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**REMARKS ON THE USE OF MATHEMATICS IN ECONOMIC THEORY**  
**THE EXAMPLE OF THE APPROPRIATE MACROECONOMIC FOUNDATIONS OF**  
**FINANCIAL ANALYSIS**

In the mathematics of finance it is, as a rule, admitted by theoreticians and practitioners of banking that the returns of shares are normally distributed. The underlying macro-model is of the conventional neoclassical type. Here, the rationality of the individuals coincides with the rationality of the system. This accounts for conventional results, that is, the normal distribution of prices and earnings in the standard mathematics of finance. Now, various writers, Mandelbrot [1-5] for instance, have found that this is not the case. In fact, earnings distributions may be heavily skewed in both directions and have so-called heavy tails. This phenomenon can be explained only through looking for an alternative macroeconomic theory underlying financial analysis. In this paper, it is suggested that unconventional results, that is, skewed distributions of earnings, can be explained by a classical-Keynesian model exhibiting the interaction between the real and the financial sector of an economy. These unusual results occur because in classical-Keynesian models the rationality of the economic system is opposed to the rationality of the economic agents. Keynes's paradox of thrift is a telling example.

The paper is divided into three sections. First, the basic neoclassical view is presented very briefly, and its relationship to the normal distribution of earnings is established. The second section presents a fundamental critique of the neoclassical model. In the third place, a classical-Keynesian theoretical framework, comprising the real and the financial sector, is shown to produce the abnormal results in the mathematics of finance. In the concluding remarks some suggestions on the appropriate use of mathematics in economic theory are made.

## I. Neoclassical models and the normal distribution of earnings

There are various types of neoclassical models supposed to picture the functioning of free-market economies. The General Equilibrium Model, the Partial Equilibrium Model and the Rational Expectations System are particularly important. The Rational Expectations Model postulates that economic agents make use of the best possible economic theory to forecast prices and hence earnings. Market prices are supposed to contain all the relevant information required to make forecasts and to take decisions. Each agent has his own forecasts, which may deviate from the prices, which will come to be realised. However, estimated prices and thus estimated earnings will be normally distributed around the realised prices and earnings.

This is so because in the Rational Expectations System the economy is always in equilibrium. Output and employment are *given* and at their *full employment levels* in the sense that a *natural level of unemployment* prevails, comprising structural or voluntary unemployment. The equilibrium is brought about by smoothly functioning markets. On the macroeconomic level, the supply curve reflects resource restrictions, and the demand curve relies on the Real Balance Effect. With the quantity of money given, the macroeconomic demand, that is, the demand of all goods contained in the social product increases if prices decline in a situation of excess supply, and vice versa.

In the mathematics of finance this Rational Expectations procedure is also applied to particular industries and firms. Most importantly, however, the normal distribution of earnings is also applied to shares of large corporations or of industries. Share prices are supposed to reflect the value of the underlying enterprises. Earnings are the indicator of this value.

An important implication of neoclassical models in general, and of the rational expectations model in particular is that saving is always fully invested. The market for new capital goods brings saving and investment into equilibrium at the full employment level through variations of the rate of interest. The Walrasian model indeed reflects a perfectly self-regulating economy, implying that there is always a strong tendency towards equilibrium. The Rational Expectations System is based upon the very far-reaching assumption that a competitive market economy is always in equilibrium. As already suggested, this assumption seems to be implied also in the mathematics of finance associated to the postulate of the normal distribution of earnings.

In fact, stocks and flows of the real and the financial sector represent a system of interrelated markets. This market system results in a temporary general equilibrium, which is associated to differing profit rates in the various markets, that is, for the various sectors and industries located on

the supply side. The differences of earnings between markets give rise to differences in the investment strategies for hedge funds, for example.

## **II. A fundamental critique of the neoclassical model: there is no tendency towards a full employment equilibrium**

Even if there are competitive conditions, there is no necessary tendency towards a full-employment equilibrium in a market economy. This point has been argued convincingly by the neo-Ricardian economists following up the publication of Piero Sraffa's *Production of Commodities by Means of Commodities* in 1960 in the course of the capital theory debate (Harcourt 1972).

This implies that factor markets do not exist if the process of production is a social and circular process in the sense of Wassily Leontief (who deals with quantities) and Piero Sraffa (determination of prices of production and of distribution). In fact, there are no well-behaved associations between factor prices and factor quantities, that is, there are no well-behaved demand curves for the factors of production. The capital-theoretic critique implies that lower interest rates are not necessarily associated with more capital, higher capital-output ratios and higher capital-labour ratios, and vice versa. Given this, the capital-theoretic critique implies that market prices do no longer convey the right signals, which would lead an economy towards a full employment equilibrium.

In the course of the capital theoretic critique (Harcourt 1972) the neo-Ricardian critiques have argued that heterogeneous capital goods cannot be measured in physical units. Real capital must be valued, in money terms for example. To be able to fix prices for real capital goods (machines, for example), money wages and the rate of profits must be known. However, according to neoclassical theory the rate of profits is an unknown to be governed on the market for new capital goods. This contradiction can only be discarded through postulating that the rate of profits is institutionally determined, for example through social and political forces involved in the process of income distribution, entrepreneurial associations and trade unions. In fact, as Joan Robinson (1956) has insisted upon, an economy simply cannot get into equilibrium because the equilibrium profit rate must prevail for some period of time. Indeed, the rate of profits used to evaluate the money value of a machine and the profit rate put to use to discount the gross earnings (profits), in terms of money again, must be the same in equilibrium. The price of a machine must equal the present value of its earnings.

Given all this, neoclassical theory, especially the rational expectations system, cannot take appropriate account of the role of money in an economy. The quantity of money is simply supposed to govern the level of absolute prices. Moreover, saving is always supposed to be invested. Given this, the differing roles of money circulating in the real and in the financial sector, and the interaction between the two sectors, are not taken account of in neoclassical economic theory. Finally, neoclassical theory cannot deal with the great crises, which have occurred in the history of capitalist development, that is, the influence of money in the form of effective demand on economic activity.

Important implications regarding neoclassical theory arise of this critique. Prices are no longer governed in the sphere of exchange through supply and demand. In fact, the fundamental prices are governed by the conditions of production and by the distribution of incomes, that is, the structure of money wages and the rate of profits. Quantities are governed by effective demand, for the economy as a whole and for individual sectors of the economy. Money is no longer neutral. In fact, in the financial sector banks can create money through the monetary financing of investment. Once investment is given, saving adjusts to investment through quantity variations. In fact, saving secures the real financing of investment (I) through making available the resources required to produce (I).

The critique of neoclassical theory clears the way for establishing the classical-Keynesian system of political economy, which is very broadly sketched in the next section.

### **III. A classical-Keynesian theoretical framework and the skewed distributions of earnings**

We first sketch how the real sector of an economy functions in principle to govern prices and quantities. In fact, classical-Keynesian long-period theory pictures the functioning of the institutional-technical system (Bortis 1997/2006 and 2007, chapter 4; and Bortis 2003) governing trend or long-period output and employment. Government expenditures, consumption and exports are shown to be the main determinants of trend output. Very importantly, the demand for consumption goods increases if the distribution of incomes is more equal. Like consumption, trend investment appears as derived demand. The prices of production are governed by the conditions of production and by income distribution. In principle, these prices emerge from the normal cost calculation within enterprises.

Subsequently, in the course of the business cycle (Bortis 1997/2006, pp. 204-20), the behaviour of entrepreneurs is coordinated by the system, that is, by effective demand. Two effects are

important. The income effect governs the upswing and the downswing of the cyclical movement, the capacity effect governs the turning points of the cycle. The income effect is based upon the cumulative post-Keynesian interaction between investment and profits. Higher profits result in higher investment levels in the upswing, and more investment means higher profits. This cumulative process drives the economy upwards. However, high investment levels means rapidly growing productive capacities. The higher output gradually depresses prices and profits. Once profits fall below the institutionally governed normal level, the economy changes in direction. Investment decline and bring about a further decline of profits. A downward spiral is initiated. The profit-investment mechanism now works in the opposite direction. Reduced profits and lower investment levels mutually reinforce each other.

The financial sector eventually reinforces the amplitude of the cycle. In fact, the banking system has the capacity to create credit money. In the upswing bankers expand the credit volume rapidly, enhancing thus the cumulative interaction between investment and profits. And, vice versa, in the downswing, banks restrain credits, enhancing thus the decline of investments and profits.

The impact of the business cycle on the distribution of earnings is now self-evident. In the business upswing the distribution of prices and earnings will be skewed to the right, and skewed to the left in the downswing. This is essentially due to the interaction between profits and investments, that is, the income effect of investment. This conclusion holds for long-period (Kondratiev) cycles as well as for medium-term (Juglar) cycles.

We are now in a position to examine a second mechanism, which might contribute to the distribution of earnings becoming increasingly skewed to the right. This factor is linked with the increasing inequality of incomes which has occurred in the 1970s and, increasingly, since the 1980s. The interaction between the financial and the real sector is crucial in this context. The problem is that more and more money is accumulated in the financial sector. This represents the amount of saving, which is no longer required to finance investment, part of which is in fact financed by bank credit. This results in the creation of new money. Given this, the part of saving no longer required to finance new investments precisely corresponds to the credit and money creation by banks ( $\Delta M$ ). This means that wealth in money form accumulates in the financial sector. A large part of this money becomes financial capital looking for investment possibilities. Some of these investments are certainly positive in that they lead on to an improved performance of the enterprises, which are acquired by financial capital. In many cases, however, the unique aim of a takeover is to realise higher profits. Indeed, profits have increased considerably at the expense of

wages in the last decades. This might also be an important factor, which contributes to making the distribution of earnings more skewed to the right in the upswing of the business cycle, and less skewed to the left in the downswing. This proposition would have to be tested empirically.

This mechanism can be understood better if the macroeconomic significance of saving is examined more closely (Bortis 2009). Here the distinction between *saving* and *finance* is crucial. Banks provide finance. This is the *monetary* financing of investment (I) through bank credits (B), which include financial means of the enterprises themselves; hence  $B = I$ . Banks provide credits on the basis of excess reserves represented by saving. Indeed, the saving of households and enterprises (S) appear on the debit (liabilities) side of bank accounts and as excess reserves on the credit (assets) side. Now, the banks must hold a fraction (r) of the credits (B) as reserves for practical or legal reasons; as a rule (r) is rather small (0.1, 0.05, or, in practice even less) which means that the potential of money creation through the banking system is enormous. Since investment is limited by effective demand, the banking system remains with excess reserves amounting to  $(S - rB)$ . This large part of saving corresponds to the newly created money ( $\Delta M = S - rB$ ). The amount  $\Delta M$  is now no longer needed to finance investment (I), as is governed by expectations in the short-term, the profit-investment mechanism in the medium term and by the institutional system in the long term. In fact, due to the capacity of the banking system to create money, the financial resources available, saving S and newly created money  $\Delta M$ , necessarily exceed investment (I) which equals saving (S):  $(\Delta M + S > I)$ .

However, in the real sector saving (S) must always equal investment (I):  $S = I$ . Indeed, with I determined, saving ( $S = s Y = s Q$ ) adjusts to investment through changes in output and employment ( $I = s Y = s Q$ ). This is the real financing of investment: saving makes available the resources (labour, capital equipment and land) required to produce investment goods. Hence since investment (I) must always equal saving (S), part of saving equivalent to  $\Delta M$ , will flow into the financial sector:  $\Delta M + (S - \Delta M) = I$ .

Now, since  $S = I$ , saving (S) and investment (I), taken by themselves, do not affect the quantity of money, which circulates in the real and the financial sector respectively. In fact, saving (S) leaves the real sector and moves to the financial sector in the form of saving and term deposits; on the other hand, an amount of money equal to investment (I) leaves the financial sector for the real sector: hence  $fM = fM + (S - I)$  and  $rM = rM + (I - S)$ ; here, (f) is the fraction of the quantity of money (M) which is in the financial sector, and r the fraction of money in the real sector ( $M = fM + rM$ ) and  $(f + r = 1)$ . Hence it is the money created by the banks in excess of saving ( $\Delta M$ ), which

moves from the real to the financial sector, bringing about a rise of the ratio of money in the financial sector to money in the real sector ( $f/r$ ). The approximate present size of this ratio is impressive. Indeed in the last thirty years, the quantity of money  $M$  in the real sector ( $rM$ ) has been multiplied by *four*, the quantity of money in the financial sphere ( $fM$ ) by *forty*! A large part of ( $fM$ ) may be considered profit-seeking financial capital. This fact goes far to explaining the rise of land prices, houses, gold, old masters, and, very importantly, share prices. As a consequence *profits have to increase absolutely* to ensure an *acceptable profit rate*. This, as has been suggested in the above, leads, as a rule, to an increasingly unequal distribution of incomes (and wealth). As a consequence, the spending power of the population diminishes, which, in turn, results in reduced effective demand and in an increase of persistent involuntary unemployment. This link between income distribution and employment is perhaps the most important aspect of the relationship between the real and the financial sector. Moreover, the banking system has the capacity to expand very quickly the credit volume in the cyclical upswing, enhancing thus the interaction between profits and investment, which is basic to the income effect of investment; in the downswing banks tend to reduce credits, accelerating thus the downswing. The financial sector thus results in making cyclical movements more pronounced. All this explains, at least in part, a famous proposition by Keynes: “Depressions arise when money is shifted from the industrial circulation [the real sector] to the financial circulation [the financial sector]” (R. Skidelsky 1992, vol. II, p. xxiv). The main reason is that money circulating in the financial sector has no value equivalent, but is constantly seeking such an equivalent through profit-seeking or value-preserving investment. (In the real sector, however, money always *represents* a value equivalent given by direct and indirect labour and by primary, intermediate and final goods.)

A second postulate related to Keynes’s monetary theory is about the link between saving and income distribution on the one hand and output and employment on the other: «[the Keynesian doctrine] can easily be made to say both that ‘who tries to save destroys real capital’ and that, via saving, ‘the unequal distribution of income is the ultimate cause of unemployment’. *This* is what the Keynesian revolution amounts to» (Schumpeter 1946, p. 517). Both propositions can be explained by  $I = sQ$ . Saving ( $S = sQ$ ) adjusts to investment ( $I$ ) through variations in output ( $Q$ ) and employment ( $N$ ), since  $Q = AN$ , where  $A$  is labour productivity. Now, if due to a more unequal income distribution the saving/income ratio  $s = S/Y$  increases, output  $Q$  will diminish. In a second step investment ( $I$ ) might recede. As already suggested, a downward spiral may come into being, implying that increased saving ( $s$  rises) may destroy real capital. Hence an increasing saving ratio

(s), due to income distribution becoming more unequal, will reduce output and employment. In a Keynesian vein, an unequal income distribution thus emerges as the basic cause of unemployment. All this confirms Ricardo's insight: The distribution of income and wealth is the most important and the most fundamental problem in political economy. This is all the more true as, in a Sraffian vein, the – fundamental - prices of production depend upon the condition of production and on income distribution (Pasinetti 1977, pp. 72-73). Indeed, in Sraffa's model income distribution, that is, the real wage rate *or* the real wage rate in terms of some good may be chosen outside the technical relations of production within the social relations of production.

### **Conclusions: Suggestions on the use of mathematics in economic theory**

In complex macroeconomic matters, mathematics may be very useful to detect problems, that is, to discover *quantitative* particularities of phenomena, for instance the fact that earnings are not normally distributed. However, to interpret and to explain these particularities, the *qualitative* aspect of these phenomena has to be considered. This requires very solid economic theory, dealing not only with the economic system as does neoclassical Walrasian General Equilibrium Theory and the Rational Expectations System, but with the entire socio-economic and political system as is the case with classical-Keynesian political economy. It is impossible to discriminate between the two approaches by mathematical and empirical means (econometrics and statistics). Theoretical arguments must decide which approach is more plausible. Here the capital-theoretical debate is crucially important (Harcourt 1972). The outcome of this debate was entirely in favour of classical-Keynesian political economy, which gradually emerges as the economic theory of the future. Indeed, in the above it has been suggested that the skewed distribution of earnings can be explained on the basis of conceptions taken from classical-Keynesian political economy.

On the empirical level one cannot proceed in this way, that is, attempting to establish the most plausible approach. Indeed, in the spirit of Popper, econometric methods may only be used to empirically *falsify* some mathematically formulated theoretical hypotheses, based on some economic theory. Given this, it is possible to empirically support specific theoretical hypotheses, like the existence of equilibrium relations that can be found between economic variables through cointegration analysis in multiple time-series analysis.



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