period 1990-2003 (OECD, 2007b, Table B2.1, p. 205).<sup>16</sup> As not all education expenditure is on salary of teaching personnel, we use  $\omega^H = 0.05$ . For  $\omega^A = 0.02$ , we then find from (46) that  $\omega = 0.71$  and therefore  $\mathfrak{h}^{H*}(= \mathfrak{h}_{opt}^H) = \frac{5}{71} = 0.071$  and  $\mathfrak{h}^{A*} = \frac{2}{71} = 0.028$ .<sup>17</sup> For  $\omega^A = 0.07$ , we obtain  $\omega = 0.68$ ,  $\mathfrak{h}^{H*}(= \mathfrak{h}_{opt}^H) = 0.074$  and  $\mathfrak{h}^{A*} = 0.104$ .

### 5.2.6 Human Capital Production

Finally, we calibrate the human capital accumulation process (4). Elasticity parameters  $\gamma$  and  $\eta$  are not independent from each other when assuming that the economy initially is in steady state. According to (23), given  $\rho + \sigma g = (1 - \tau_r)r$ ,  $g$ ,  $n$ ,  $\delta_H$ ,  $s_H = \tau_w$  and  $h^H/h = \mathfrak{h}^{H*}$ , we obtain the relationship

$$\gamma = \frac{(1 - \tau_r)r - n - g + \delta_H(1 - \eta)}{\delta_H} \mathfrak{h}^{H*}. \quad (48)$$

As baseline calibration we take  $\eta = 0.15$  and obtain  $\gamma$  by using (48). The steady state analysis does not require to fix  $\eta$  (and  $\gamma$ ). In section 7, we also consider  $\eta = 0.3$ . It turns out that results are basically insensitive to variations in  $\eta$  (and  $\gamma$ ).

## 6 Optimal Long Run Policy Mix

Before analyzing the optimal policy program that maximizes intertemporal welfare by calculating the entire transition path in response to a policy reform, we numerically compare the long run social optimum to the decentralized steady state equilibrium, under existing US tax policy and assuming by setting  $s_H = \tau_w$  that there are no distortions of the accumulation of human capital. This also allows us to compare the

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<sup>16</sup>Although there is no publicly provided education in our model, it is more appropriate to take such expenditure into account, in addition to private education spending. An underlying assumption which justifies that choice is that credit constraints are negligible for advanced economies, such that publicly provided education and private education are almost perfect substitutes. In fact, recent studies find no evidence for the relevance of educational borrowing constraints in the US (see, e.g., Cameron and Taber, 2004, and the references therein).

<sup>17</sup>We shall note that  $h^A/h$  does not necessarily correspond to the fraction of workers in R&D and thus cannot be readily observed even under the assumption that the economy is in steady state. Although there is a representative agent, there may well be heterogeneity, such that not all individuals possess the same level of human capital. Thus, an implied  $h^A/h$  which exceeds the fraction of R&D workers (equal to about 1 percent) is consistent with the notion that the average R&D worker has a higher level of human capital than the average worker in the labor force.

results with the previous literature, which exclusively focussed on a long run analysis and did not consider human capital accumulation (e.g., Jones and Williams, 2000; Steger, 2005; Strulik, 2007). Importantly, we also derive the optimal subsidy rates targeted to R&D and capital costs, by employing Proposition 3.

## 6.1 R&D Investment

We start with R&D investment. We first consider the role of the R&D technology by considering variations in the degree of the duplication externality,  $\theta$  (thus varying the standing on shoulders parameter,  $\phi$ ) and alternative R&D intensities,  $\omega^A$ , to which we match our parameters in steady state. We then investigate how optimal R&D subsidies vary to changes in the strength of intellectual property rights,  $EPL$ , and the return to R&D, captured by  $\psi$ . Finally, we discuss the role of calibrating the interest rate,  $r$ .

### 6.1.1 R&D Technology and Optimal R&D Subsidies

According to panel (a) of Table 3, which is based on an R&D intensity of two percent in long run market equilibrium, there is dramatic underinvestment in R&D in the case where the duplication externality is not very high. We find that for  $\theta \leq 0.9$ , the long run socially optimal human capital fraction  $\mathfrak{h}_{opt}^A$  is in the wide range of about 5 – 16 times higher than the market equilibrium fraction  $\mathfrak{h}^{A*}$ . What we would like to know, however, is how to improve the allocation of labor and to what extent which kind of tax policy should be used. Interestingly, due to the effectiveness of R&D subsidies for the equilibrium fraction  $\mathfrak{h}^{A*}$ , the necessary R&D policy to restore the social optimum does not so much depend on  $\theta$ , if  $\theta \leq 0.9$ . Our results suggest that the R&D sector should be able to deduct from pre-tax profits to obtain the corporate income tax base about 1.27 – 1.48 the amount invested in R&D. As pre-tax profits in the sense of the model are already net of R&D costs, this suggests that firms should be allowed to deduct up to 2.5 times their R&D costs from sales revenue to obtain the tax base. The current R&D subsidy policy in the US thus seems insufficient. Only if  $\theta$  is very high, about 0.98 or higher, there is overinvestment in R&D such that current R&D subsidies should be cut. Such a large degree of the duplication externality does not seem to be realistic, however.

$\theta$	$\mathfrak{h}_{opt}^A$ (in %)	$\mathfrak{h}^{A*}$ (in %)	$\mathfrak{h}_{opt}^A/\mathfrak{h}^{A*}$	$s_A^{opt}$	$s_R^{opt}$
0	45.4	2.8	16.1	0.97	1.48
0.25	41.7	2.8	14.8	0.96	1.48
0.5	35.9	2.8	12.7	0.95	1.46
0.75	25.3	2.8	8.9	0.92	1.41
0.9	13.4	2.8	4.7	0.83	1.27
0.95	7.5	2.8	2.7	0.67	1.02
0.99	1.7	2.8	0.6	-0.61	-0.93

(a) Parameters matched to R&D intensity of two percent (i.e.  $\omega^A = 0.02$ ).

$\theta$	$h_{opt}^A$ (in %)	$h^{A*}$ (in %)	$h_{opt}^A/h^{A*}$	$s_A^{opt}$	$s_R^{opt}$
0	45.3	10.4	4.4	0.88	1.34
0.25	41.6	10.4	4.0	0.86	1.31
0.5	35.8	10.4	3.5	0.81	1.25
0.75	25.2	10.4	2.4	0.69	1.05
0.9	13.3	10.4	1.3	0.30	0.46
0.95	7.5	10.4	0.7	-0.34	-0.52
0.99	1.7	10.4	0.2	-5.45	-8.35

(b) Parameters matched to R&D intensity of seven percent (i.e.  $\omega^A = 0.07$ ).

**Table 3:** Human capital in R&D (social optimum and decentralized) and optimal R&D policy in the long run: the role of the R&D technology

Note: Underlying set of parameters (except  $\theta$  and  $\phi$ ) as in Table 2. Results are independent of  $\rho, \sigma, \gamma, \eta, \delta_H$ ;  $\phi$  is implied by equ. (44).

Panel (b) of Table 3 shows that when we assume an R&D intensity of 7 percent, the R&D underinvestment problem is less dramatic, but still substantial. For  $\theta \leq 0.9$  there should be 1.3 – 4.4 times higher human capital investment in R&D. Interestingly and importantly for a robust policy implication, the optimal R&D subsidy is not that different to the previous case. For  $\theta \leq 0.75$ , firms should be able to deduct 1.05 – 1.34 times the amount of R&D costs from pre-tax profits. Thus, our analysis suggests that US firms should be allowed to deduct not less than twice their R&D costs from sales revenue for calculating taxable corporate income.

### 6.1.2 Intellectual Property Rights, the Return to R&D and Optimal R&D Subsidies

The degree of intellectual property rights protection, as captured by the effective patent life,  $EPL$ , could be viewed as a policy parameter. Since three subsidy instruments are enough, however, to implement the first best optimum, we study the role of changes in  $EPL$  (set to 10 years in the baseline calibration) for underinvestment and optimal subsidies. Moreover, it is interesting to look at the sensitivity of results to the (implicitly assumed) return to R&D,  $\frac{\partial Y}{\partial A}$ . Assuming  $\frac{\partial Y}{\partial A} \frac{A}{Y} = \varphi = 0.23$  and  $\frac{A}{Y} = 0.26$  (Griliches, 1992),  $\varphi = 0.23$  implies a social rate of return to R&D of about 100 percent. One may argue that the social rate of return of 70 percent is also in line with empirical evidence and hence  $\varphi = 0.18$ .

Table 4 reports (for the baseline calibration  $\theta = 0.5$ ) the results of the associated sensitivity analysis analogously to Table 3, involving a re-calibration of the model according to the lower part of Table 2. (Recall that We find that the extent of the R&D underinvestment bias is largely insensitive with regard to lowering  $\varphi$ . The reason is that the calibration strategy implies an inverse relation between  $\alpha$  and  $\beta$ . Similarly, our results are largely insensitive to variations in  $EPL$ .)

$EPL$	$\varphi$	$\mathfrak{h}_{opt}^A$ (in %)	$\mathfrak{h}^{A*}$ (in %)	$\mathfrak{h}_{opt}^A/\mathfrak{h}^{A*}$	$s_A^{opt}$	$s_R^{opt}$
5	0.18	35.9	2.8	12.7	0.95	1.46
10	0.18	35.9	2.8	12.7	0.95	1.46
20	0.18	35.9	2.8	12.7	0.95	1.46
5	0.23	35.9	2.8	12.7	0.95	1.46
10	0.23	35.9	2.8	12.7	0.95	1.46
20	0.23	35.9	2.8	12.7	0.95	1.46

(a) Parameters matched to R&D intensity of two percent (i.e.  $\omega^A = 0.02$ ).

$EPL$	$\varphi$	$\mathfrak{h}_{opt}^A$ (in %)	$\mathfrak{h}^{A*}$ (in %)	$\mathfrak{h}_{opt}^A/\mathfrak{h}^{A*}$	$s_A^{opt}$	$s_R^{opt}$
5	0.18	35.8	10.3	3.5	0.81	1.25
10	0.18	35.8	10.3	3.5	0.81	1.25
20	0.18	35.8	10.4	3.4	0.81	1.24
5	0.23	35.8	10.3	3.5	0.81	1.25
10	0.23	35.8	10.4	3.5	0.81	1.25
20	0.23	35.7	10.4	3.4	0.81	1.24

(b) Parameters matched to R&D intensity of seven percent (i.e.  $\omega^A = 0.07$ ).

**Table 4:** Human capital in R&D (social optimum and decentralized) and optimal R&D policy in the long run: the role of intellectual property rights and the return to R&D

### 6.1.3 The Interest Rate and Optimal R&D Subsidies

One may argue that a gross interest rate of seven percent,  $r = 0.07$ , is too high. However, if the steady state interest rate is assumed to have a smaller value, the observation of massive underinvestment in R&D and substantial welfare gains in response to an increase in R&D subsidies would even be reinforced. To see this, notice that  $\rho$  and  $\sigma$  are positively related, via (28), to the after-tax interest rate,  $(1 - \tau_r)r$ . Setting  $r$  to a lower value, given  $g$ , means that individuals are assumed to be more patient. Moreover, according to (34) in Proposition 2, the optimal fraction of human capital in R&D,  $\mathfrak{h}_{opt}^A$ , is decreasing in both preference parameters,  $\rho$  and  $\sigma$ . That is, if individuals are more patient, the social planner devotes more resources to R&D. Hence, a lower value for  $r$  implies a larger R&D underinvestment gap in the market economy.

## 6.2 Physical Capital Investment

For an R&D intensity of 2 percent (see Table 2, especially the lower part, for the implied calibration), we find that the US economy underinvests in physical capital. Employing Proposition 2, the optimal long run investment rate,  $sav_{opt}$ , is equal to 31.3 percent, whereas in market equilibrium the investment as a fraction of GDP,  $sav^*$ , is 21.3 percent (used for the calibration of capital depreciation rate  $\delta_K$  in (39)). According to Proposition 3, this means that US firms should be allowed to deduct

about one and a half of their capital costs from sales revenue (i.e.  $s_d^{opt} = 0.49$ ) rather than being allowed to deduct their capital costs by 100 percent (i.e.  $s_d = 0$ ) for calculating corporate income. For an R&D intensity of 7 percent (again, see Table 2 for the implied calibration), the gap between the decentralized and the socially optimal investment rate is even larger ( $sav^* = 0.213$ ,  $sav_{opt} = 0.38$ ,  $s_d^{opt} = 0.68$ ).

## 6.3 Comparison to the Literature

### 6.3.1 The Role of Distortionary Taxes for the Calibration

Previous analyses suggest that the R&D underinvestment problem is considerably less dramatic than implied by our study. There are two main differences between our analysis and the literature. First, we explicitly capture tax/subsidy policy and calibrate the economy accordingly. Second, our calibration strategy does not use some empirical measure of the labor share (or human capital income share) to calibrate the output elasticity of labor/human capital,  $1 - \alpha$ . Our baseline calibration rather uses evidence on the R&D intensity to calibrate  $\alpha$  for the long run, in turn determining the human capital income share,  $\omega$ , endogenously.

We will now demonstrate, exemplarily, that if we followed the strategy of the important and prominent contribution of Jones and Williams (2000), we obtain results which are similar to theirs. First, one can show how abstracting from the tax system leads to a downward bias of the extent of R&D underinvestment. According to (34) in Proposition 2, the optimal fraction of human capital in R&D,  $\mathfrak{h}_{opt}^A$ , is decreasing in both preference parameters,  $\rho$  and  $\sigma$ . That is, if individuals are less patient, the social planner devotes less resources to R&D. According to (28),  $\rho$  and  $\sigma$  are positively related – by the Keynes-Ramsey rule – to the after-tax interest rate,  $(1 - \tau_r)r$ . Setting the tax rate on capital income,  $\tau_r$ , to zero rather than to its actual value means that individuals are assumed to be less patient. This brings the socially optimal R&D resources closer to the market equilibrium. Moreover, as discussed after Proposition 1, both corporate income taxation and the capital gains tax distort R&D incentives. R&D subsidies should account for these distortions. To see the effects numerically, suppose again  $n = 0.01$ ,  $g = 0.021$ ,  $r = 0.07$ ,  $k/y = 3$ ,  $\delta_K = 0.04$ ,  $EPL = 10$ ,  $\omega^H = 0.05$  and recalibrate the model by assuming that there are no taxes and subsidies. For an R&D intensity  $\omega^A = 0.02$ , we then obtain, like for the case with tax distortions,  $\alpha = 0.36$ , implying  $\beta = 2.58$ . Thus, the equilibrium fraction of human capital in R&D,  $\mathfrak{h}^{A*}$ , is again about 2.8 percent ( $= \frac{2}{71}$ ); moreover,  $\mathfrak{h}^{H*} = \mathfrak{h}_{opt}^H = 0.07$  ( $= \frac{5}{71}$ ). However, with  $\sigma g + \rho$  equal to 0.07 rather than 0.042 as for Tab. 3 (a), the optimal R&D effort,  $\mathfrak{h}_{opt}^A$ , is now given by 28 percent for  $\theta = 0$ , by 18 percent for  $\theta = 0.5$ , and by 10.3 percent for  $\theta = 0.75$ . Thus, the relative gap to the market equilibrium shrinks considerably compared to the case with taxes and subsidies shown in Table 3 (a); for instance, if  $\theta = 0.75$ ,  $\mathfrak{h}_{opt}^A/\mathfrak{h}^{A*}$  is now equal to 3.7 instead of 8.9.

If, in addition to abstracting from taxes and subsidies, we assume  $\omega^A = 0.07$  instead of  $\omega^A = 0.02$  (calibration as for Tab 3 (b)), then  $\alpha$  becomes 0.42 and the implied labor share,  $\omega = 1.12 - \alpha$ , is 70 percent, i.e., almost equal to the labor share in the case where  $\omega^A = 0.02$ . Consequently, we obtain very similar values for  $\mathfrak{h}_{opt}^H$  and therefore for

$h_{opt}^A$  as in the case where the R&D subsidy is 2 percent. However, now  $h^{A*} = \frac{7}{70}$ , i.e., 10 percent of human capital is allocated to R&D in market equilibrium. This means that  $h_{opt}^A/h^{A*}$  is equal to 2.8 for  $\theta = 0$ , to 1.8 for  $\theta = 0.5$ , and to 1.03 for  $\theta = 0.75$ ; that is, for  $\theta = 0.75$  the long run equilibrium R&D intensity is about socially optimal. Interestingly, these figures almost match the results of Jones and Williams (2000) who also assume an interest rate of seven percent and an effective patent life of 10 years in their baseline calibration. In fact, they set the output elasticity of labor such that the R&D intensity is about seven percent and abstract from taxes or subsidies – the case just examined. As a result, for the same extent of the duplication externality which corresponds to  $\theta = 0$ ,  $\theta = 0.5$  and  $\theta = 0.75$ , they obtain an R&D investment in social optimum relative to the equilibrium investment of 2.2, 1.7 and unity, respectively. This demonstrates that the different results of our study, shown in Table 3, stem from the public finance side in the model, which is supposed to capture the key elements of the US tax-transfer system.

### 6.3.2 The Role of the Mark Up Factor

Steger (2005) employs a general, semi-endogenous R&D-based growth model to investigate the allocative bias in the R&D share and the saving rate along the balanced growth path. The main finding is that the market economy slightly underinvests in R&D but heavily underinvests in physical capital accumulation. Compared to our results, the underinvestment gap in R&D is much smaller, whereas the underinvestment gap in physical capital is larger. These differential findings are mainly driven by the fact that Steger (2005) does not disentangle the mark-up factor, governing the pricing decisions of intermediate goods producers, from the elasticity of physical capital in final output production. We do so by employing the concept of a competitive fringe. In Steger (2005), the elasticity of physical capital in final output production is specified to match the capital income share such that the implied price mark up is comparably high. This implies higher profits of intermediate goods producers inducing strong incentives to conduct R&D in the market economy. At the same time, the comparably high price of intermediate goods results in a small amount of physical capital in decentralized equilibrium, which enlarges the gap between the first-best and the decentralized capital investment rate.

### 6.3.3 Fully Endogenous vs. Semi-Endogenous Growth

A natural question is whether our results are robust with respect to the chosen model. There is recent empirical literature testing the implications of semi-endogenous growth models against those of Schumpeterian fully endogenous growth models (Ha and Howitt, 2007, Zachariadis, 2003, Madsen, 2008, Venturini, 2012a, 2012b). Most of the literature finds evidence that a higher R&D intensity has a positive impact on patenting and economic growth and interprets this as evidence for fully endogenous growth, i.e. an elasticity of knowledge creation with respect to the stock of knowledge equal to one ( $\phi = 1$  in our notation). Comparing the degree of R&D underinvestment between our model and a fully endogenous growth model is beyond the scope of this paper.

However, in a related paper (Grossmann, Steger and Trimborn, 2013b) we compare for the steady state the degree of R&D underinvestment between the plain vanilla Jones (1995) and Romer (1990) model under separate calibration of both and find that it is considerably higher in the Romer model. In particular, we find that in order install the first-best allocation the optimal long run subsidy rate  $s_A$  (same notation as in our paper) has to increase to  $s_A = 0.79$  in the Jones-framework, whereas it has to increase to  $s_A = 0.91$  in the Romer model. (The optimal long run subsidy on capital costs is the same in both models.) Since the Jones-model is a stripped-down version of our model in the present paper (abstracting from the tax-transfer system in particular) we are confident that the result of higher R&D underinvestment in a fully endogenous model compared to a semi-endogenous model would translate to our framework. In other words, we would expect that modifying our model to a fully endogenous growth model should increase the degree of R&D underinvestment at the market solution and also the R&D subsidy rate needed to obtain the first-best solution.

## 7 Dynamically Optimal Policy Reform

Like the related literature, the analysis in the previous section has ignored transitional dynamics. However, there may be very slow adjustment to the new steady state in response to policy shocks. For instance, implementing the optimal long run policy mix implies a half-live of over 100 years in our model. We now examine which policy reform maximizes the intertemporal welfare gain, starting from an initial balanced growth path. The resulting change in intertemporal welfare ( $U$ ) is measured by the permanent consumption-equivalent change in intertemporal welfare, denoted by  $\Theta$  (see appendix for details). The transitional dynamics are simulated by applying the relaxation algorithm (Trimborn et al., 2008).<sup>18</sup> For tractability reasons we restrict the attention to the case where subsidy rates are time-invariant.<sup>19</sup> That is, we start from an initial steady state under the status quo policy and calculate the time path of consumption in response to a one-time change in the subsidy rates. We first consider the benchmark case, where the government budget is balanced by a lump-sum tax/transfer before restricting ourselves to the case where lump-sum finance is infeasible.

### 7.1 Benchmark Case

The policy mix which maximizes the welfare gain is denoted by  $(\bar{s}_R^{opt}, \bar{s}_d^{opt}, \bar{s}_H^{opt})$ . The results are presented in Table 5. We find that the dynamically optimal subsidy rates, when restricted to be time-invariant, are not much different from those suggested by the steady state analysis  $(s_R^{opt}, s_d^{opt}, s_H^{opt})$ . Both  $\bar{s}_R^{opt}$  and  $\bar{s}_d^{opt}$  are slightly higher than optimal long run values  $s_R^{opt}$  and  $s_d^{opt}$ , respectively. Deviation of  $\bar{s}_H^{opt}$  from the optimal

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<sup>18</sup>Details of the numerical evaluations presented in this section are discussed in supplementary material available on request.

<sup>19</sup>The optimal subsidies may be time-variant. However, Grossmann et al. (2013) have shown that the welfare loss from setting the R&D subsidy to its optimal long run level is negligible compared to the case where the time varying, first-best subsidy rates are implemented.

long run education subsidy ( $s_H^{opt} = 0.3$ ) is overall negligible and does not seem to follow a pattern. Moreover, the results do not critically depend on  $\eta$  (and thus not on  $\gamma$ ). However, the extent of the duplication externality  $\theta$  is an important parameter we could not satisfactorily calibrate, which requires a careful sensitivity analysis. Fortunately, the optimal policy mix does not critically depend on  $\theta$  for intermediate values of this parameter. Thus, we can safely conclude that the underinvestment problem is severe for R&D and substantial for physical capital. The policy implications outlined in Section 6 roughly apply.

$\theta$	$\eta$	$\phi$	$\gamma$	$s_R^{opt}$	$\bar{s}_R^{opt}$	$s_d^{opt}$	$\bar{s}_d^{opt}$	$s_H^{opt}$	$\bar{s}_H^{opt}$	$\Theta$
0.5	0.15	0.91	0.09	1.46	1.49	0.49	0.54	0.3	0.27	4.16
0.5	0.3	0.91	0.08	1.46	1.49	0.49	0.54	0.3	0.31	4.16
0.75	0.15	0.95	0.09	1.41	1.44	0.49	0.52	0.3	0.30	0.98
0.75	0.3	0.95	0.08	1.41	1.44	0.49	0.52	0.3	0.32	0.98

(a) Parameters matched to R&D intensity of two percent (i.e.  $\omega^A = 0.02$ ).<sup>20</sup>

$\theta$	$\eta$	$\phi$	$\gamma$	$s_R^{opt}$	$\bar{s}_R^{opt}$	$s_d^{opt}$	$\bar{s}_d^{opt}$	$s_H^{opt}$	$\bar{s}_H^{opt}$	$\Theta$
0.25	0.15	0.84	0.09	1.31	1.37	0.68	0.71	0.3	0.30	1.62
0.25	0.3	0.84	0.08	1.31	1.37	0.68	0.71	0.3	0.28	1.62
0.5	0.15	0.90	0.09	1.25	1.31	0.68	0.71	0.3	0.31	0.86
0.5	0.3	0.90	0.08	1.25	1.31	0.68	0.71	0.3	0.30	0.86
0.75	0.15	0.95	0.09	1.05	1.10	0.68	0.70	0.3	0.33	0.32
0.75	0.3	0.95	0.08	1.05	1.10	0.68	0.70	0.3	0.27	0.32

(b) Parameters matched to R&D intensity of seven percent (i.e.  $\omega^A = 0.07$ ).

**Table 5:** Optimal growth policy mix and welfare gain,  $\Theta$ .

The potential welfare gains when implementing the optimal growth policy mix are remarkable. For instance, for  $\theta = 0.5$ , the intertemporal welfare gain is equivalent to a permanent annual increase in the consumption level per capita,  $\Theta$ , of about 86 percent if we start out with an R&D intensity of  $\omega^A = 0.07$ ; it even equals 416 percent for the case  $\omega^A = 0.02$ . Unlike the optimal policy mix, the welfare gain from implementing an appropriate policy reform critically depends on both  $\theta$  and  $\omega^A$ . As discussed in Section 5, it may make more sense to view R&D activity in a broader way as measured by the officially reported R&D intensity. Therefore, we prefer the case  $\omega^A = 0.07$  to the case  $\omega^A = 0.02$ . This suggests that for an intermediate value of  $\theta \approx 0.5$ , the welfare gain from an appropriate policy reform is roughly equivalent to a permanent doubling of per capita consumption.

<sup>20</sup>For  $\omega^A = 0.02$  and  $\theta = 0.25$  the algorithm does not converge. The gap between the decentralized allocation and the socially optimal solution and, hence, the implied optimal policy change is so large that a numerical solution cannot be found in this case. This indicates that the implied  $\bar{s}_d^{opt}$  and  $\Theta$  are even larger compared to the case  $\omega^A = 0.02$  and  $\theta = 0.5$ .

## 7.2 The Role of the Lump-Sum Transfer

So far we assumed that any change in subsidy rates  $(s_R, s_d, s_H)$ , e.g. by implementing the optimal policy program, is associated with a change in the lump-sum transfer  $(T)$  in order to keep the government's budget balanced. The distortionary taxes were kept constant at observed rates  $(\tau_w, \tau_r, \tau_c, \tau_g)$ . One concern about the results in the previous subsection 6.1 (Table 4), which suggest a large optimal R&D subsidy  $(\bar{s}_R^{opt})$  and a large welfare gain from implementing the optimal policy program  $(\Theta)$ , is that they are driven by lump-sum finance. To address this concern, we conduct the following experiment. We assume, first, that the tax rates  $(\tau_w, \tau_r, \tau_c, \tau_g)$  and the subsidy rates  $(s_R, s_d, s_H)$  are as reported in Table 2, second, that the economy is initially in its steady state and, third, that the government's budget is balanced by imposing an appropriate lump-sum transfer  $(T)$ . Now, as the optimal policy program is being implemented,  $T$  is held constant and higher subsidy rates are financed by adjusting the distortionary wage tax,  $\tau_w$ , such that the government's budget remains balanced. The other tax rates are held fixed at observed levels. We focus on the wage tax because labor income taxation is in advanced countries the largest source of tax revenue (which is also true in our model). Because we allow for endogenous human capital accumulation, it is distortionary in our framework. Focussing on one tax parameter is a rather strict robustness test for the welfare gains found in Table 4. Welfare gains would be even higher, if the government budget could be balanced by a larger set of tax instruments.

Table 6 displays the dynamically optimal R&D subsidy rate  $(\bar{s}_R^{opt})$  and the welfare gain  $(\Theta)$  implied by this experiment, assuming, inter alia  $\theta = 0.5$  and  $\eta = 0.15$ . In Panel (a), we match the R&D intensity to  $\omega^A = 0.02$  whereas  $\omega^A = 0.07$  in Panel (b). The first rows in Panels (a) and (b) report, for convenience, the baseline scenario #0, taken from Panel (a) and (b) of Table 5, respectively. Scenarios #1-3 show the implied dynamically optimal R&D subsidy  $(\bar{s}_R^{opt})$  and the resulting welfare gain  $\Theta$  assuming that a time-varying wage tax  $\tau_w$  ensures that the government's budget remains balanced. In scenario #1, the education subsidy is held constant at  $s_H = 0.3$ . We find that the optimal subsidy rates  $\bar{s}_R^{opt}$  and  $\bar{s}_d^{opt}$  do not significantly change compared to the baseline scenario with adjustment of the lump-sum transfer. According to Panel (a), the welfare gain in the case where  $\omega^A = 0.02$  declines only slightly from 4.16 to 4.12, i.e. by about four percentage points. According to Panel (b), where  $\omega^A = 0.07$ , it declines from 0.86 to 0.83. The main reason for this rather negligible reduction in  $\Theta$  is that a change in  $\tau_w$  has only a small distortionary impact on education decisions, and hence on the level of human capital, as long as we take seriously our calibration strategy. For instance, for the steady state level of human capital,  $h^*$ , we can derive the following elasticity:<sup>21</sup>

$$E \equiv \frac{d \ln h^*}{d \ln(1 - \tau_w)} = \frac{\gamma}{1 - \gamma - \eta}. \quad (49)$$

Given  $\gamma = 0.087$  and  $\eta = 0.15$  we find that a reduction in the after-tax wage income share  $(1 - \tau_w)$  by 1 percent (resulting from  $\Delta\tau_w > 0$ ) reduces the level of human capital in the steady state by  $E \approx 0.11$  percent. In addition, given our calibration

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<sup>21</sup>Use (70) and (71) as stated in the appendix (proof of Proposition 1).

strategy, this effect remains small even if we change  $\gamma$  and  $\eta$ . According to (48), for instance,  $\gamma$  can be increased by setting  $\eta = 0$ . Maintaining our calibration of the fraction of human capital devoted to teaching,  $\mathfrak{h}^{H*} = 0.071$  (see Section 5.2), this leads to a minor increase of  $\gamma$  from  $\gamma = 0.087$  to  $\gamma = 0.098$ . However, since elasticity  $E$  depends positively on  $\gamma$  and  $\eta$ , an increase in  $\gamma$  (brought about by a reduction in  $\eta$ ) actually reduces  $E$ . An additional reason for the small reduction in the welfare gain between scenario #1 and the baseline scenario #0 is that the sizable education subsidy  $s_H = 0.3$  partially eliminates the allocative bias induced by increasing tax rate  $\tau_w$ . In scenario #2, we therefore reduce the education subsidy to  $s_H = 0$ . The implied welfare gain from an optimal policy reform reduces to  $\Theta = 4.02$  when  $\omega^A = 0.02$  and to  $\Theta = 0.80$  when  $\omega^A = 0.07$ . Quantitatively, however, the differences in these results to the baseline scenario are still small. Finally, in scenario #3, we assume that also the education subsidy can be set at its dynamically optimal level (which equals  $\bar{s}_H^{opt} = 0.54$ ). Interestingly, neither the welfare gain in response to the implementation of the optimal policy program nor the optimal subsidy rates  $\bar{s}_R^{opt}$  and  $\bar{s}_d^{opt}$  change compared to the baseline scenario.

#	scenario	$\bar{s}_R^{opt}$	$\bar{s}_d^{opt}$	$\tau_w(0)$	$\tau_w(\infty)$	$\Theta$
0	lump-sum transfer, $\bar{s}_H^{opt} = 0.27$	1.49	0.54	0.3	0.3	4.16
1	$\tau_w$ endogenous, $s_H = 0.3$	1.49	0.51	0.85	0.49	4.12
2	$\tau_w$ endogenous, $s_H = 0$	1.48	0.49	0.83	0.47	4.02
3	$\tau_w$ endogenous, $\bar{s}_H^{opt} = 0.54$	1.49	0.54	0.87	0.53	4.16

(a) Parameters matched to R&D intensity of two percent (i.e.  $\omega^A = 0.02$ ).

#	scenario	$\bar{s}_R^{opt}$	$\bar{s}_d^{opt}$	$\tau_w(0)$	$\tau_w(\infty)$	$\Theta$
0	lump-sum transfer, $\bar{s}_H^{opt} = 0.31$	1.31	0.71	0.3	0.3	0.86
1	$\tau_w$ endogenous, $s_H = 0.3$	1.30	0.68	0.80	0.45	0.83
2	$\tau_w$ endogenous, $s_H = 0$	1.30	0.66	0.78	0.43	0.80
3	$\tau_w$ endogenous, $\bar{s}_H^{opt} = 0.57$	1.31	0.71	0.83	0.49	0.86

(b) Parameters matched to R&D intensity of seven percent (i.e.  $\omega^A = 0.07$ ).

**Table 6:** Optimal policy in the absence of lump-sum transfers.

Note: Parameters matched to R&D intensity  $\omega^A = 0.02$ . All other parameters as in Table 2. In scenario #1-3, the time path of  $\tau_w$  is determined such that government budget balances and welfare is being maximized.  $\tau_w(0)$  and  $\tau_w(\infty)$  denote the wage tax rates which balance the government budget at  $t = 0$  and  $t \rightarrow \infty$ , respectively.

## 8 Conclusion

This paper has employed a comprehensive endogenous growth model to derive the optimal growth policy mix. Our analysis represents a first step to examine the growth policy implications of distortions resulting from income taxation, R&D externalities, and product market imperfections. It is consciously based on well-understood and widely-used ingredients in endogenous growth theory. The analysis has accounted for the US tax system as well as transitional dynamics in response to policy shocks.

Calibrating our model to the US by assuming the US was in steady state under the status quo investment incentives prior to the recent crisis, the results suggest that the current policy leads to severe underinvestment in both R&D and physical capital. Our preferred calibration implies that firms should be allowed to deduct between 2-2.5 times their R&D costs and about 1.5-1.7 times their capital costs from sales revenue for calculating taxable corporate income. The results on the optimal policy mix are not sensitive to reasonable changes in the calibration. A policy reform targeted to all three growth engines simultaneously may entail an intertemporal welfare gain which is equivalent to a permanent doubling of per capita consumption.

Should we take these results at face value? One may object that the welfare gain is too large to appear plausible. We basically agree on this point in the sense that one should be sceptical at this stage of the research process. Our reading of the results is that there is strong indication for the welfare significance of the quest for the optimal growth policy. Therefore, we believe that there should be more research on this important topic, e.g. by investigating alternative growth frameworks. For instance, whereas standard growth theory assumes that human capital is general and thus can be reallocated between R&D and production sectors without frictions, future research should study the implications of imperfect intersectoral labor mobility for optimal growth policy.<sup>22</sup> It also seems indicated to incorporate potential risks associated with innovations in an analysis of dynamically optimal growth policy.<sup>23</sup>

## 9 Appendix

### 9.1 Proof of Proposition 1

The current-value Hamiltonian which corresponds to the household optimization problem (19) is given by

$$\begin{aligned} \mathcal{H} = & \frac{c^{1-\sigma} - 1}{1-\sigma} + \mu [\xi (h^H)^\gamma h^\eta - \delta_H h] + \\ & \lambda [(1-\tau_r)r - n]a + (1-\tau_w)wh - (1-s_H)wh^H - c + T, \end{aligned} \quad (50)$$

where  $\lambda$  and  $\mu$  are multipliers (co-state variables) associated with constraints (4) and (5), respectively. Necessary optimality conditions are  $\partial\mathcal{H}/\partial c = \partial\mathcal{H}/\partial h^H = 0$  (control variables),  $\dot{\mu} = (\rho - n)\mu - \partial\mathcal{H}/\partial h$ ,  $\dot{\lambda} = (\rho - n)\lambda - \partial\mathcal{H}/\partial a$  (state variables), and the corresponding transversality conditions. Thus,

$$\lambda = c^{-\sigma} \quad (51)$$

$$\mu\xi\gamma(h^H)^{\gamma-1}h^\eta = \lambda(1-s_H)w, \quad (52)$$

$$\frac{\dot{\mu}}{\mu} = \rho - n - \xi (h^H)^\gamma \eta h^{\eta-1} + \delta_H - \frac{\lambda}{\mu}w(1-\tau_w), \quad (53)$$

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<sup>22</sup>Grossmann (2007) investigates an endogenous growth model in which workers choose their type of education (production vs. R&D skills) ex ante and are immobile across occupations.

<sup>23</sup>See Jones (2013) for a first analytical framework in this direction.

$$\frac{\dot{\lambda}}{\lambda} = \rho - (1 - \tau_r)r, \quad (54)$$

$$\lim_{t \rightarrow \infty} \mu_t e^{-(\rho-n)t} h_t = 0, \quad (55)$$

$$\lim_{t \rightarrow \infty} \lambda_t e^{-(\rho-n)t} a_t = 0. \quad (56)$$

Differentiating (51) with respect to time and using (54), we obtain the Euler equation

$$\frac{\dot{c}}{c} = \frac{(1 - \tau_r)r - \rho}{\sigma}. \quad (57)$$

Now, define  $\tilde{z} \equiv z A^{-\frac{\alpha}{(1-\alpha)(\beta-1)}}$  for  $z \in \{w, c, a, T\}$ ; we will show that the adjusted values ( $\tilde{z}$ ) of these variables are stationary in the long run. From (57),

$$\frac{\dot{\tilde{c}}}{\tilde{c}} = \frac{(1 - \tau_r)r - \rho}{\sigma} - \frac{\alpha}{(1 - \alpha)(\beta - 1)} \frac{\dot{A}}{A}. \quad (58)$$

Differentiating (52) with respect to time and making use of (4), (52), (53) and (54) we obtain:

$$\frac{\dot{h}^H}{h^H} = \frac{1}{1 - \gamma} \left[ (1 - \tau_r)r - n + (1 - \eta)\delta_H - \frac{1 - \tau_w}{1 - s_H} \frac{\xi \gamma h^\eta}{(h^H)^{1-\gamma}} - \frac{\dot{w}}{w} \right]. \quad (59)$$

Moreover, with  $H^A = N h^A$ , (2), (4), (5) can be written as

$$\frac{\dot{A}}{A} = \frac{\nu}{1 + \psi} A^{\phi-1} (N h^A)^{1-\theta}, \quad (60)$$

$$\frac{\dot{h}}{h} = \xi (h^H)^\gamma h^{\eta-1} - \delta_H, \quad (61)$$

$$\frac{\dot{\tilde{a}}}{\tilde{a}} = (1 - \tau_r)r - n + (1 - \tau_w) \frac{\tilde{w}h}{\tilde{a}} - (1 - s_H) \frac{\tilde{w}h^H}{\tilde{a}} - \frac{\tilde{c}}{\tilde{a}} - \frac{\alpha}{(1 - \alpha)(\beta - 1)} \frac{\dot{A}}{A} + \frac{\dot{T}}{\tilde{a}}. \quad (62)$$

Next, substitute (11) and (12) into (8) and use both (13) and  $R = r + \delta_K$  to obtain the following expression for the profit of each intermediate goods producer  $i$ :

$$\pi_i = \pi = A^{\frac{\alpha}{(1-\alpha)(\beta-1)-1}} (1 - \tau_c)(\kappa - 1) \left( \frac{\alpha}{\kappa} \right)^{\frac{1}{1-\alpha}} [(1 - s_K)(r + \delta_K)]^{-\frac{\alpha}{1-\alpha}} H^Y. \quad (63)$$

Now define  $\tilde{q} \equiv P^A A^{1 - \frac{\alpha}{(1-\alpha)(\beta-1)}} / N$  and differentiate  $\tilde{q}$  with respect to time; then use the resulting expression as well as (63) to rewrite (15) as

$$\frac{\dot{\tilde{q}}}{\tilde{q}} = \left( 1 - \frac{\alpha}{(1 - \alpha)(\beta - 1)} \right) \frac{\dot{A}}{A} - n + \frac{1}{1 - \tau_g} \times \left( (1 - \tau_r)r + \psi \frac{\dot{A}}{A} - \frac{(1 - \tau_c)(\kappa - 1) \left( \frac{\alpha}{\kappa} \right)^{\frac{1}{1-\alpha}} [(1 - s_K)(r + \delta_K)]^{-\frac{\alpha}{1-\alpha}} h^Y}{\tilde{q}} \right) \quad (64)$$

The capital market clearing condition reads  $Na = K + P^A A$ ; it implies, by using (14) and  $R = r + \delta_K$  (as well as the definitions of  $\tilde{a}$  and  $\tilde{q}$ ), that

$$\tilde{a} = \left( \frac{\alpha}{\kappa(1-s_K)(r+\delta_K)} \right)^{\frac{1}{1-\alpha}} h^Y + \tilde{q}. \quad (65)$$

The wage rate equals the marginal product of human capital in the final goods sector, i.e.,  $w = (1-\alpha)Y/H^Y$ . Using (13) we obtain

$$\tilde{w} = (1-\alpha) \left( \frac{\alpha}{\kappa(1-s_K)(r+\delta_K)} \right)^{\frac{\alpha}{1-\alpha}}. \quad (66)$$

Moreover, in equilibrium,  $\Pi = 0$  holds. This leads to

$$w = \frac{P^A \tilde{v} A^\phi}{1-s_A}, \quad (67)$$

according to (18). Combining (67) with (2) and using both  $\tilde{q} = P^A A^{1-\frac{\alpha}{(1-\alpha)(\beta-1)}}/N$  and  $\tilde{w} = A^{-\frac{\alpha}{(1-\alpha)(\beta-1)}} w$ , we can write

$$h^A = \frac{\tilde{q}(1+\psi)\frac{\dot{A}}{A}}{(1-s_A)\tilde{w}}. \quad (68)$$

We next derive steady state values. In steady state, the growth rate of  $\dot{A}/A$  must be equal to zero. Differentiating the right-hand side of (60) with respect to time and setting the resulting term to zero leads to  $\dot{A}/A = g_A$  as given by (21), provided that  $\dot{h}^A = 0$ . In the following we show that  $\dot{h}^A = 0$  indeed holds if  $\dot{A}/A = g_A$ ; we therefore set  $\dot{A}/A = g_A$  to derive the following (candidates of) steady state values. Setting  $\dot{\tilde{c}} = 0$  in (58) and using  $g = \frac{\alpha g_A}{(1-\alpha)(\beta-1)}$ , we find

$$r = \frac{\sigma g + \rho}{1-\tau_r}. \quad (69)$$

Note that substituting (69) into (66) also gives us a stationary value for  $\tilde{w}$  in terms of exogenous parameters only. According to (61) and  $\dot{h} = 0$ , we obtain

$$h = \left( \frac{\xi}{\delta_H} \right)^{\frac{1}{1-\eta}} (h^H)^{\frac{\gamma}{1-\eta}}. \quad (70)$$

Setting  $\dot{h}^H = 0$  in (59) (which holds in steady state, as will become apparent) and employing both  $\dot{w}/w = g$  and (69) implies<sup>24</sup>

$$h^H = \left( \frac{1-\tau_w}{1-s_H} \frac{\gamma}{(\sigma-1)g + \rho - n + (1-\eta)\delta_H} \right)^{\frac{1-\eta}{1-\gamma-\eta}} \left( \frac{\xi}{(\delta_H)^\eta} \right)^{\frac{1}{1-\gamma-\eta}}. \quad (71)$$

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<sup>24</sup>That the wage rate grows with rate  $g$  in steady state follows from  $w = \tilde{w} A^{-\frac{\alpha}{(1-\alpha)(\beta-1)}}$  and  $\dot{\tilde{w}} = 0$ .

Combining (70) and (71) gives us expression (23) for the equilibrium fraction of human capital devoted to education.

Using  $\dot{A}/A = g_A$  in (68), we furthermore obtain

$$h^A = \frac{(1 + \psi)g_A\tilde{q}}{(1 - s_A)\tilde{w}} \quad (72)$$

To find the steady state values for  $h^Y$  and  $\tilde{q}$ , first substitute (72) into labor market clearing condition  $h^Y = h - h^H - h^A$ , which gives us

$$h^Y = h - h^H - \frac{(1 + \psi)g_A\tilde{q}}{(1 - s_A)\tilde{w}}. \quad (73)$$

Also set  $\dot{\tilde{q}} = 0$  in (64) and use  $\dot{A}/A = g_A$  to find  $\tilde{q} = \Omega h^Y$  with

$$\Omega \equiv \frac{(1 - \tau_c)(\kappa - 1) \left(\frac{\alpha}{\kappa}\right)^{\frac{1}{1-\alpha}}}{[(1 - \tau_r)r + \psi g_A - (n + g - g_A)(1 - \tau_g)][(1 - s_K)(r + \delta_K)]^{\frac{\alpha}{1-\alpha}}}. \quad (74)$$

Substituting  $\tilde{q} = \Omega h^Y$  into (73) and solving for  $h^Y$  yields

$$h^Y = \frac{h - h^H}{1 + \frac{(1 + \psi)g_A\Omega}{(1 - s_A)\tilde{w}}} \quad (75)$$

and thus

$$\tilde{q} = \frac{\Omega(h - h^H)}{1 + \frac{g_A\Omega}{(1 - s_A)\tilde{w}}}. \quad (76)$$

Substituting (76) into (72) yields

$$h^A = \frac{h - h^H}{\frac{(1 - s_A)\tilde{w}}{\Omega(1 + \psi)g_A} + 1}. \quad (77)$$

Dividing by both sides of (77) by  $h$ , substituting into it both expressions (66) for  $\tilde{w}$  and (74) for  $\Omega$  as well as using  $(1 - \tau_r)r = \sigma g + \rho$  from (69) gives us expression (24) for the steady state fraction of human capital devoted to R&D.

Equations (70), (75), (76) and (77) give us explicit expressions for  $h$ ,  $h^Y$ ,  $\tilde{q}$  and  $h^A$ , respectively, noting that  $h^H$  is explicitly given by (71) and  $\tilde{w}$  by (66), using (69) for the latter. Setting next  $\dot{\tilde{a}} = 0$  in (62) and using (69),  $\dot{A}/A = g_A$  as well as  $g = \frac{\alpha g_A}{(1 - \alpha)(\beta - 1)}$  yields

$$\tilde{c} = [(\sigma - 1)g + \rho - n]\tilde{a} + (1 - \tau_w)\tilde{w}h - (1 - s_H)\tilde{w}h^H + \tilde{T}. \quad (78)$$

We also need to show that the adjusted lump-sum transfer per capita,  $\tilde{T}$ , is stationary in the long run when  $r$ ,  $h$ ,  $h^A$ ,  $h^Y$ ,  $h^H$ ,  $\tilde{w}$ ,  $\tilde{c}$ ,  $\tilde{a}$ ,  $\tilde{q}$  are stationary. Under a balanced government budget it must hold that the sum of education subsidy payments ( $s_H w N h^H$ ) and lump-sum transfer payments ( $T N$ ) is equal to the sum of revenue from labor income taxation ( $\tau_w w N h$ ), taxation of capital income from asset holding ( $\tau_r r K$ ),

taxation of capital gains ( $\tau_g \dot{P}^A A$ ), and corporate income taxation of intermediate good firms after depreciation allowances ( $\int_0^A \tau_c (p_i x_i - R x_i - s_d R x_i) di$ ), and of R&D firms after R&D subsidy ( $\tau_c (P^A \dot{A} - w H^A - s_R w H^A)$ ). Hence, using  $p_i = \kappa(1 - s_K)R$  for all  $i$ ,  $K = \int_0^A x_i di$ ,  $R = r + \delta_K$  as well as expressions (15) and (63), we have

$$\begin{aligned} \tilde{T} = & \tau_w \tilde{w} h + \tau_r r \tilde{k} + \tau_c [\kappa(1 - s_K) - (1 + s_d)] (r + \delta_K) \tilde{k} + \frac{\tau_g}{1 - \tau_g} \times \\ & \left( \left[ (1 - \tau_r) r + \psi \frac{\dot{A}}{A} \right] \tilde{q} - (1 - \tau_c) (\kappa - 1) \left( \frac{\alpha}{\kappa} \right)^{\frac{1}{1-\alpha}} [(1 - s_K)(r + \delta_K)]^{-\frac{\alpha}{1-\alpha}} h^Y \right) \\ & \tau_c \left[ \tilde{q} (1 + \psi) \frac{\dot{A}}{A} - (1 + s_R) \tilde{w} h^A \right] - s_H \tilde{w} h^H, \end{aligned} \quad (79)$$

where  $\tilde{k} \equiv A^{-\frac{\alpha}{(1-\alpha)(\beta-1)}} k$ . According to (14),  $\tilde{k}$  is stationary in the long run if  $h^Y$  is; thus, provided that  $\dot{A}/A = g_A$  as claimed,  $\tilde{T}$  is stationary. We also see that, in steady state both per capita capital stock  $k$  and, according to (13), per capita income grow with rate  $g$  as given by (22).

The investment share is given by  $sav = (\dot{K} + \delta_K K)/Y = (\dot{K}/K + \delta_K)k/y$ . Using  $\dot{K}/K = n + g$  together with expressions (14) and (13) for  $k$  and  $y$ , respectively, we obtain

$$sav = \frac{\alpha(n + g + \delta_K)}{\kappa(1 - s_K)R}. \quad (80)$$

Using  $R = r + \delta_K$  and expression (69) for  $r$  confirms (26).

Finally, it remains to be shown that the transversality conditions (55) and (56) hold under assumption (A1). Differentiating (52) with respect to time and using  $\dot{h} = \dot{h}^H = 0$  as well as  $\dot{w}/w = g$  implies that, along a balanced growth path,  $\dot{\mu}/\mu = \dot{\lambda}/\lambda + g$ . From (54) and (69) we find  $\dot{\lambda}/\lambda = -\sigma g$  and thus  $\dot{\mu}/\mu = (1 - \sigma)g$ . As  $h$  becomes stationary, (55) holds if  $\lim_{t \rightarrow \infty} e^{[(1-\sigma)g+n-\rho]t} = 0$ , or  $\rho > (1 - \sigma)g + n$ . Using the expression for  $g$  in (22) shows that the latter condition is equivalent to (A1). Similarly, using  $\dot{\lambda}/\lambda = -\sigma g$  and the fact that  $a$  grows with rate  $g$  in the long run, we find that also (56) holds if  $\rho > (1 - \sigma)g + n$ . This concludes the proof. ■

## 9.2 Proof of Proposition 2

The current-value Hamiltonian which corresponds to the social planning problem (32) is given by

$$\begin{aligned} \mathcal{H} = & \frac{c^{1-\sigma} - 1}{1 - \sigma} + \lambda_k \underbrace{\left( A^{\frac{\alpha}{\beta-1}} k^\alpha (h^Y)^{1-\alpha} \right)}_{=y} - (\delta_K + n)k - c) + \\ & \lambda_h \left[ \xi (h^H)^\gamma h^\eta - \delta_H h \right] + \lambda_A \bar{\nu} A^\phi \underbrace{N^{1-\theta} \overbrace{(h - h^Y - h^H)}^{=\dot{A}}}}_{=h^A}^{1-\theta}, \end{aligned} \quad (81)$$

$\bar{\nu} \equiv \frac{\nu}{1+\psi}$ , where  $\lambda_k$ ,  $\lambda_h$  and  $\lambda_A$  are co-state variables associated with constraints (30), (4) and (31), respectively. Necessary optimality conditions are  $\partial\mathcal{H}/\partial c = \partial\mathcal{H}/\partial h^H = \partial\mathcal{H}/\partial h^Y = 0$  (control variables),  $\dot{\lambda}_z = (\rho - n)\lambda_z - \partial\mathcal{H}/\partial z$  for  $z \in \{k, h, A\}$  (state variables), and the corresponding transversality conditions. Thus,

$$\lambda_k = c^{-\sigma} \quad (82)$$

$$\lambda_h \xi \gamma (h^H)^{\gamma-1} h^\eta = \lambda_A (1 - \theta) \bar{\nu} A^\phi N^{1-\theta} (h^A)^{-\theta}, \quad (83)$$

$$\lambda_k (1 - \alpha) A^{\frac{\alpha}{\beta-1}} k^\alpha (h^Y)^{-\alpha} = \lambda_A (1 - \theta) \underbrace{\bar{\nu} A^\phi N^{1-\theta} (h^A)^{-\theta}}_{=\dot{A}/h^A}, \quad (84)$$

$$\frac{\dot{\lambda}_k}{\lambda_k} = \rho - \frac{\alpha y}{k} + \delta_K, \quad (85)$$

$$\frac{\dot{\lambda}_h}{\lambda_h} = \rho - n - \xi (h^H)^\gamma \eta h^{\eta-1} + \delta_H - \frac{\lambda_A}{\lambda_h} (1 - \theta) \bar{\nu} A^\phi N^{1-\theta} (h^A)^{-\theta}, \quad (86)$$

$$\frac{\dot{\lambda}_A}{\lambda_A} = \rho - n - \frac{\lambda_k}{\lambda_A} \frac{\alpha}{\beta-1} A^{\frac{\alpha}{\beta-1}-1} k^\alpha (h^Y)^{1-\alpha} - \phi \frac{\dot{A}}{A} \quad (87)$$

$$\lim_{t \rightarrow \infty} \lambda_{z,t} e^{-(\rho-n)t} z_t = 0, \quad z \in \{k, h, A\}. \quad (88)$$

( $\lambda_{z,t}$  denotes the co-state variable associated with state variable  $z$  at time  $t$ .)

We exclusively focus on the long run. In steady state, with  $h^A$  being stationary,  $A$  must grow with rate  $g_A$ . Moreover,  $y$ ,  $k$ , and  $c$  must grow at the same rate  $g$ , if  $h^Y$  is stationary. Differentiating (82) with respect to time, we obtain

$$\frac{\dot{\lambda}_k}{\lambda_k} = -\sigma \frac{\dot{c}}{c} = -\sigma g, \quad (89)$$

where we used  $\dot{c}/c = g$  for the latter equation. Combining (89) with (85) implies a capital output ratio

$$\frac{k}{y} = \frac{\alpha}{\rho + \delta_K + \sigma g}. \quad (90)$$

Next, differentiate (83) with respect to time to find that in steady state, under a stationary allocation of human capital,

$$\frac{\dot{\lambda}_h}{\lambda_h} = \frac{\dot{\lambda}_A}{\lambda_A} + g_A \quad (91)$$

holds, where we used  $\dot{A}/A = g_A$ ,  $\dot{N}/N = n$  and the fact that  $(1 - \theta)n = (1 - \phi)g_A$ , according to (21). Making use of the same properties, differentiating (84) with respect to time leads to

$$\frac{\dot{\lambda}_k}{\lambda_k} + \left( \frac{\alpha}{\beta-1} - 1 \right) g_A + \alpha g = \frac{\dot{\lambda}_A}{\lambda_A}. \quad (92)$$

Using (89) and the definition of  $g$  in (22), we can rewrite (92) to

$$\frac{\dot{\lambda}_A}{\lambda_A} = (1 - \sigma)g - g_A \quad (93)$$

and thus, according to (91),

$$\frac{\dot{\lambda}_h}{\lambda_h} = (1 - \sigma)g. \quad (94)$$

Moreover, substituting the right-hand side of (83) into (86) as well as using both (94) and the fact that  $\xi (h^H)^\gamma h^\eta = \delta_H h$  when  $\dot{h} = 0$ , eventually confirms the expression for  $h^H/h$  in (33).

Next, rewrite (84) to

$$\frac{\lambda_k}{\lambda_A} = \frac{(1 - \theta) \frac{\dot{A}}{A} \frac{h^Y}{h^A}}{(1 - \alpha) A^{\frac{\alpha}{\beta-1}-1} k^\alpha (h^Y)^{1-\alpha}}. \quad (95)$$

Substituting (95) into (87) and using  $\dot{A}/A = g_A$  together with the definition of  $g$  in (22) leads to

$$\frac{\dot{\lambda}_A}{\lambda_A} = \rho - n - (1 - \theta)g \frac{h^Y}{h^A} - \phi g_A. \quad (96)$$

Combining (96) with (93) and using the fact that  $(1 - \phi)g_A = (1 - \theta)n$  leads to

$$\frac{h^Y}{h^A} = \frac{\rho - \theta n + g(\sigma - 1)}{(1 - \theta)g} = \Gamma. \quad (97)$$

Using  $h^Y = h - h^A - h^H$  then confirms the expression for  $h^A/h$  in (34).

To confirm the socially optimal savings and investment rate ( $sav = 1 - c/y$ ) as well, note from (30) that

$$sav = \left( \frac{\dot{k}}{k} + \delta_K + n \right) \frac{k}{y}. \quad (98)$$

Using  $\dot{k}/k = g$  and expression (90) for  $k/y$  confirms (35).

Finally, it is easy to see from (89), (94) and (93) that, under assumption (A1), transversality conditions (88) hold for  $k$ ,  $h$  and  $A$ , respectively (using  $\dot{k}/k = g$ ,  $\dot{h} = 0$  and  $\dot{A}/A = g_A$ ). This concludes the proof. ■

### 9.3 Consumption-equivalent change in intertemporal welfare - derivation of $\Theta$

First, adjust per capita consumption to  $\tilde{c} \equiv cA^{-\frac{\alpha}{(1-\alpha)(\beta-1)}}$ , which is stationary in the long run (Proposition 1). Moreover, denote the change in life-time utility by  $\Delta U$  and the (hypothetical) permanent change in adjusted steady per capita consumption by  $\Delta \tilde{c}$ . Initially, there is an adjusted steady consumption stream  $\tilde{c}_0$ , as we start from an initial balanced growth path. Then we have

$$\Delta U = \int_0^\infty \frac{((\tilde{c}_0 + \Delta \tilde{c})e^{gt})^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho-n)t} dt - \int_0^\infty \frac{(\tilde{c}_0 e^{gt})^{1-\sigma} - 1}{1 - \sigma} e^{-(\rho-n)t} dt \quad (99)$$

which we can solve to find

$$\Theta \equiv \frac{\Delta\tilde{c}}{\tilde{c}_0} = \frac{(\tilde{c}_0^{1-\sigma} + \Delta U(\sigma - 1)(g(1 - \sigma) + n - \rho))^{\frac{1}{1-\sigma}}}{\tilde{c}_0} - 1. \quad (100)$$

We numerically find  $\tilde{c}_0$  under the status quo policy and obtain the change in welfare  $\Delta U$  which results from a policy reform. In turn, we get  $\Theta = \Delta\tilde{c}/\tilde{c}_0$  from (100).

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