Influence of social networks on spatial diffusion of innovation

sous la direction du Prof. Thierry Madiès
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Summary

This study analyzes the relation between social networks and distance, and their influence on the spatial diffusion of innovation. It is composed of two parts: a theoretical one and an empirical one. In the first part, we examine the formation of social networks and determine which kinds of networks structures are stable and efficient. The empty, the star and the complete network are stable and efficient. Then, we characterize three particular network structures (the regular graphs, the small worlds and the random graphs) and determine which one fosters more knowledge creation and/or diffusion. The small world network structure promotes the most knowledge creation and diffusion, and thus maximizes the average long run knowledge level. In the second part, we analyze the influence of social networks on the spatial spread of innovation. We examine in detail three empirical studies on this problematic and conclude that innovation is spatially concentrated as social networks are. Finally, we briefly examine the influence of New Information and Communication Technologies (NICT) on the spatial diffusion of innovation. We find that the diffusion of innovation remains concentrated in space.
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List of abbreviations

Abbreviations from equations and models are not taken into account in the list of abbreviations.
SFP: Fifth Framework Programme
b.e.: backward eliminated
chi-sq.: chi-square
CRENoS: Centro Ricerche Economiche Nord Sud
df: degrees of freedom
EAE: French Annual Company Survey
e.g.: exempli gratia
EPO: European Patent Office
et al.: et alii
EU: European Union
i.e.: id est
INSEE: Institut National de la Statistique et des Etudes Economiques
IPC: International Patent Classification
ISI: Institute of Scientific Information
Km: Kilometer
NICT: New Information and Communication Technologies
Non-zero corr: Non-zero correlation
NUTS: Nomenclature des Unités Territoriales Statistiques
Obs.: Observations
OLS: Ordinary Least Squares
OR: Odds ratio
OST: Observatoire des Sciences et des Techniques
n.: numbers
R&D: Research and Development
SCI: Science Citation Index
ULP: University Louis Pasteur
vs.: versus
Introduction

The purpose of this study is to explain the influence of social networks (i.e. innovation communities) on the spatial diffusion of innovation. More precisely, we will analyze whether geographic proximity and social interactions favor spatial diffusion or spatial concentration of knowledge. This question is of great importance as governments and other institutions want to foster innovation and, consequently, have to know whether innovation diffuses globally in space, and what is the role of social networks in this problematic.\footnote{In this study, both the notion of innovation and the notion of knowledge have the same meaning and implications, but this is not always true for other studies.} More precisely, if innovation is highly concentrated in space, regions that benefit from innovative activities will develop faster and better than regions in which innovation is not present. Consequently, this will increase disparities in the economic growth of distinct regions of a country (or other geographical areas). On the contrary, if innovation diffuses globally in space, there will be an equal economic development between regions.\footnote{This is a simplified assumption under which the economic development of a region only depends on innovative activities. However, in the reality, the economic development of a region or a country depends on many other factors (e.g. technical infrastructures, political stability, quality of the education system, etc.). Hence, in spite of the fact that innovation can be high and diffuses globally in space, there can be some disparities in the economic development across regions, because of a possible lack of one of the factors described above.} Hence, we can now well understand the importance for governments and other institutions to know how innovation diffuses in space and what the influence of social networks is in order to apply the adequate economic policies which do not generate disparities within their countries, but which foster innovation rapidly and for a long time.

This study is elaborated on the basis of theoretical and empirical studies. Theoretical studies are mostly utilized for the first part of this study as it analyzes the formation of social networks and links with innovation in a theoretical way. Empirical studies on the influence of social networks on the spatial diffusion of innovation are examined to elaborate the second part of this study as it is
essentially empirically oriented. We will now describe the exact structure of the first and second part of this study.

In the first part of this study, we will define what exactly a social network is, how it is constructed, etc. More precisely, we will first briefly describe the sociological and economic theories of social networks. Secondly, we will analyze in detail the construction of networks using a game theoretic approach, and examine which kinds of network structures arise in equilibrium (are stable) and are efficient. In the third place, we will characterize three particular types of network structures, and determine which one fosters more knowledge creation and/or diffusion.

In the second part of this study, we will analyze the relation between social networks and distance, and their influence on the spatial spread of innovation. First, we will define the notions of distance and innovation while focusing on the concepts of tacit and codified knowledge. Secondly, we will analyze in detail the relation between distance and social networks, and their influence on the spatial diffusion of innovation. We will introduce the concepts of technological and pecuniary externalities, co-authorships, and patents citations to examine the influence of social networks on the spatial spread of innovation. In the third place, we will discuss the results achieved in the second part of this study. Finally, we will briefly analyze the influence of NICT on the spatial diffusion of innovation.

To conclude, we can assert that the combination of these two parts brings a complete characterization of what social networks exactly are, and how they influence the spatial diffusion of innovation.
I Social networks theory and innovation

The first part of this study analyzes the formation of social networks in a theoretical way. It examines as well three particular network structures and determines which kind of network structure fosters more knowledge creation and/or diffusion. In fact, the structure of social networks affects the diffusion of information, knowledge, etc. and we will see in which way (Strogatz, 2001, p. 268). To start with, the following chapter briefly describes different approaches to social networks, namely the sociological and economic theories of social networks.

1 Concept of social network

There are many theories of social networks according disciplines (sociology, mathematics, statistical physics, computer science, business strategy, geography, and organization theory) (Goyal, 2007, p. 7). We will discuss now very briefly the sociological and economic approach to social networks.

1.1 Sociological theory

According to Wasserman and Robins (2006), “a social network is a set of \( n \) actors and a collection of \( r \) social relations that specify how these actors are related to one another.” (Wasserman and Robins, 2006, p. 148). In sociology, social structure is a central concept. The social structure is a configuration of social relations and positions, namely it describes a social network (Cook and Whitmeyer, 1992, pp. 109-10). The social structure is commonly viewed “as a pattern of particular ties between actors, where variation in the network in the existence or strength of ties is meaningful and consequential” (Cook and Whitmeyer, 1992, p. 118). Hence, we can assert that the social structure is a
composition of particular links. However, the studies that analyze the formation or maintenance of networks take links as consisting only of exchange of value. But much of the network theorists study a great variety of types of ties. In fact, there is no theoretical condition of the content and type of ties that describe the social relationship between agents (Cook and Whitmeyer, 1992, pp. 118-19, 123). Actually, there is no formal sociological network theory at all. The sociologists use the words “network theory” simply to describe several concepts employed in network analysis or “they are simply postulating certain connections between behavior on the one hand and the characteristic, both morphological and interactional, of the social networks of the people concerned” (Mitchell, 1974, p. 283). The latter point can further be clarified in the sense that the behavior of an agent is interpreted in the way the pattern of links of the network is. Thus, we can assert that transactions are the result of the network structure (Mitchell, 1974, p. 285). To sum up, we can affirm that the social network analysis, in agreement with the “structuralist” view in sociology, states that all significant social phenomena can be explained by social structure (Cook and Whitmeyer, 1992, p. 114).

1.2 Economic theory

The economic theory of social networks formation accentuates the incentives of an agent to form connections with other agents in ways that are beneficial for her. This point of view stresses the idea of rationality, preferences and knowledge of agents. They will make a trade-off between the costs and benefits of forming connections. As a result, links creation has strategic aspects. In fact, links formation will influence the payoffs of the agents concerned and also of other agents, i.e. create externalities, in ways that are responsive to the structure of the network. This implies that a change in the structure of the network will have a great influence on individual’s payoffs and behavior. It is also worth noting that considerations of social efficiency are significant in the economic theory of social networks. We can conclude that, in this economic approach, a network is a
collection of nodes and links that influence, and are influenced by, strategic behavior of agents (Goyal, 2007, pp. 7, 25, 143-46).

The next two chapters will be based on the economic theory of social networks. In chapter 2, we will present a basic model of strategic networks formation with quantitative aspects. In chapter 3, we will analyze which kind of network structure fosters knowledge creation and/or diffusion.

2 Strategic networks formation: a game theoretic approach

This chapter presents a model of networks formation based on a game theoretic approach which accentuates the concepts of equilibrium, stability, efficiency and, to a lesser extent, equity. Both static and dynamic features will be discussed. This model is built on the economic theory introduced above. Nevertheless, this chapter analyzes the formation of network without an explicit link with innovation. The structures of social networks and links with innovation will be presented in the next chapter.

The purpose of this model is to reach stable and efficient networks. A network is said to be stable when there are no more incentives for agents to delete existing links, and for unconnected agents to create new links and modify the network

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Another literature on networks formation exists. It corresponds to an evolutionary approach and it is the opposite of a game theoretic approach on strategic networks formation. We will very briefly characterize this evolutionary approach. In this theory, every agent is connected to every other individual, but the links are chosen probabilistically. This probability depends on whether the link concerned had importance in the past. Thus, if an agent had positive experience with another agent, the probability that she will connect with her again will increase. Moreover, since agents can gain experience with distinct links, the probability of activating them will change. Although models within the evolutionary approach do not have a standard baseline, they mostly conclude that networks evolve forever, but the latter can also sometimes settle down and exhibit aggregate regularities. We will not develop further this approach as our study will present in detail the game theoretic approach. We briefly presented this evolutionary approach so that you know it exists another theory on the networks formation as the game theoretic approach (Cowan, 2005, pp. 44-46).
A network is efficient, i.e. socially desirable, when the payoffs of the network are higher than the payoffs of another network, and/or when the network maximizes aggregate welfare (defined as the addition of individual payoffs). It is important to note that the game on networks formation will focus on single- or two-agent incentives, because it is far too complicated to analyze incentives concerning links formation among large groups. Indeed, networks are subjected to subtle and rapid transformations via local linking and de-linking activities. Therefore, as networks are complicated objects, it is not possible to analyze incentives and coordination among large groups of individuals (Goyal, 2007, pp. 144, 158, 170).

We will now describe a static game with one-sided link formation and two-sided link formation, and define a Nash equilibrium. We will then present a dynamic game with one-sided link formation and two-sided link formation, and discuss the relation between absorbing networks and Nash equilibrium networks (Goyal, 2007, pp. 144-45).

2.1 Static

In this section, we will analyze networks formation in the short run. The same analysis for the long run will be provided in the dynamic section. In the first static subsection, we will analyze a simple model of networks formation with one-sided link formation. We will discuss this model with two-sided link formation in the second static subsection.

2.1.1 One-sided link formation

In this model, there are \( n \) agents and each of them can create links unilaterally with any subset of other agents. The links are one-sided in the sense that they are created on the initiative of one agent which incurs the costs of links creation. All
agents are homogenous in terms of value and costs of forming connections.\footnote{This assumption is very strong; in the reality, agents are heterogeneous. Other theoretical works include this feature in their models of networks formation, but we will not treat these works in this study. We will also not introduce competition among agents or effects of congestion. Nevertheless, in different contexts, these features are important (Goyal, 2007, pp. 163, 185). If you want information about studies that incorporate competition among agents, you can read Goyal, S. and Joshi, S. (2002) “Networks of collaboration in oligopoly”, Games and Economic Behavior, 43, pp. 57-85.} The payoffs to an agent depend on his own links, but also on the links that other agents form. Another important assumption is that the value can flow without any friction in a network.\footnote{In many contexts, however, the value flow is negatively affected by several factors; one of them being the distance between agents (Goyal, 2007, p. 164).} Finally, an agent can be a member of several groups or networks at the same time. So what would be the structure of the resulting network (Goyal, 2007, pp. 146-47, 163-65)?

Here is a clarification of the notations of this model.\footnote{These notations also hold for the next sections.} \(N = \{1, \ldots, n\}\) with \(n \geq 3\) is the set of agents and \(i\) and \(j\) are members of this set. A strategy of the agent \(i\) is 
\[
s_i = (s_{i1}, \ldots, s_{ii-1}, s_{ii+1}, \ldots, s_{in}),\]
where \(s_{ij} \in \{0,1\}\) for each \(j \in N \setminus \{i\}\). The agent \(i\) has a link with \(j\) only if \(s_{ij} = 1\). The set of (pure) strategies of the agent \(i\) is 
\[
\mathcal{S}_i = \{0,1\}^{n-1}.
\]
A strategy profile for all agents is \(s = (s_1, \ldots, s_n)\), with the set of all strategies being 
\[
\mathcal{S} = \prod_{i=1}^n \mathcal{S}_i.
\]
There is a similarity between a strategy profile and a directed network.\footnote{According to Gross and Yellen (1999), “A directed graph (or digraph) is a graph each of whose edges is directed.” (Gross and Yellen, 1999, p. 3). And, according to Koehly and Pattison (2006), in a directed graph, a vertex corresponds to the sender of a link, and is connected by a directed arrow or an arc to another vertex which corresponds to the receiver of the link (Koehly and Pattison, 2006, p. 164).} \(\mathcal{G}\) is the set of directed networks on \(n\) nodes.

\[
N_i^d(g) = \{j \in N \mid g_{ij} = 1\}
\]
is the set of agents with whom agent \(i\) creates a link and 
\[
\eta_i^d(g) = |N_i^d(g)|
\]
is the number of connections of agent \(i\) in the network \(g\). Similarly, 
\[
N_{-i}^d(g) = \{j \in N \mid g_{ji} = 1\}
\]
is the set of agents who create a link with agent \(i\) and 
\[
\eta_{-i}^d(g) = |N_{-i}^d(g)|
\]
is the number of agents who create links with agent \(i\). \(\eta_i^d(g)\) is the out-degree\footnote{The degree of a node in a network is the number of links incident with that node (Harary, 1972, p. 14). In other words, it is the number of direct connections of that node (Goyal, 2007, p. 12).} and \(\eta_{-i}^d(g)\) is the in-degree of the agent \(i\) in the network \(g\). In the directed network \(g\), \(\mathcal{N}_i(g) = \{k \mid i \xrightarrow{g} k\}\) is the set of agents to
whom agent \(i\) has a path.\(^9\) We assume that an agent \(i\) can access herself, thus the number of individuals the agent \(i\) can reach is \(\eta_i(g) \equiv |N_i(g)| + 1\) (Goyal, 2007, p. 147).

As an example of a static game with one-sided link formation, we will discuss the two-way flow model.\(^{10}\) The latter describes a networks formation game in which the links are created unilaterally, but the flow of advantages is independent of who supported the cost for the link. In this situation, there are advantages of accessing a large number of people, but links are costly to sustain. \(\mathbb{Z}_+\) is the set of nonnegative integers. \(\phi : \mathbb{Z}_+^2 \to \mathcal{R}\) is such that \(\phi(x,y)\) is strictly increasing in \(x\) and strictly decreasing in \(y\). Each agent’s payoff function \(\Pi_i : \mathcal{G} \to \mathcal{R}\) is

\[
\Pi_i(g) = \phi(\hat{n}_i(g), \eta_i^d(g)). \tag{2.1.1.1}
\]

\(\hat{n}_i(g)\)\(^{11}\) is the number of agents that are reached by the agent \(i\) in the undirected network\(^{12}\) \(g\), and \(\eta_i^d(g)\) is the number of links created by the agent \(i\) in the network \(g\). Given the above properties of \(\phi\), \(\hat{n}_i(g)\) can be understood as the benefits that the agent \(i\) receives from the network (e.g. increasing the number of agents accessed by paths amplifies payoffs), and \(\eta_i^d(g)\) can be interpreted as the costs of sustaining links that reduce payoffs. Therefore, the hypothesis on the payoff function \(\phi(\cdot,\cdot)\) allow for increasing and decreasing marginal returns from

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\(^9\) A path is a trail in which every node or point is different. A trail is a walk in which every link is distinct (Bollobás, 2002, p. 5; Goyal, 2007, p. 14 and Harary, 1972, p. 13). Finally, a walk is a sequence of nodes in which two consecutive nodes have a link in the network. Thus, the length of a path between \(i\) and \(j\) is the number of links that are in the path (Goyal, 2007, pp. 14, 91).

\(^{10}\) This two-way flow model is due to Bala, V. and Goyal, S. (2000) “A non-cooperative model of network formation”, Econometrica, 68, pp. 1181-230. This model assumes that the flow of benefits between two agents depends only on the existence of a path, but not on the length of the path, and that all agents have the same payoff functions (the agents are symmetric). It is important to briefly mention these hypotheses, because the two-way flow model can also be analyzed with the introduction of heterogeneity and decay (Goyal, 2007, p. 171). We will not introduce heterogeneity and decay in our analysis. However, we will introduce the notion of decay in the next two-sided link formation section.

\(^{11}\) \(\hat{n}_i(g)\) can be explained as follows. \(\hat{g} = \max\{g_{ij}, g_{ji}\}\) captures the two-way flow idea, and thus we can simply rename the concepts of neighbors and paths. \(\hat{N}_i^d(g) = \{i \in \mathcal{N} \mid \hat{g}_{ij} = 1\}\) and \(\hat{\eta}_i^d(g) = |\hat{N}_i^d(g)|, \hat{N}_i(g) = \{k \mid i \rightarrow k\}\) and \(\hat{n}_i(g) \equiv |\hat{N}_i(g)| + 1\) (Goyal, 2007, p. 148).

\(^{12}\) An undirected network has the following definition: it is “a graph for which the relations between pairs of vertices are symmetric, so that each edge has no directional character (as opposed to a directed graph). Unless otherwise indicated by context, the term “graph” can usually be taken to mean “undirected graph.”” (Weisstein, E.W. “Undirected Graph”, from MathWorld—A Wolfram Web Resource).
linking activity (Goyal, 2007, pp. 147-48, 165-66). Hence, it is important to underline the following key assumption:

The function $\phi(\hat{r}_i (g), \eta_{i}^{d} (g))$ increases strictly in $\hat{r}_i (g)$ and decreases strictly in $\eta_{i}^{d} (g)$  \hspace{1cm} (2.1.1.1)

(Goyal, 2007, p. 166).

What is the structure of networks that will arise in equilibrium in this example, and more generally under static games with one-sided link formation? These kinds of games can be solved with the concept of Nash equilibrium strategies. A strategy profile $s^* = (s^*_i, ..., s^*_n)$ is a Nash equilibrium in the network $g$, if $s^*_i$ maximizes the payoffs of each agent $i$, given the strategy profile of all individuals other than agent $i$ ($s^*_{-i}$). A strategy profile $s^* = (s^*_i, s^*_{-i})$ is a Nash equilibrium in the network $g$ if:

$$\Pi_i(s^*_i, s^*_{-i} | g) \geq \Pi_i(s_i, s^*_{-i} | g).$$  \hspace{1cm} (2.1.1.2)

If the payoffs follow the equation 2.1.1.1 and they satisfy the assumption 2.1.1.1, then a Nash network will be either minimally connected$^{13}$ or empty.$^{14}$ Several networks fulfill the requisite of minimal connectedness. Figure 1 illustrates some of them (Goyal, 2007, pp. 31, 148, 166-67).

---

$^{13}$ The minimality of a network means that every link in the network is necessary, i.e. they are not superfluous. A network is said to be connected, if there is a path between any pair of agents $i, j \in N$ (Goyal, 2007, pp. 15, 91 and Harary, 1972, p. 13). Thus, a minimally connected network has only one undirected path between every pair of agents. There is an undirected path between $i$ and $j$, with $i \neq j$, in a network $g$, if there is a set of different agents $i_1, ..., i_q$ such that $\hat{g}_{i_{1} i_{2}} = \hat{g}_{i_{2} i_{3}} = \cdots = \hat{g}_{i_{q} j} = 1$ (Goyal, 2007, pp. 148, 166).

$^{14}$ The proof of this statement is given in Goyal (2007, p. 189). An illustration of an empty network is provided on page 20.
A center-sponsored star is a directed network with \( n - 1 \) links created by a single agent who is situated at the center of the star. A periphery-sponsored star is a directed network with \( n - 1 \) links. In the latter star, \( n - 1 \) agents create each one link with the same agent who is situated at the center of the star (Goyal, 2007, p. 167).

To develop the intuition, we will discuss an example of linear payoffs. The linear payoffs function is

\[
\Pi_i(g) = \tilde{h}_i(g) - n_i^d(g)c. \tag{2.1.1.3}
\]

This function satisfies the assumption 2.1.1.1. The parameter \( c \) (the cost of forming links) has an important function in this model. If \( c \in (0, 1) \), the agent \( i \) will create a link with the agent \( j \) only for \( j \)'s value alone. If \( c \in (1, n - 1) \), the agent \( i \) will require that the agent \( j \) access some other individuals to induce her to create a link with \( j \). And if \( c > n - 1 \), the costs of forming links will be superior to the benefits of accessing other agents. In the latter case, the optimal strategy for the agent \( i \) is to create no link with nobody, no matter what the other agents are doing (Goyal, 2007, p. 166). This example will also be used to illustrate some statements in the dynamic one-sided link formation section.
Finally, we will briefly discuss the notion of efficiency using the above linear payoffs example. We already characterized what an efficient network is, so we will not describe it again. If the payoff function exhibits marginal gains from accessing an agent that are superior to marginal costs of creating a link, then an efficient network will be connected. This point directs us to the following result.

Assume that the payoffs follow the equation 2.1.1.1 and that they satisfy the assumption 2.1.1.1. Any efficient network is minimal (this statement follows from the lack of decay), and if \( \phi(x + 1, y + 1) \geq \phi(x, y) \) for all \( y \in \{0, 1, \ldots, n - 2\} \) and \( x \in \{y + 1, y + 2, \ldots, n - 1\} \), then an efficient network will be connected (this statement follows from the conditions on payoffs). A complete characterization of efficient networks with the above linear payoffs example is possible. Thus, simple calculations can be utilized to assert that an efficient network can be minimally connected or empty. It follows by direct calculations that a minimally connected network is efficient for \( c < n \), and an empty network is efficient for \( c > n \) (Goyal, 2007, pp. 170, 193).

To sum up, the characteristic of “one-sided link formation” means that links are created on the initiative of one agent who bears costs of links creation. In a static game with one-sided link formation, the kinds of networks that arise in equilibrium (are stable) and are efficient are Nash networks which are empty or minimally connected networks.

We will now analyze a simple model of networks formation with two-sided link formation in a static game.

2.1.2 Two-sided link formation

The notion of two-sided link means that a link between two agents needs the assent of both agents concerned to be created. Therefore, agents create links deliberately and shape the network to their proper goals. Thus, what is the structure of the networks that will arise in equilibrium? Are these networks
socially efficient? What are the distributional properties of the networks? We will answer that type of questions in this subsection (Goyal, 2007, pp. 150, 199).

To explain the idea of two-sided link formation we will describe an announcement game. In this game, each agent declares a set of intended links. These are binary variables, \( s_{ij} \in \{0, 1\} \), where \( s_{ij} = 1 \) (\( s_{ij} = 0 \)) indicates that the agent \( i \) intends to (does not intend to) create a connection with the agent \( j \). A (pure) strategy for the agent \( i \) is \( s_i = \{s_{ij}\}_{j \in N \backslash \{i\}} \), with \( s_i \) being the strategy set of the agent \( i \). A strategy profile for all agents is \( s = (s_1, ..., s_n) \), with the set of all strategies being \( \mathcal{S} = \prod_{i=1}^n s_i \). Define \( g_{ij} = \min\{s_{ij}, s_{ji}\} \). \( g_{ij} = 1 \) only if \( s_{ij} = s_{ji} = 1 \). Every strategy profile brings on an equivalent undirected network \( g(s) \). In the undirected network \( g, \mathcal{N}_i(g) = \{k \mid i \xrightarrow{g} k\} \) is the set of agents to whom the agent \( i \) has a link. \( \Pi_i : \mathcal{S} \rightarrow \mathcal{R} \) is the agent \( i \)'s payoff function in network \( g \). To clarify this payoff function we will provide an example with the connections model.

This model captures the idea that social links give access to information and advantages, but forming links is costly (time, effort and material resources involved). The discussion of this model will concentrate on stability, efficiency and payoff distributions. \( \delta \in [0, 1] \) is the rate of decay, i.e. fall in value, due to a move across links. Given a strategy profile \( s \), the agent \( i \)'s payoff in a network \( g(s) \) is the following:

\[
\Pi_i(s) = 1 + \sum_{j \in \mathcal{N}_i(g(s)) \backslash \{i\}} \delta^{d(i,j;g(s))} - \eta_i(g(s))c,
\]

(2.1.2.1)

where \( c > 0 \) is the cost of creating a link, \( d(i,j;g(s)) \) is the geodesic distance\(^{18}\) between two nodes \( i \) and \( j \) in the undirected network \( g(s) \) (Goyal, 2007, pp. 15, 150-51, 171, 200), and \( \delta^{d(i,j;g(s))} \) are the benefits that arise from the connection (Cowan, 2005, p. 44). If \( \delta = 0 \), there is full decay and no flow of value, and if

\(^{15}\) This game is along the lines of the game drafted by Myerson, R. (1991) *Game Theory: Analysis of Conflict*, Harvard University Press.

\(^{16}\) The links are binary variables (either they are present or absent), but in many contexts, the quality of links is more important as just the existence of them (Goyal, 2007, p. 159).


\(^{18}\) The geodesic distance between two nodes \( i \) and \( j \) is the length of the smallest path between them (Goyal, 2007, p. 15).
\( \delta = 1 \), there is an absence of decay. Therefore, this payoff function means that links ease the flow of benefits (information, etc.), but for values of \( \delta \in (0, 1) \) there is delay in flow of information, because paths that connect agents are becoming longer. Subsequently, this will induce agents to connect with more people to shorten the length of paths, but this must be balanced by the costs of creating links (Goyal, 2007, p. 151).

What is the structure of the network that will arise in equilibrium in such an example? In the present static game with two-sided link formation, a link is created only if both agents agree to form a tie. If every agent announces that she will not create a connection, then the best response of the agent \( i \) is also not to engage in links formation. It follows that, in the present model, the empty network is a Nash equilibrium. More generally, for any pair of agents, it is always a best response to announce to form no link, even though it can seem more natural that agents can communicate and agree to create a link, if both benefit from it. More precisely, we can distinguish several cases in which networks are stable. Assume that payoffs are explained by the equation 2.1.2.1. For \( c < \delta - \delta^2 \), the pairwise stable network is the complete network \( g^c \) (because connections are cheap, i.e. costs are smaller than benefits which arise from a connection). For \( \delta - \delta^2 < c < \delta \), the pairwise stable network is a star. And for \( \delta < c \), the pairwise stable network is the empty network (because the costs are very high and no link is created). Figure 2 provides an illustration of some pairwise stable, equilibrium networks\(^{19} \) (Goyal, 2007, pp. 151, 201).

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\(^{19}\) The proof of these statements follows from simple computations (Goyal, 2007, p. 229).
A star network can be characterized as follows. It has small average degree, short average distances and an unequal degree distribution. As a result, a star network has unequal payoff distribution. These characteristics imply that the star seems to be like the random network since the latter has short path lengths and low "cliquishness". The star is also the unique minimal network, and thus it minimizes the number of connections necessary to have a path between every pair of agents, because it is minimally connected. With regard to the complete and empty networks, we can state that they are regular. A network is regular, if every node has identical number of links, namely $\eta_i(g) = \eta, \forall i \in N$. The complete network is a regular network with $\eta = n - 1$, and the empty network is a regular network with $\eta = 0$ (Goyal, 2007, pp. 10, 183, 201).

We will now clarify the notion of pairwise stability. This concept formalizes the idea of two-person deviation in a stationary state (Goyal, 2007, p. 151 and Cowan, 2005, p. 44). A network $g$ is pairwise stable if:

\begin{align}
(i) \quad & \forall g_{ij} = 1, \Pi_i(g) \geq \Pi_i(g - g_{ij}) \text{ and } \Pi_j(g) \geq \Pi_j(g - g_{ij}); \\
(ii) \quad & \forall g_{ij} = 0, \Pi_i(g + g_{ij}) > \Pi_i(g) \Rightarrow \Pi_j(g + g_{ij}) < \Pi_j(g).
\end{align}

The concept of pairwise stability relates to the attractiveness of links in network $g$ one at a time. The first condition requires that links in a pairwise stable network have to be profitable for every agent involved in the link. The second condition

\footnote{We will present in detail the regular graphs, the small worlds and the random graphs in the next chapter. Briefly, a “clique” is a network in which all possible edges are realized; a “clique” is a complete network (Mercklé, 2004, p. 27). If the agent $i$ and the agent $j$ are in a same “clique”, then there are many short paths through which the knowledge can flow between them (Cowan and Jonard, 2004, p. 1569).}
states that for links not present in the network, if one agent is better off them, then the other agent must be worse off them. It is very important to know whether a pairwise stable network is always present. A sufficient condition for the existence of pairwise stable networks is that there is no improving path that begins from every network. In a game with a finite number of agents and networks, a sufficient condition is that there does not exist any cycles\(^{21}\) of improving paths in the network. Therefore, the existence of a pairwise stable network is linked to the existence of cycles of improving paths. An improving path from a network \(g\) to another network \(g'\) is a finite sequence of networks \(g^1, ..., g^k\), with \(g^1 = g\) and \(g^k = g'\) that, for \(l \in \{1, ..., k - 1\}\), either:

\[
\begin{align*}
(i) & \quad g^{l+1} = g^l - g_{ij} \text{ for some } g^l_{ij} = 1 \text{ and } \Pi_k(g^l - g_{ij}) > \Pi_k(g^l) \text{ for } k \in \{i, j\} \text{ or} \\
(ii) & \quad g^{l+1} = g^l + g_{ij} \text{ for some } g^l_{ij} = 0 \text{ and } \Pi_l(g^l + g_{ij}) > \Pi_l(g^l) \text{ for } l \in \{i, j\}.
\end{align*}
\tag{2.1.2.4, 2.1.2.5}
\]

And a set of networks \(\tilde{G} \subset G\) define a cycle of improving paths, if for any \(g, g' \in \tilde{G}\) (including \(g = g'\)) there is an improving path from \(g\) to \(g'\). Explicitly, an improving path is a sequence of networks that can come out, when agents create or delete links, based on issues of individual payoff. Therefore, the concept of pairwise stability is helpful to check for strategic stability, but that notion rules out only a small set of potential deviations\(^{22}\) (Goyal, 2007, pp. 151-52, 159-60).

Finally, we will briefly discuss the notion of efficiency in the connections model. Assume that the payoffs are explained by the equation 2.1.2.1. Thus, the unique efficient and stable network is the complete network, if \(c < \delta - \delta^2\) (because connections are cheap), the star, if \(\delta - \delta^2 < c < \delta + \frac{(n-2)}{2}\delta^2\), and the empty network if, \(c > \delta + \frac{(n-2)}{2}\delta^2\) (because costs are too high and no link is formed) (Goyal, 2007, p. 201 and Cowan, 2005, p. 44). The considerations that underlie

\(^{21}\) A cycle is a trail in which there are at least three nodes. The first and last nodes have to be similar (Goyal, 2007, p. 14). According to Harary (1972), a cycle is a closed walk with \(n\) distinct nodes and \(n \geq 3\) (Harary, 1972, p. 13).

\(^{22}\) For more information on ways of strengthening the notion of pairwise stability, you can read Goyal (2007, pp. 152-56).
these results are closed to those discussed in the above one-sided link model. The analysis of the two-sided link formation model shows that the properties of centrality and short distances of a star are strong characteristics of pairwise stable networks, and also maximize welfare. However, in the context in which decay is introduced, the star is efficient only if the costs of creating links are not too small or too high. Finally, in the connections model with small costs of linking, we can state that efficient networks are as well stable, but in the connections model with moderate costs of linking, efficient networks are not stable. This is due to externality effects (Goyal, 2007, pp. 201, 216).

To sum up, we can state that the characteristic of “two-sided link formation”, contrary to the one of “one-sided link formation”, means that a link between two agents needs the assent of both agents concerned to be created. In a static game with two-sided link formation, the empty, the star and the complete networks are pairwise stable and efficient.

In the next section, we will analyze a dynamic game theoretic model of networks formation. We will discuss the one-sided link formation in the first subsection, and then the two-sided link formation the second subsection.

2.2 Dynamic

The dynamic section analyzes networks formation in the long run focusing on the relation between absorbing networks and Nash equilibrium networks (the static section examined networks creation only in the short run, and focusing strictly on Nash equilibrium networks). It is important to study what happens in the long run as networks are changing when agents delete or add links. Thus, the addition and deletion of links lead to a dynamic process of networks formation. Therefore, does this process lead to networks that are specific? What is the relation between the kinds of networks that arise in equilibrium in the long run and the static networks

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23 The proof of these statements is the same as the one in the above one-sided link model (Goyal, 2007, p. 229).
based on Nash equilibrium which arise in the short run? Does the process of revising links converge? If yes, what are the networks that are stable? We will answer these questions in the present section (Goyal, 2007, pp. 149, 168).

2.2.1 One-sided link formation

The dynamics of network can be explained in the following manner. Imagine that an agent can get the opportunity to revise her links, namely she can form new links, and maintain or delete existing links. The decision on linking or not will depend on several factors. First, the structure of the existing network will have an influence on the potential rewards from linking or not. Second, it is possible that the present linking activity will have a bearing on the incentives for other agents to create links and so shape the future linking activity. This will depend on the rapidity of the network’s changes and the extent to which agents are patient. Finally, the decision of an agent on linking will also depend on her past history of actions, and on information from her neighbors (Goyal, 2007, pp. 33, 88, 149).

To analyze this dynamic one-sided link formation model, we will explain the case in which agents are perfectly impatient. They only care about present returns and ignore the above second consideration. They also believe that actions of other agents in the current period are identical as the ones in the preceding period. Hence, agents are boundedly rational. These characteristics lead to the myopic best-response model of dynamics.24 In this model, we assume a rule of behavior in the case of indifference: if two links are uniformly attractive, then the agent must choose each of the strategies with strictly positive probability. The features of the dynamic process are the following: time is a discrete variable \( t = 1, 2, 3, ... \). \( g^t \) is the network/strategy profile at the beginning of time \( t \). In each period, an agent has an occasion to modify her strategy with a probability

---

If $p < 1$, an agent reveals inertia with positive probability. Agents are assumed to know perfectly the network structure at any point in time. An agent chooses a set of links that maximizes her payoff, under the hypothesis that links of other agents do not change. If several actions are optimal, then the agent chooses one at random. The strategy of an agent $i$ in the period $t$ is $g_t^i$. If the agent $i$ is inactive in the period $t$, then $g_t^i = g_t^{i-1}$. This best-response strategy revision rule creates a transition probability function $P_{gg^i}: \mathcal{G} \times \mathcal{G} \to [0, 1]$, with $\sum_{g^i} P_{gg^i} = 1$, for every $g \in \mathcal{G}$. The dynamics of networks $g^t$ follow the latter transition probability function. A strategy profile $g$ is absorbing, if the dynamic process cannot change from a state once it achieves it, namely $P_{gg} = 1$. It is also possible that, in some contexts, a multiple absorbing states exist. Therefore, it is necessary to analyze the relative stability of different networks. The concept of strict equilibrium deals with that question. A Nash equilibrium is strict, if every agent chooses a strategy that generates payoffs that are strictly higher than the payoffs of another strategy, namely

$$\Pi_i(s_t^i, s_{-t}^i | g) > \Pi_i(s_t^i, s_{-t}^i | g).$$

(2.2.1.1)

If the payoffs follow the equation 2.1.1.1, and if they satisfy the assumption 2.1.1.1, then a strict Nash equilibrium is either a center-sponsored star or an empty network. An illustration of these networks is provided by Figure 3 (Goyal, 2007, pp. 31, 33, 149-50, 168-69).

**Figure 3. Strict equilibrium networks**

![Figure 3](image)  
Source: Goyal (2007, p. 168)

---

25 This assumption is very strong. In fact, it is not possible to know exactly the structure of the network at any point in time, because it is complicated even with a few agents. However, we will not introduce a model with incomplete information in our study (Goyal, 2007, pp. 159, 185).

26 The proof of this statement is provided in Goyal (2007, pp. 189-90).
Following the linear payoffs example explained on page 16, we can assert that, in this dynamic one-sided link section, the center-sponsored star network (empty network) is the sole strict equilibrium, if $0 < c < 1$ ($c > 1$). This statement points out that strategic consideration in links creation lead to networks that have small average degree, small average distances between agents and an unequal degree distribution. Thus, it leads to a star network (Goyal, 2007, p. 168).

However, it is important to note that, if we introduce equity considerations\textsuperscript{27} (inequity aversion), as the star network gives unequal payoffs, this kind of network rarely emerges in reality; and in cases where the star network emerges, it is not stable (Goyal, 2007, p. 183).

Finally, a comparison between efficient networks (explained in the static section), and center-sponsored star and empty network (strict Nash networks) leads to the following two remarks: (i) the center-sponsored star is efficient for low costs of creating links ($c < 1$), (ii) the empty equilibrium network is under-connected when compared to efficient networks for middle and high costs of links creation ($1 < c < n$). The intuition of this under-connectedness is that the links create externalities for other agents, and so the individuals underrate the social value of links (Goyal, 2007, p. 170).

We can conclude that there are many strategic inter-temporal concerns in linking decisions (Goyal, 2007, p. 150). To summarize, we can state that, in a dynamic game with one-sided link formation, a strict Nash equilibrium is either a center-sponsored star or an empty network, and they are stable and efficient.

In the next section, we will analyze the two-sided link formation of the dynamic game theoretic model.

\textsuperscript{27} The equity considerations are under the assumption of homogenous agents (Goyal, 2007, p. 183).
2.2.2 Two-sided link formation

As in the previous section, we will consider myopic agents that add and cancel links. The problematic and notation is exactly the same as above so we will not re-explain the notation and the similitude. We will only explain the differences (Goyal, 2007, p. 156).

In this dynamic two-sided link formation model, the probability of any pair of agents to be selected is the same and does not vary over time, namely \( p_{ij} = \frac{2}{n(n-1)} \) for all \( t \). This probability is independent across the periods. The basic decision rule for agents to form links in network \( g \) is the following:

\[
\Pi_i(g + g_{ij}) > \Pi_i(g) \quad \text{and} \quad \Pi_i(g + g_{ij}) \geq \Pi_j(g).
\]

(2.2.2.1)

and the basic rule for deleting a link \( g_{ij} \) is

\[
\Pi_i(g - g_{ij}) > \Pi_i(g)
\]

(2.2.2.2)

This decision rule reflects a myopic best response to existing networks, and it is the same as in the previous section. The addition and deletion of links lead to a dynamic process of network creation. The interest is to know whether this dynamic process of network creation converges, and if this is the case, what are the limit networks. Every pairwise stable network is an absorbing state of the dynamic process, and if the process converges, then the limit has to be a pairwise stable network. In different contexts, there are many pairwise stable networks, and it is important to know, if they are all similarly robust to minor perturbations. To achieve this, we will use the notion of stochastically stable networks (Goyal, 2007, p. 157).

A description of stochastic stability is the following. Assume that \( g \) and \( g' \) are two absorbing states of the myopic best-response model of dynamics analyzed on pages 23 and 24. Because \( g \) is an absorbing state, a shift from \( g \) to \( g' \) is possible only if it is a mistake on the part of one or several agents. Similarly, a shift from \( g' \) to \( g \) is a mistake (called a mutation) made by some subset of agents. The state \( g \) is stochastically stable, if more mutations are needed to shift from \( g \) to \( g' \) than
from \(g'\) to \(g\). If the same number of mutations is required to move between the two states, then they are both stochastically stable. We will assume now that agents make occasionally errors, experiments, etc. We also suppose that, depending on the fact that agents can have a possibility to modify links at any point in time \(t\), an agent chooses her strategy arbitrarily with a small probability of mutation \(\varepsilon > 0\). Given a network \(g\), and for any \(\varepsilon > 0\), the process of mutation defines an aperiodic and irreducible Markov chain,\(^{28}\) and has a probability distribution \(\mu^\varepsilon_g\) that is unique and invariant. We will consider the support of \(\mu^\varepsilon_g\) as the probability of errors turns out to be very small, namely as \(\varepsilon\) converges to 0. Define \(\lim_{\varepsilon \to 0} \mu^\varepsilon_g = \mu_g\). Hence, a state \(g\) is stochastically stable, if \(\mu_g(s) > 0\). Therefore, the concept of stochastic stability defines states or networks that are stable to such mutations or perturbations (Goyal, 2007, pp. 70-71).

We will now discuss the network’s effects on the rates of convergence of the dynamic process of networks formation. The invariant distribution \(\mu_g\) is significant only if the rate of convergence of the dynamic process is fast. In our model, the dynamics are Markovian, and since there is a single invariant distribution, then results of standard mathematics assert that the rate of convergence is exponential.\(^{29}\) The interaction structure of the network can shape the rate of convergence of the dynamic process of network change. As an illustration, according to Ellison (1993; cited by Goyal (2007, p. 73)), in a complete network, the shift between strict Nash equilibria due to mutations would be very long with large populations, because the number of mutations which is needed is of the order of magnitude of the population. But with another structure of network, it can be fully different. We can conclude that, if agents choose best responses, and the perturbations are random and equiprobable, then the stochastic stability of diverse actions is dependent on the structure of interaction (Goyal, 2007, pp. 72-73, 80).

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\(^{28}\) A Markov chain is a collection of variables which are random and have the property that the future is conditionally independent of the past, given what happens in the present (Weisstein, E.W. “Markov Chain”, from MathWorld—A Wolfram Web Resource).

\(^{29}\) In other words, there is a number \(\rho < 1\) such that the probability distribution of actions at the time \(t\), \(\mu^t\), moves toward the invariant distribution \(\mu^*\) at a rate \(\rho^t\) (Goyal, 2007, p. 73).
Finally, according to the study of Robson and Vega-Redondo (1996; cited by Goyal (2007, p. 75)) about an imitate the best action rule taken together with random matching, we can state that an efficient action is the single stochastically stable action (Goyal, 2007, pp. 75-76).

To sum up, we can state that, in a dynamic game with two-sided link formation, the stochastic stability of different networks is dependent on the structure of interaction of the network. Moreover, an efficient network will always be stochastically stable.

The purpose of the analysis presented in this chapter was to describe in detail the formation of networks based on a game theoretic approach, and to characterize which types of networks arise in equilibrium (are stable) and are efficient. The following table summarizes, in a nutshell, the results obtained for each subsection of this chapter.

Table 1. Results for the model of networks formation based on a game theoretic approach

<table>
<thead>
<tr>
<th>Stability and efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static:</strong></td>
</tr>
<tr>
<td>One-sided link formation</td>
</tr>
<tr>
<td>Empty and minimally connected networks</td>
</tr>
<tr>
<td>Two-sided link formation</td>
</tr>
<tr>
<td>Empty, star and complete networks</td>
</tr>
<tr>
<td><strong>Dynamic:</strong></td>
</tr>
<tr>
<td>One-sided link formation</td>
</tr>
<tr>
<td>Empty and center-sponsored star networks</td>
</tr>
<tr>
<td>Two-sided link formation</td>
</tr>
<tr>
<td>The stochastic stability of different networks is dependent on the structure of interaction of the network. An efficient network is always stochastically stable.</td>
</tr>
</tbody>
</table>

*Source:* by author

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30 This imitation rule states that an agent chooses an action which gives the highest payoffs among the actions that other agents choose (Goyal, 2007, p. 75).
In the next chapter, we will analyze three particular structures of networks whose properties ("cliquishness" and path lengths) are often discussed in our field of study. It is the Ising models, the small worlds and the random graphs. These network structures will then be linked to the concept of innovation.

3 Ising models, small worlds and random graphs

This chapter describes in detail the three network structures mentioned above, and make a link between these network structures and innovation. More precisely, we will analyze which kind of networks fosters more knowledge creation and/or diffusion. The Ising models, the small worlds and the random graphs can be linked to the networks structures described in the preceding chapter. The Ising model is locally very dense, i.e. it is "cliquish", and has long average path lengths between agents. These characteristics are those of the regular network. More precisely, a network is said to be regular, if every node has identical number of links. The complete and empty networks analyzed in the previous chapter are regular networks as they fulfill the latter condition (in spite of the fact that the empty network has no connections). The random network has short path lengths and low "cliquishness". Thus, it corresponds to the star network which has short average distances and small average degree. Concerning the small world network, the link with the kinds of networks described in the preceding chapter is not very obvious. In fact, the small world has the characteristic of short path lengths and is locally very dense or "cliquish". The first characteristic of the small world corresponds to the star network, but it is not true for the second one. Furthermore, only the second characteristic of the small world matches with those of the complete network. Therefore, it is difficult to make an explicit link with the small world network structure and a specific network structure described in the previous chapter.
In the first section, we will discuss in detail the structure of the Ising models, the small worlds and the random graphs. In the second section, we will make a link between these network structures and the concept of innovation.

3.1 Network structure

In terms of network structure, both regular graphs from Ising models and random graphs reach the extremes, whereas the small worlds lie in between (Cowan, 2005, p. 35). More details will be provided in the following subsections.

3.1.1 The Ising models

In an Ising model, the individuals are positioned at fixed points in a regular integer space. They are connected to the same \( n \) nearest neighbors; this is why this structure is also called nearest-neighbor networks or graphs. An Ising model has a completely regular interaction structure, it is locally very dense and it has long average paths between agents (Cowan, 2005, p. 34 and Cowan and Jonard, 2004, pp. 1557-58). Figure 4 illustrates a regular graph from the Ising model.

**Figure 4. Regular graph**

Source: Cowan and Jonard (2004, p. 1560)

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31 It is not possible to characterize this model very precisely, because it was created and mostly analyzed in physics and not in economics. Nevertheless, more information on this subject can be found in the study of Ising which is the main reference on this subject. Ising E. (1925) “Beitrag zur theorie des ferromagnetismus”, Zeitschrift Physik, 31, pp. 253-58.
More precisely, a network is said to be regular, if every node has identical number of links. The complete network analyzed in the previous chapter is a regular network, because it fulfills the latter condition. In fact, the complete network is a regular network with $\eta = n - 1$ connections. A strongly regular graph with the parameters $(n, k, \lambda, \mu)$ is a network on $n$ nodes with degree $k$ and with the following properties:

(i) Two contiguous nodes have exactly $\lambda$ common neighbors;

(ii) Two noncontiguous nodes have exactly $\mu$ common neighbors.


Finally, in many real-world networks, the probability of a connection between two agents is higher, if they have a mutual acquaintance (Watts and Strogatz, 1998; cited by Newman et al. (2002, p. 2566)). Therefore, we can notice the importance of the property of “cliquishness” of this kind of network.

We will now present another network structure, called the small world.

3.1.2 The small worlds

Small world structures are mostly due to Watts and Strogatz (1998). They develop a one-parameter class of random graph models in which the parameter is employed to scale between the regular graph from the Ising model, and the random graph from the random graph theory. They use the following algorithm for creating a graph, called the random rewiring procedure. They fix the parameter $p$ (the degree of randomness) which defines the network structure, so as to start with a regular ring of $n$ nodes. Each node is linked to its $k$ nearest neighbors by undirected edges. They study each link in the network in a clockwise sense and rewire one end of this link with the probability $p$. They connect it to a randomly chosen agent, and make sure that this agent is not connected to herself and that there are no duplications, namely no two nodes are connected more than once.

---

32 An edge is a relation between two nodes or agents, i.e. it is a link (Mercklé, 2004, p. 23).
Otherwise, the link is left in place. This process ends when a lap is completed. Afterwards, the process begins again by considering only edges that connect nodes to their second-nearest neighbors clockwise. This process continues to more distant neighbors, after each lap, until each link or edge in the original ring is treated once. There are several possible realizations of this process for distinct values of $p$. Therefore, the algorithm varies between the regular graph (if $p = 0$) and the random graph (if $p = 1$). In the former case, the original ring is not modified. In the latter case, as $p$ becomes higher, the network becomes more disordered until every edge is rewired arbitrarily, i.e. $p = 1$. For intermediate values of $p$ ($0 < p < 1$), the small world network appears. It is important to note that this shift happens without changing the density or total number of links in the network. Thus, the average number of edges per node is stable (Cowan, 2005, p. 36; Watts and Strogatz, 1998, pp. 440-41; Cowan and Jonard, 2004, p. 1560 and Cowan and Jonard, 2000, p. 6).

As already stated, the small world network structure lies between the regular graph and the random graph (in terms of network structure). The small world network has the characteristic of short path lengths, and is locally very dense or “cliquish”. Most of the connections of the small world are local, but some 10 percent of them are long distance connections so that the knowledge can reach distant parts of the network (Cowan, 2005, p. 35; Cowan and Jonard, 2004, p. 1557 and Cowan and Jonard, 2000, p. 4). Small world networks represent well real-world networks, because of the characteristic of constant vertex degree (Newman, 2000, pp. 823-24, 826). Figure 5 illustrates the small world structure.

**Figure 5. Small world**

![Small World Network](Source: Cowan and Jonard (2004, p. 1560))
The features of “cliquishness” and short path lengths can be characterized more precisely. Average “cliquishness” can be defined as follows:

$$C(p) = \frac{1}{N} \sum_i \sum_{j \in \Gamma_i} \frac{X(j, l)}{||\Gamma_i|| (||\Gamma_i|| - 1)/2},$$

where $X(j, l) = 1$ if $j \in \Gamma_i$, and if not, $X(j, l) = 0$. $\Gamma_i$ is the set of agents to whom the agent $i$ is directly connected, and $||\Gamma_i||$ is the size of this neighborhood (Cowan, 2005, p. 36 and Watts and Stogatz, 1998, p. 441). The “cliquishness coefficient” $C(p)$ can be characterized as follows. “Suppose that a vertex $v$ has $k_v$ neighbours; then at most $\frac{k_v(k_v-1)}{2}$ edges can exist between them (this occurs when every neighbour of $v$ is connected to every other neighbour of $v$). Let $C_v$ denote the fraction of these allowable edges that actually exist. Define $C_v$ as the average of $C_v$ over all $v$” (Watts and Stogatz, 1998, p. 441). Then, the average path length can be defined as:

$$L(p) = \frac{1}{N} \sum_i \sum_{j \neq i} \frac{d(i, j)}{N - 1},$$

where $d(i, j)$ is the geodesic distance between $i$ and $j$ (it is the length of the shortest path between $i$ and $j$) (Cowan, 2005, pp. 36-37). “$L$ is defined as the number of edges in the shortest path between two vertices, averaged over all pairs of vertices.” (Watts and Stogatz, 1998, p. 441). To give an intuition of the meaning of $C(p)$ and $L(p)$, we provide an example with friendship networks. $C_v$ represents the extent to which the friends of the agent $v$ are also friends with each other (Watts and Stogatz, 1998, p. 441) ($C$ is a measure of the density or local coherence of that friendship) (Cowan and Jonard, 2004, p. 1560), and $L$ is the average number of friends in the shortest path that connects two agents (Watts and Stogatz, 1998, p. 441).

Finally, we will study the evolution of the characteristics of path length and “cliquishness” with $p$. If $p$ increases, the “cliquishness” will decrease since the local coherence will fall with the introduction of random links. The average path length will also decrease since a random link is a possible shortcut that can connect vertices which would not be connected otherwise. However, the latter
decreases at a faster rate than the former. Small world networks issue from this rapid drop in the path length \( L(p) \) since it creates an interval within which the “cliquishness” is high, and the path length is short (Cowan, 2005, p. 37 and Watts and Strogatz, 1998, p. 440). These statements are illustrated by Figure 6.

**Figure 6. Cliquishness and path length as a function of \( p \)**

![Graph showing cliquishness and path length as a function of p](image)

*Source: Cowan and Jonard (2004, p. 1561)*

In this figure, we can observe that for values of \( p \) which are between 0.01 and 0.1, the “cliquishness” comes close to the one of the regular graph, and the path length is near to that of the random graph. Therefore, in this region, small worlds networks can emerge, and the conditions are positive both for knowledge creation and diffusion (Cowan, 2005, p. 37).

We will introduce, in the last subsection, a network structure called the random graph.

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33 The proof of this statement is the following. “For small \( p \), each [shortcut] has a highly nonlinear effect on \( L \), contracting the distance not just between the pair of vertices that it connects, but between their immediate neighbourhoods, neighbourhoods of neighbourhoods and so on. By contrast, an edge removed from a clustered neighbourhood to make a [shortcut] has, at most, a linear effect on \( C \); hence \( C(p) \) remains practically unchanged for small \( p \) even though \( L(p) \) drops rapidly. The important implication here is that at the local level (as reflected by \( C(p) \)), the transition to a small world is almost undetectable” (Watts and Strogatz, 1998, pp. 440-41).
3.1.3 The random graphs

The study of random graphs was initiated by Erdős and Rényi\(^{34}\) in 1959. They laid the foundations of the random graph theory by proving a lot of fundamental results. Erdős and Rényi found out that the random graph has several sharply delineated properties with a high probability. Another great discovery of Erdős and Rényi was that all standard properties of the random graphs (connection, etc.) happen suddenly (Bollobás, 2002, p. 216).

We will now define exactly what the random graph is. According to Newman et al. (2002), the random graph is constructed by taking some vertices, and then placing links between them in such a way that each pair of nodes has a connecting edge with an independent probability (Newman et al., 2002, p. 2567). And, according to Bollobás (2002), “every probability space whose points are graphs gives us a notion of a random graph.” (Bollobás, 2002, pp. 216-17). A simple way to define the random graph is the following. The random graph can be denoted by \(G_{n,m}\), where \(n\) and \(m\) are two integers with \(0 \leq m \leq \binom{n}{2}\). This graph is achieved by taking a group of \(n\) elements as the set of vertices, and then by choosing randomly (and with no replacement) \(m\) of the \(\binom{n}{2}\) possible edges. The interest is mainly when \(n\) is very high \((n \to \infty)\) and \(m\) is a function of \(n\). In other words, the random graph evolves by a stochastic process, beginning with a set of vertices without edges (i.e. it is an empty network), and then adding edges up to the achievement of a network that is complete. One way to achieve this is to add new edges randomly at times 1, 2, etc. This provides a process that results in a random graph of type \(G_{n,m}\) at time \(m\) (Janson, 2000, pp. 1-2).\(^{35}\)

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\(^{35}\) In this case, the random rewiring procedure from Watts and Strogatz (1998) from the previous section is also valid to create the random graph.
More precisely, in the random graph, agents are connected to \( n \) randomly chosen individuals (Cowan and Jonard, 2004, p. 1557). This model is characterized by a lack of local coherence; i.e. it is not locally “cliquish”. This kind of graph has also short average path lengths between vertices, but they increase logarithmically with the size of the network\(^{36}\) (Cowan, 2005, p. 35 and Newman \textit{et al.}, 2002, p. 2569). As already stated, a star network has characteristics that are near of the ones of the random graph. In fact, a star network has small average degree and short average distances, and this corresponds to the characteristics of a lack of local “cliquishness” and short average path lengths of random graphs. Figure 7 depicts the random graph.

\textbf{Figure 7. Random graph}

\begin{center}
\includegraphics[width=0.2\textwidth]{random_graph.png}
\end{center}

\textit{Source:} Cowan and Jonard (2004, p. 1560)

It is important to note that there is no spatial structure imposed on the connections in these kinds of graphs. The interactions are spatially global. Every vertex corresponds to other vertices, i.e. they are equivalent. Moreover, there is no distance between the vertices before the arbitrary choice of edges (Janson, 2000, p. 1 and Cowan and Jonard, 2004, p. 1558). We can therefore assert that, in the random graph, an agent is linked to any other agent in the population with some probability, not considering the location. As a consequence, the emerging network has no spatial patterns in the space where individuals are located physically. In other words, agents are located in a network space, but not in a physical space. Thus, when we talk about the notion of distance, it is only a distance in networks (Cowan, 2005, p. 35).

\(^{36}\)The proof of this statement is provided in Newman (2002, pp. 2569-70).
Finally, a property of the random graph is that it can exist in two distinct regimes. The regime transition depends on the degree distribution. Thus, the network can either be made of several small components of nodes linked together by edges or it can contain a “giant component”\(^{37}\) (Newman et al., 2002, p. 2568); this depends on the number of links. If there is a small number of links, the network will be divided into many small clusters of nodes, i.e. components. If the number of links in the graph increases, then the components will grow by linking to isolated vertices and also by combining with other components. As a consequence, a “giant component” will emerge. This regime transition occurs precisely at \(m = \frac{n}{2}\), where \(m\) is the number of links, and \(n\) is the number of nodes (Strogatz, 2001, p. 272). The existence or not of a “giant component” in the graph has significant repercussions for social networks. A component in a graph is a set of agents who have paths between them and can hence communicate. Therefore, the extent to which a component is large will determine the level of communication between agents in that component. If there is a “giant component”, then there will be many paths between many agents, and the communication level will be high. On the contrary, if all components of the network are small, then the communication can only take place within small groups of agents, and the average communication level of the entire network will be low. In real-world, it appears that the networks have a giant component, and therefore the communication level is high (Newman et al., 2002, pp. 2566-69).

After having described in detail the formation and the characteristics of the regular graphs, the small worlds and the random graphs, we will analyze, in the next section, the link between these kinds of graphs and innovation.

\(^{37}\) A “giant component” is a “group of connected vertices that fills a significant portion of the whole network and whose size scales up with the size of the whole network-in addition to a number of small components” (Newman et al., 2002, p. 2568). We will not discuss here under which conditions a “giant component” can emerge or not, and what the size of the “giant component” can be. This is done in many of the studies about the related literature, and particularly in Newman et al. (2002, p. 2569).
3.2 Innovation and the optimal network structure

In this section, we will determine which network structure (from the three kinds of networks described in the previous section) fosters more knowledge creation and diffusion. We will remark that the distinct characteristics of these graphs (especially the extent to which a graph is “cliquish”, and has long or short path lengths) have an impact on knowledge creation and diffusion.

3.2.1 Introduction

It is important to focus on the two extreme network structures analyzed above (regular graph and random graph) to examine knowledge creation and diffusion (Cowan, 2005, p. 35). If a network is locally very dense or “cliquish”, i.e. local redundancy exists (Cowan and Jonard, 2004, p. 1569), then many agents will work on related issues. This can create a critical mass of knowledge workers, and then become an epistemic community with a common language, a standardization of problems solving, etc. that will foster innovation. In other words, innovation is eased by the agglomeration of human capital, and this is enhanced, if a network is highly “cliquish”. A network is said to be “cliquish” when individuals are highly and closely interconnected. A network that satisfies the latter condition is a regular one, e.g. a complete network; it is not a random graph, e.g. a star network since it has the opposite characteristics. This discussion relates to knowledge creation and not to knowledge diffusion. Nevertheless, a problem persists. It is possible that there is unequal knowledge growth across groups of agents. As a result, some groups advance rapidly, and others are left behind. Thus, the contribution to aggregate knowledge growth will be low. However, this can be softened, if the network is highly “cliquish”. In fact, within a “clique”, agents are more or less similar, and few agents are left behind. Therefore, the “cliques” advance together quickly. Consequently, we can state that a “cliquish” or locally dense structure gives the highest knowledge growth rates (Cowan, 2005, pp. 35, 42 and Cowan and Jonard, 2000, p. 13).
If we consider the diffusion of knowledge (and not knowledge creation), it will spill over faster when path lengths are short. The diffusion of knowledge can be understood as a spread of a piece of knowledge to every agent in the economy, and this we will be faster and with the smallest fall in value when the distance between the originator of the knowledge and the recipients is short. Thus, we can say that the diffusion of knowledge will be more rapid, if the network has the structure of the random graph thanks to its shortcut property (Cowan, 2005, p. 35). It follows from these statements that there is a tendency of knowledge homogenization throughout the network, and this inhibits knowledge creation (Cowan and Jonard, 2000, p. 13). Therefore, it confirms that when path lengths are short and the property of “cliquishness” is not present, only diffusion of existing knowledge is possible.

Therefore, it seems that the improvement of both knowledge creation and diffusion at the same time is not possible as regular networks foster knowledge creation, but not knowledge diffusion, and random networks improve knowledge diffusion, but not knowledge creation. Nevertheless, this difficulty can be resolved by the small world network structure since it is “cliquish” and has short path lengths (Cowan, 2005, p. 35). The small world has the property of connecting remote and probably heterogeneous parts of the network. This produces a tendency of homogenization of overall knowledge levels, but without decreasing too much the local cumulativeness stemming from high “cliquishness” as there are few shortcuts. As a result, this generates knowledge levels that are higher than in the random graph (Cowan and Jonard, 2000, pp. 13, 15). To conclude, we can assert that with the small world network structure, conditions are positive both for knowledge creation and diffusion (Cowan, 2005, p. 37). Moreover, it will be proven below that the small world network is the structure that maximizes the knowledge growth (Cowan and Jonard, 2000, p. 13).

In the next section, we will introduce a model of knowledge diffusion; more precisely, it is a barter exchange model. We will discuss the properties and
shortcomings of this model, and we will study which kind of network structure maximizes the average knowledge level.

3.2.2 Barter exchange of knowledge

In this section, we will discuss a model of myopic barter exchange of knowledge. The diffusion of knowledge throughout the economy will be achieved in a barter way. In this model, the aggregate performance will be measured by the average knowledge level over all individuals. In this barter model, an agent is characterized by the individuals to whom she is directly connected, and a knowledge stock that evolves over time. The decision for an agent, when she meets an individual, is either to trade or not. This decision depends only on whether the other has something to trade or not, and does not depend on the quality and quantity of what the other person has to offer in exchange. Therefore, when two agents meet, all possible trades between them take place. This process continues until all trades are done. This is the case when for all possible pairs, one individual weakly dominates the other in all possible categories of knowledge (Cowan, 2005, p. 38 and Cowan and Jonard, 2004, pp. 1557-59, 1561-63).

We will now discuss the issue of the speed at which the knowledge diffuses through a barter arrangement. Figure 8 illustrates the time series of the average knowledge level for three values of $p^{38}$ (0.001, 0.09 and 1). The results are given from 1 to 1′000′000 trading rounds (Cowan and Jonard, 2004, pp. 1565-66).

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38 Recall that $p$ is the rewiring probability or the degree of randomness (Cowan and Jonard, 2004, p. 1566 and Watts and Strogatz, 1998, p. 441).
When $p = 1$, the graph has short path lengths, but little local structure. This is the random graph structure. At the beginning of the barter exchange the diffusion is very rapid as can be seen in Figure 8. However, as time goes by, the diffusion process ends at relatively low knowledge levels. When $p = 0.001$, the graph has high local structure, and long path lengths. We recognize here the regular graph. At the beginning of the barter trade for the regular graph, and contrary to the random graph, the diffusion is slow, but the process continues for a longer time, and attains a higher aggregate knowledge level. When $p = 0.09$, both advantages of short path lengths and local structure are present (we can recognize here the small world network structure). Thus, the early diffusion of knowledge is rapid, and continues for a long time. In other words, the characteristic of short path lengths implies a fast initial knowledge growth (but not as rapid as the random network), and the characteristic of “cliquishness” implies continued growth. This network structure produces the highest knowledge levels, and, consequently, the average knowledge level is maximized. However, the small world region ($0.01 \leq p \leq 0.1$, in Figure 6) maximizes also the dispersion of knowledge among agents, if we use the variance as a measure of the equity in knowledge among individuals. As a consequence, there is a dilemma between efficiency, in terms of the quantity and quality of knowledge diffused, and equity in the allocation of this knowledge.
Nevertheless, if we use another measure of the disparity in knowledge levels among individuals, this dilemma can disappear. This is true, if we use the coefficient of variation of knowledge levels instead of the variance. With this new measure, the normalized dispersion of knowledge levels decreases in the small world region (measured by the rewiring probability $p$). This suggests that an increase in heterogeneity is compensated by an increase in overall knowledge levels. Therefore, the trade-off between efficiency and equity disappears (Cowan, 2005, p. 40; Cowan and Jonard, 2004, pp. 1565-67 and Cowan and Jonard, 2000, p. 4).

We will now discuss a difficulty that arises from the barter exchange model. When agents meet their directly connected neighbors, they trade their proper knowledge as a barter arrangement. Nevertheless, it is possible that the knowledge transfer will be incomplete, i.e. the absorptive capacity $\alpha^{39}$ from the different individuals is imperfect.\(^{40}\) In other words, when $\alpha < 1$, the post-trade knowledge levels will not converge totally. This means that the agent $i$ is not able to learn all what the agent $j$ has to teach. Furthermore, if path length increases, the knowledge will degrade even more, because it travels along an extended multiagent chain. In that case, $\alpha$ will be very low. Consequently, it will be costly to transmit knowledge in terms of time, and also in terms of the diminution of the quantity and quality of knowledge. We can remark that the short path length property of small worlds and also of random graphs is very useful to soften this problem, and gives the possibility of a rapid and wide diffusion of knowledge with low costs of transmission. Therefore, short path lengths can facilitate and accelerate the aggregate knowledge growth (Cowan, 2005, pp. 38-39).

The small world network results can be influenced by the value of the absorptive capacity. In fact, the optimal network structure, measured by $p$ in the above rewiring algorithm, turns out to be more random as $\alpha$ increases. In other words,

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\(^{39}\) Absorptive capacity is the ability to recognize the value of new information and assimilate it. The absorptive capacity is usually a function of the level of prior related knowledge (Cohen and Levinthal, 1990, p. 128).

\(^{40}\) It follows from this statement that no new knowledge is created (Cowan, 2005, p. 39).
the optimal value of $p$ goes up with $\alpha$, because the relative importance of “cliquishness” and path length changes as the absorptive capacity changes. If $\alpha = 1$, when the agent $i$ trades with the agent $j$, they will become identical. Therefore, an agent $k$ will be indifferent between trading with $i$ or $j$, and if she has a link with $i$, she does not need to have a link with $j$; hence, the agent $k$ will be better off having a link with someone who is not connected with $i$. This proves that the value of “cliquishness” decreases, and the relative value of short paths augments as $\alpha$ increases. However, an amount of “cliquishness” is still needed for knowledge diffusion. Moreover, the importance of a “clique” for knowledge diffusion grows, if there is an expert with a high level of knowledge within it. If there is no expert, the value of “cliquishness” will be attenuated, and the characteristic of path length will dominate. The absorptive capacity has two additional effects. It can modify the efficiency of the process of barter exchange, and it can influence the speed of convergence of the economy. The first effect can be explained as follows. In a barter economy, it is possible that significant trading opportunities are not taken into account. Thus, if $\alpha$ is high, then each trade will bring more knowledge and increase the final knowledge levels. The second effect is the following. If $\alpha$ increases, then a trade will bring the partners closer in terms of knowledge levels. This implies that the trading opportunities are exhausted more rapidly, and the economy will converge more quickly (Cowan, 2005, p. 40 and Cowan and Jonard, 2004, pp. 1567-68).

Finally, in a barter exchange model, there is another difficulty. It is the double coincidence of wants constraint. The property of “cliquishness” can help to resolve this problem. In fact, within a “clique” there are indirect, but still short paths between agents. Thus, if the trade between the agent $i$ and $j$ does not take place because of a failed double coincidence of wants, then the transaction can still occur, if $i$ and $j$ share a common neighbor. We can remark that the property of local redundancy of the “cliquishness” is very useful as it provides a solution to this problem. In the random graph, path lengths are short. As a consequence, the diffusion of knowledge is very rapid, but the process ends at relatively low knowledge levels. This is due to the problem of double coincidence of wants as
the random graph has no “cliquishness” property which can circumvent this problem. The regular network can avoid the double coincidence of wants’ constraint as it benefits from the “cliquishness” property, but path lengths are long. Despite this diffusion problem, this kind of network can foster aggregate knowledge for a long time, thanks to the property of local “cliquishness” (Cowan, 2005, pp. 39-40).

As a conclusion, we can state that in this barter exchange model the small world network is the unique structure that can foster knowledge diffusion. Therefore, the average knowledge level is maximized with this network structure. The next section will present another model of knowledge diffusion and creation, namely the broadcast exchange model. The structure of this model is the same as in the previous section, but the knowledge dynamic is distinct (Cowan, 2005, p. 40). We will analyze the properties of this model, and we will study which kind of network structure arises that maximizes the average knowledge level.

3.2.3 Exchange of knowledge as a broadcast

This section introduces a model of knowledge transfer as a broadcast. In this model, an agent is characterized by the individuals to whom she is directly connected and a knowledge endowment. In each period, an agent innovates, and broadcasts her knowledge to agents who are directly connected to her. This agent broadcasts randomly and sequentially. If the absorptive capacity of the neighbors is less than 1 ($\alpha < 1$), i.e. it is not perfect, then the knowledge will be partially absorbed with effort, and tacit knowledge will be important. On the contrary, if $\alpha > 1$, i.e. the knowledge is super-additive, then complementarities in knowledge of the agent $i$ and $j$ involve that when $j$ receives $i$’s knowledge, then $j$ will be able to improve upon it. In fact, neighbors are distinct in their ability to innovate, and to absorb knowledge that is broadcasted. The knowledge

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41 The model of the previous section studied only diffusion of knowledge. The model of this section analyzes both knowledge diffusion and creation.
42 It is worth noting that the agent who broadcasts her knowledge is not subjected to a loss of her knowledge (Cowan and Jonard, 2000, p. 5).
endowment of an agent can increase due to her ability to receive new knowledge from a recent innovator, but also due to her innovation ability. However, if agents are very dissimilar in their knowledge endowment, then no knowledge transmission will be possible. Nevertheless, if the trade could still take place, then the dissimilarity between the sender and the receiver will decrease (Cowan, 2005, p. 40 and Cowan and Jonard, 2000, pp. 4-5, 10).

In this broadcast model, the efficiency of the network structure is analyzed in terms of average knowledge levels in the long run. To analyze both knowledge creation and diffusion, we divide the parameter space into two regions, namely when $\alpha < 1$ and when $\alpha > 1$. In the first region, there is no new knowledge which is created, because the absorptive capacity is low. Hence, an agent can hardly understand the knowledge flow she receives, and she can not improve upon it. As a result, no new knowledge can be created. In other words, the innovative potential is low, and the knowledge advances are mainly obtained through an imitation of the individuals who innovate exogenously. Therefore, the modifications in aggregate knowledge levels are only due to diffusion. Besides, the network structure has no influence on long run knowledge levels; every value of $p$ generates the same state of the economy in the long run. These statements are in contradiction with the barter model which states that the small world structure dominates other network structures (Cowan, 2005, p. 42 and Cowan and Jonard, 2000, pp. 10-11).

In the second region of the parameter space, there is both creation and diffusion of knowledge. As the absorptive capacity is high, each agent integrates the knowledge that she receives into her existing stock, and, consequently, she will become more knowledgeable and achieve higher efficiency than before the broadcast (Cowan and Jonard, 2000, p. 11). To illustrate what happens in the first and second region of the parameter space, and the transition between the two regions, we provide the following figure.
This figure represents the relationship between the long run average knowledge level, and both the rewiring probability $p$ and the absorptive capacity $\alpha$. As can be seen in this figure, for $\alpha < 1.05$ there are no differences between distinct structures of graphs for every value of $p$. All network structures can appear, from the regular one to the random one. This corresponds to the first region described above. However, the transition between regions begins already smoothly at $\alpha = 1$. When $\alpha > 1.05$, then clear patterns of the parameter space appear, and a sharp efficiency peak emerge in the small world region. The value of $p$ (degree of randomness) that maximizes the average knowledge level augments monotonically with $\alpha$, though staying in the small world region, namely the part between $p = 0.01$ and $p = 0.1$. The maximizing $p$-value is always in this region, and most of the distribution of knowledge is also located in this area. Outside the small world region, for every value of $\alpha$, the long run average knowledge levels are smaller than the maximum. Therefore, we can affirm that the small world network structure maximizes the average knowledge level over agents, and consequently achieves the best aggregate performance in the broadcast economy. This is in line with the barter model explained above. However, in this broadcast model, the dispersion of knowledge levels (measured with the variance) is also maximized in the small world region. In other words, there is the highest disparity in knowledge levels between agents for a degree of randomness $p$ that increases monotonically with $\alpha$ in the area between $p = 0.01$ and $p = 0.1$. The source of
this variance comes from the fact that some individuals are left behind when knowledge is created. In every network structure people are left behind, but in the small world structure the agents who advance make it very rapidly, and this creates a big gap between distinct agents (Cowan and Jonard, 2000, pp. 4, 11-12, 16). Therefore, in the broadcast model, there is a trade-off between efficiency and equity.

As a conclusion, we can state that in this broadcast model the small world network is the unique structure that fosters knowledge diffusion and creation, and thus maximizes the average long run knowledge level. Nevertheless, the disparity of knowledge levels among agents is also maximized in the small world region. Therefore, we can remark that a trade-off between efficiency and equity persists. In the next section, we will provide an analysis of the spatial allocation of knowledge. More precisely, we will study which structure of network produces a spatial clustering of knowledge.

3.2.4 Spatial location of knowledge

In this section, we will study the spatial correlation of knowledge levels. This can be considered either in geographical space or in network space. A positive spatial correlation of knowledge is present, if agents that are close to each other have similar knowledge characteristics. On the other hand, a negative spatial correlation of knowledge appears, if knowledgeable agents are near agents that are “left behind”; in that case, no clustering of knowledge emerges. For small values of $p$, the geographic space has an analogous topology to network space. Therefore, if there are significant correlations for small values of $p$ in the network space, there will probably be echoes of them in the geographic space (Cowan and Jonard, 2000, p. 17). As an illustration, we provide Figure 10.
Figure 10. Spatial correlation in the geographical space

Source: Cowan and Jonard (2000, p. 17)

Figure 10 depicts the long run geographical correlation of knowledge levels as a function of $p$ and $\alpha$. It can be seen that there is almost no spatial correlation in this space. All values are small, and not statistically significantly different from 0. However, one region emerges for high $\alpha$ and low $p$. In this area, the correlations are still small, but differ significantly from 0. Nevertheless, the relation between $\alpha$, $p$ and the spatial correlation of knowledge levels has a lot of randomness (Cowan and Jonard, 2000, pp. 17-18).

If we consider the network space, we obtain a distinct picture as illustrated by Figure 11 (Cowan and Jonard, 2000, p. 18).

Figure 11. Spatial correlation in the network space

Source: Cowan and Jonard (2000, p. 18)
This figure indicates the Moran coefficient $S^{43}$ as a function of $p$ and $\alpha$. We obtain three different regions. The first is a large band that begins in the lower left corner of the $(p, \alpha)$-space (the structure is regular with low absorptive capacity), and continues to the upper right corner (the structure is random with high absorptive capacity) where the knowledge is distributed randomly among agents. There is a second region in which the structure is random and the absorptive capacities are not perfect, and in which a small negative correlation appears. Finally, we can observe positive spatial correlation in a third region where the relations between agents are very “cliquish”, and where there are high innovation rates. Thus, we can confirm that the correlation in the network space for small values of $p$ is echoed in the geographic space, but the echo is weak. Therefore, a small divergence between the geographic space (the space in which the knowledge shifts), and the network space (the space in which correlations are assessed) generates a distinct impression on the existence and strength of clustering (Cowan and Jonard, 2000, p. 18).

In the two above figures, there is clearly no small world effect on knowledge clustering or spatial correlation. The knowledge clustering present in these figures is only due to the property of “cliquishness” as the spatial correlation is at its highest level when $p = 0.001$. Besides, trying to introduce shortcuts to distant parts of the network has the only effect of impeding clustering (Cowan and Jonard, 2000, p. 18). Therefore, when we consider spatial allocation of knowledge, only network structures with the property of high “cliquishness” can produce clustering of knowledge. More precisely, the regular graph produces the highest clustering effects, but not the small world network, because the latter has also the property of shortcuts that reduce clustering.

In this chapter, we have characterized three important network structures: the regular graph, the small world and the random graph. Afterwards, we have

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$^{43}$ The Moran coefficient represents the spatial dependence. It ranges from $-1$ to $1$. If the values are close to $0$, this indicates that there is no spatial dependence, i.e. there is spatial randomness. A positive Moran coefficient represents positive spatial dependence, namely comparable values are located close to each other. Finally, a negative Moran coefficient indicates negative spatial dependence (Wieczorek and Hanson, 1997, p. 336).
analyzed these three network structures with a link with innovation. The analysis provides the following results. The small world network is the most efficient network structure in terms of knowledge creation and diffusion. The average long run knowledge level is maximized with this kind of network structure. Moreover, it seems that the small world network structure represents well real-world networks. The regular and random graphs only foster either creation or diffusion of knowledge, but not both at the same time. Therefore, it confirms that the small world is the unique efficient network structure both for knowledge creation and diffusion. However, the small world maximizes also the disparity in knowledge levels among agents. This is true, if we use the variance as the measure of the equity in knowledge among individuals, but this problem disappears, if we use the coefficient of variation of knowledge levels instead of the variance. Finally, if we consider the spatial allocation of knowledge, as it is the property of “cliquishness”, but not the property of short path length that produces the clustering of knowledge, it is not the small world network that produces the highest spatial clustering of knowledge, but it is the regular network. Nevertheless, we can definitely conclude that in terms of knowledge growth, the small world network is the most efficient network structure.

To conclude the first part of this study, we will briefly summarize the main results achieved. This first part analyzed the formation of networks, based on the economic theory of network. The purpose of this analysis was to find out which structures of networks arise in equilibrium (are stable) and are efficient. It has been shown that the empty network, the star network and the complete network are the network structures that arise mostly in equilibrium and are efficient, depending on the particular contexts and assumptions (static, dynamic, one-sided link formation or two-sided link formation). This study also analyzed in detail three important network structures, the regular graphs, the small worlds and the random graphs, and a link was established between these graphs and the issues of knowledge creation and diffusion. It has been proven that the structure of network that fosters more knowledge creation and diffusion, and thus maximizes the average long run knowledge level is the small world network structure. The first
part of this study analyzed social networks and innovation in a theoretical way, and the second part will analyze the relation between social networks and innovation in a more empirical way.

In the second part of this study, we will examine the influence of social networks on the spatial spread of innovation. The focus will be on the tight relation between distance and social networks, and their influence on the spatial diffusion of innovation. Nevertheless, the analysis will not concentrate on the different structures of networks that have the most important impact on the diffusion of innovation. We will not try to find out which structures of networks have the appropriate characteristics to determine empirically the spatial diffusion of innovation. The focus will be on the influence of social networks in general. Moreover, we will concentrate exclusively on the diffusion of innovation, and not on the creation of innovation. We will make an in-depth analysis of the influence of the distance and social networks on spatial diffusion of knowledge.
II Diffusion of innovation: relation between distance and social networks

The focus of the second part of this study is on the relation between distance and social networks, and their influence on the spatial diffusion of innovation. We will analyze whether geographic proximity and social interactions favor spatial diffusion of knowledge or spatial concentration of knowledge. The structure of the second part of this study is as follows. First of all, we will define the concepts of distance and innovation while focusing on the notions of tacit and codified knowledge. Afterwards, we will analyze in detail the relation between distance and social networks, and their influence on the spatial diffusion of innovation. To achieve this, we will introduce the concepts of technological and pecuniary externalities, co-authorships and patents citations. In the subsequent chapter, we will discuss the results obtained. Finally, in the last chapter, we will briefly examine the influence of NICT on the spatial diffusion of innovation. For now, we will analyze the notions of distance and innovation.

4 Concepts of distance and innovation

We already explained in detail in the first part of this study how exactly a social network is constructed, which ones are stable, etc. Therefore, we will not clarify it again. In this section, we will give an explanation of the notions of distance and innovation while focusing on the concepts of tacit and codified knowledge. To start with, we will analyze the concept of distance in relation with the theory of “New Economic Geography”, mostly investigated by Krugman in the 1990s. In a second place, we will define the notion of innovation.

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44 In this study, both the notion of innovation and the notion of knowledge have the same meaning and implications.
4.1 Notion of distance

When we consider geographic proximity, implicitly, we introduce the notion of distance. Therefore, does geographic proximity increase social interactions, and does the latter foster innovation? Does innovation spread locally or globally? We will try to answer these questions throughout the second part of this study.

4.1.1 Some insights from the New Economic Geography

To have a characterization of the notion of distance, we will introduce the agglomeration model of Krugman (1991). The conclusion of this model is that, if different conditions are satisfied and according to values of distinct parameters, economic activities will concentrate in space and produce a divergence in the economic growth of distinct regions. There will normally be a concentration of economic activities in space given that the following assumptions are satisfied: exchange costs are low (i.e. the economic integration is rather high), there are imperfect competition, labor mobility, increasing returns to scale, pecuniary externalities and product differentiation (which limits price competition). Under the assumption of increasing returns to scale, benefits arise from locating the production of goods and services in a specific region and from increasing the production in this location. The location will have the largest nearby demand, minimizing transportation costs, while other locations will be deserved from this central one (Krugman, 1991, pp. 485-86, 488, 493). This concentration in space can lead to a critical mass of knowledge workers with specialized skills, and then become an epistemic community that will foster innovation which diffuses mainly locally. Moreover, this innovation activity will create knowledge externalities among knowledge workers which arise more intensely at a local level (Fujita and Mori, 2005, p. 12). Thus, we can state that under the assumption of increasing

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46 We will not develop in detail this agglomeration model. For precise information on this model, also called “core-periphery” model, you can read Krugman (1991).

47 We will analyze, in the next chapter, which kind of externalities (technological or pecuniary) are mostly present in the problematic of knowledge diffusion.
returns to scale, the spatial diffusion of innovation seems to be mostly concentrated in space.

The concentration of activities fosters the emergence of both positive pecuniary and technological externalities. In other words, they are a consequence of the process of concentration of activities (Krugman, 1998, p. 8). Technological externalities can, for example, improve the communication between knowledge workers, thus developing tacit knowledge. Another advantage is an enhanced formation of workers that can reduce production costs. There are many advantages stemming from technological externalities, and these advantages foster knowledge diffusion which seems to occur mainly at a local level. As a result, we can say that both technological and pecuniary externalities have an influence on the spatial diffusion of innovation which becomes mostly concentrated in space.

But what is exactly assumed under the notion of distance? This concept is a black box. In Krugman’s agglomeration model, the distance is reflected in terms of transportation costs (Krugman, 1991, p. 498). Besides, behind the notion of distance, the concept of social networks appears. These accentuate mostly the spatial concentration of knowledge diffusion. In other words, social networks foster innovation diffusion, but the latter appears to remain concentrated in space. However, we need further investigations to confirm this preliminary conclusion.

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48 In Krugman’s agglomeration model, only pecuniary externalities are taken into account. However, in the problematic of innovation diffusion, it is necessary to consider also technological externalities, and especially knowledge externalities which are an element of technological externalities. More precisely, technological externalities are interaction effects which occur off market, and affect consumers or producers’ utility. In particular, technological externalities occur when a company benefits from the research and development (R&D) undertaken by other firms without compensating them for the advantage received (Breschi et al., 2005, p. 344). Finally, according to Griliches (1992, p. 36; cited by Breschi and Lissoni (2003, p. 2)), technological externalities are “ideas borrowed by research teams”. Pecuniary externalities are interaction effects that arise through usual market’s mechanisms (e.g. price). In particular, pecuniary externalities occur when new or improved inputs are sold, but the producer is not able to completely capture the increased quality of the sold inputs (Breschi et al., 2005, p. 344). Finally, according to Griliches (1992, p. 36; cited by Breschi and Lissoni (2003, p. 2)), pecuniary externalities are “R&D intensive inputs […] purchased at less than their full “quality” price”. 
4.1.2 Physical distance vs. social distance

It is also important to note that two kinds of distance exist, namely physical distance and social distance. These two types of distance have an influence on the problematic of innovation diffusion. We will now briefly comment the differences between these two concepts. Social distance between two agents or firms can be measured through the network position of each agents or firms, i.e. the number of direct and indirect partners of each individual or firm. Social distance can account for network effects which are due to prior relationships or preferential attachments. Concerning the notion of physical distance between two agents or firms, its signification is already rather clear. Physical distance between each pair of agents or firms can be measured, for example, by the number of kilometers or meters between them. The physical distance accounts for spatial effects (Autant-Bernard et al., 2007a, pp. 2-3). It is important to distinguish these two notions as their effects on the spatial diffusion of knowledge are distinct.

4.2 Notion of innovation

We will now define the notion of innovation, focusing on the concepts of tacit and codified knowledge. The process of innovation can be tacit or codified, or both at the same time. Consequently, it is important to know which type of knowledge is most likely to arise in the problematic of spatial diffusion of innovation.

4.2.1 Introduction

According to Loilier and Tellier (2001), innovation is the utilization, the transformation or the improvement of existing resources that result in new products or processes which have a substantial impact on potential users (Loilier and Tellier, 2001, p. 561). According to Rogers (1998), a technological product innovation is a new or improved product, i.e. goods and services, which results from new materials, technologies or knowledge, and a technological process
innovation is the adoption of new or improved production methods (Rogers, 1998, p. 7). Finally, according to Brown and Duguid (1991), “the source of innovation lies on the interface between an organization and its environment. And the process of innovating involves actively constructing a conceptual framework, imposing it on the environment, and reflecting on [...] interaction.” (Brown and Duguid, 1991, p. 51). We can remark that innovation is related to interactions and social context. Innovation is a collective and interactive process, namely it can be further created and spread, if many interconnected scientists work on it. In other words, knowledge can be created and diffused easier, if increasing interactions appear between agents in scientific and technological networks (Autant-Bernard et al., 2007a, p. 2). It is quite simple to characterize innovation, but the difficulties arise when we try to measure it. In fact, it is not possible to measure innovation perfectly. There are two kinds of indicators of innovation, namely output indicators such as number of patents, citations of patents, number of scientific and technical publications, process innovations, new or improved products introduced on the market, and input indicators such as R&D expenditures, number of researchers, managerial and organizational expenditures. Most of the studies that assess the spatial spread of innovation introduce the data collected for the indicators just described in a knowledge production function. However, it is possible that the results of a study change due to the nature of the innovation’s indicator chosen. Hence, it is important to evaluate the degree of correlation of the above indicators. The degree of correlation depends on the industries analyzed and whether they are high-tech. For high-tech manufacturer sector, there is no disparity between the different indicators of innovation. Therefore, any above indicator can be used in a study without changing the results. The opposite is true for low-tech and services sectors. In this case, the correlation between the different indicators of innovation is rather weak. In other words, changing the indicator of innovation in a study will affect the results (Autant-Bernard and Massard, 2004a, pp. 2-4 and Rogers, 1998, pp. 10, 17). Finally, we can state that it is difficult to measure innovation exactly, because the way an innovation is created and how it spreads over space and time is typically modeled through a nonlinear process.
We will now describe the notion of tacit knowledge and then the concept of codified knowledge.

4.2.2 Tacit knowledge

The notion of tacit knowledge refers to information or knowledge which is in the background of our consciousness, and thus is difficult to communicate to other individuals. This can be illustrated by the famous aphorism from Polanyi\footnote{Polanyi, M. (1966) *The Tacit Dimension*, New-York: Doubleday.} “we can know more than we can tell” (Gertler, 2002, p. 77). The ways to overcome this communication difficulty, and to acquire and communicate tacit knowledge is to demonstrate it, imitate it or make shared experiences and performances. Tacit knowledge may be produced privately or collectively. It is, however, mostly produced collectively, and it is mainly defined by social context. Nevertheless, tacit knowledge can also define social context\footnote{Rules and social contexts are mainly produced through local and regional institutions (education institutions, financial systems, etc.). The ability of agents to share tacit knowledge depends on institutional proximity, namely shared values, norms, conventions which come from commonly experienced frameworks of institutions (Gertler, 2002, pp. 90-91).} (Gertler, 2002, pp. 78, 89). Tacit knowledge needs a physical transaction to be widespread as this knowledge is highly context dependent, and is mostly acquired experientially and through interactions between agents. Consequently, tacit knowledge is very costly to transmit at large distance\footnote{It is costly to transmit tacit knowledge at large distance, because culture, institutional norms, etc. from remote regions are very different from local ones. Thus, cultural and institutional barriers exist between distinct regions (Gertler, 2002, p. 95).} (Storper and Venables, 2005, p. 319 and Gertler, 2002, p. 79). Hence, tacit knowledge is dependent on face-to-face contacts. It follows from these statements that spatial proximity between agents or knowledgeable workers improves the flow of tacit knowledge, and thus creates localized, industry-specific knowledge spillovers within technology-based industries. Moreover, individuals can absorb knowledge from contacting more skilled agents. To sum up, face-to-face contacts are an appropriate channel through which tacit knowledge can flow. Face-to-face contacts are a good communication technology particularly when the transmission of knowledge can not be codified. However, even though face-to-face contacts are an efficient technology of transacting, they
are costly in terms of time consuming as we do not have the possibility to build face-to-face contacts with everybody in the world, and thus we have to screen out individuals with whom we want to have relations. Moreover, the screening of individuals is quite difficult, because we have to know if this person is valuable, and this is mostly reflected in her tacit knowledge which is not easily communicated (Storper and Venables, 2005, pp. 319-21, 325, 335).

Tacit knowledge is not only enhanced through face-to-face contacts. It is also promoted by routines and established practices shaped by communities of practice. In this case, organizational or relational proximity is more important than spatial proximity in supporting the diffusion of tacit knowledge. We can recognize here the distinction made in the previous chapter between social and physical distance. If relational proximity that stems from communities of practice is strong enough, then social distance will overcome physical distance. In other words, tacit knowledge can flow outside geographical boundaries as long as it is mediated within these communities. This phenomenon is named by Bunnell and Coe (2001; cited by Gertler (2002, p. 86)) the “de-territorialisation of closeness”. But what are the forces that define this relational proximity which is able to surpass physical, cultural and institutional divisions? How are shared experiences or shared understandings produced? What is the role of social context in the transfer of tacit knowledge between members of a community? The literature on communities of practices does not give a clear answer to these questions. Nevertheless, the works of Brown and Duguid stand out (Gertler, 2002, pp. 86-87). “In their view, the narratives and social ties so crucial to the flow of knowledge within communities of practice are deeply embedded within the social systems in which they arise.” (Gertler, 2002, p. 88).

52 Communities of practice are groups of workers which are informally tied together by expertise, shared experiences, etc. (Gertler, 2002, p. 86). They are inherently innovative (Brown and Duguid, 1991, p. 50).
As a conclusion, we can state that, if the kind of knowledge that exists is mostly tacit, then spatial proximity is required so as tacit knowledge can flow, and face-to-face contacts are the appropriate channel through which tacit knowledge can spread. However, tacit knowledge does not need spatial proximity to diffuse, if communities of practice are present. In that case, organizational or relational proximity stemming from communities of practice can overcome spatial proximity. Both notions of face-to-face contacts and communities of practice refer to social networks. Therefore, it can be seen that the influence of social networks on the spatial spread of tacit knowledge is rather equivocal. Consequently, we need further investigations to know if the spatial diffusion of tacit knowledge is mostly concentrated in space, and what is the influence of social networks. We will provide an analysis on this subject in the upcoming sections. We will now discuss the notion of codified knowledge.

4.2.3 Codified knowledge

According to Cowan et al. (2000), “its obvious reference is to codes, or to standards – whether of notation or of rules, either of which may be promulgated by authority or may acquire “authority” through frequency of usage and common consent, i.e. by de facto acceptance.” (Cowan et al., 2000, p. 225). Codified knowledge is a kind of knowledge which is expressed in a specific symbol system that can be linguistic, mathematical or visual. A codified knowledge or information is relatively cheap to transfer as its underlying symbol system is broadly broadcasted through information infrastructure, and this reduces the marginal cost of individual information. However, the setting up of the symbol system can be relatively expensive and long (Storper and Venables, 2005, pp. 321-22), because an agent that will understand the symbol system must acquire great specialized knowledge (Cowan et al., 2000, p. 225). It follows from the above statements that codified knowledge diffuses mainly globally in space, and thus does not need face-to-face contacts to spread. Therefore, geographic proximity is not required in this case. If knowledge that exists is mostly codified, then social networks, through face-to-face contacts or communities of practice,
have less influence on its spatial diffusion. Finally, an advantage of the diffusion of codified knowledge over the diffusion of tacit knowledge is that the former is more easily measurable than the latter (Gertler, 2002, p. 82).

The boundary between tacit and codified knowledge is, however, not easily detectable. Are R&D expenditures and patents tacit or codified knowledge? At first sight, it seems that R&D expenditures are rather tacit knowledge and patents are rather codified knowledge. However, as it is very difficult to define and measure tacit knowledge, it is also possible that patents are of tacit kind. As a result, there can be tacit and codified knowledge at all stages of the innovation process. Thus, it is difficult to exactly differentiate these two kinds of knowledge so as to know whether R&D expenditures and patents are of tacit or codified kind.

Having described what is meant by the notions of distance and tacit and codified knowledge, we will analyze, in the next chapter, the relation between distance and social networks and their influence on the spatial diffusion of knowledge. Is innovation diffusion concentrated or disseminated in space? We stated above that it appears to be mostly geographically bounded. In order to confirm this first conclusion, we will analyze, in the next chapter, several econometric studies about this problematic.

5 Spatial diffusion of innovation and social networks

In this chapter we will analyze the spatial diffusion of externalities. As they are at the same time a cause and a consequence of the creation and diffusion of knowledge, they are of great importance in the problematic of the spatial spread of innovation. In the first section, we will examine whether externalities are either technological or pecuniary ones, and how they diffuse throughout space. We will analyze the diffusion of externalities with the help of the econometric study of Autant-Bernard and Massard (2004b), and then conclude whether the diffusion of knowledge is geographically bounded. In the second section, we will analyze two
econometric studies that test the influence of social networks on the spatial spread of externalities, using patent citations and co-authored publications. The influence of social networks included, we will then be able to conclude whether knowledge is mainly concentrated in space.

5.1 Spatial diffusion of externalities: technological or pecuniary externalities?

This section introduces the notion of externalities, and analyzes, with the help of an econometric study, if they are bounded in space. The main focus is to examine which type of externalities (technological or pecuniary) mostly arises in the problematic of spatial knowledge diffusion.

5.1.1 Introduction

We already described in the previous chapter what exactly technological and pecuniary externalities are. To recall, technological externalities are interaction effects that occur off market, and pecuniary externalities are externalities which are based on market interactions. In the economic geography models, the interdependence between the location of consumers and firms is due to pecuniary externalities and not technological externalities. The agglomeration of both firms and consumers is dependent on the local character of pecuniary externalities (Autant-Bernard and Massard, 2004b, pp. 2, 6 and Breschi *et al.*, 2005, p. 344). However, in the endogenous growth theory, knowledge externalities are factors of agglomeration (Autant-Bernard and Massard, 2001b, p. 2). Therefore, we can state that pecuniary externalities, as well as technological externalities, are agglomeration forces (Autant-Bernard and Massard, 2004b, p. 3). Nevertheless, technological and pecuniary externalities are not only agglomeration forces. As already stated in the subsection 4.1.1, they are also a consequence of the agglomeration of economic activities. Nonetheless, the existence of knowledge
externalities, which are an element of technological externalities, is not only linked to agglomeration process. Knowledge externalities also have a proper existence thanks to the innovation process by itself. Finally, according to Audretsch and Feldman (1999; cited by Autant-Bernard and Massard (2004, p. 4)), externalities are affected positively by geographical proximity and by technological diversity. However, the relation between externalities and geographic proximity is not automatically obvious. For example, spillovers can also be international. Consequently, it is necessary to test empirically, if externalities are bounded in space, and which type of externalities mostly appears (Autant-Bernard and Massard, 2001b, p. 3).

During the 1990’s and the beginning of 2000’s, empirical studies of knowledge externalities have progressed. Originally, knowledge externalities were revealed through a concentration or coincidence index. Then, knowledge externalities have been analyzed more directly with the use of patent citations, or with the insertion of interregional flows in the estimation of knowledge production functions. More recently, the measure of the spatial dimension of knowledge externalities, which are an element of technological externalities, is not only linked to agglomeration process. Knowledge externalities also have a proper existence thanks to the innovation process by itself. Finally, according to Audretsch and Feldman (1999; cited by Autant-Bernard and Massard (2004, p. 4)), externalities are affected positively by geographical proximity and by technological diversity. However, the relation between externalities and geographic proximity is not automatically obvious. For example, spillovers can also be international. Consequently, it is necessary to test empirically, if externalities are bounded in space, and which type of externalities mostly appears (Autant-Bernard and Massard, 2001b, p. 3).

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externalities has been enhanced by spatial econometrics tools (Autant-Bernard et al., 2007b, p. 342). “In particular, the modelling of externalities in knowledge production functions has been refined and the tests for spatial autocorrelation have permitted to assess more precisely their geographical range.” (Autant-Bernard et al., 2007b, p. 342). All the above studies conclude that spatial proximity fosters the diffusion of knowledge. Nevertheless, the geographical diffusion of knowledge can differ from different countries and from distinct industries (Autant-Bernard et al., 2007b, p. 342), and the local dimension of knowledge externalities is not always evident (Autant-Bernard and Massard, 2001b, p. 19). In order to confirm the conclusion of the above studies, we will analyze in detail the econometric study of Autant-Bernard and Massard (2004b). They examine which kind of externalities (technological or pecuniary) mostly arise in the problematic of knowledge diffusion, and if they are spatially bounded.

5.1.2 Econometric study of technological and pecuniary externalities

Autant-Bernard and Massard (2004b) provide series of equations to analyze the relation between pecuniary and knowledge externalities, productive (and consequently innovative) capability of a firm, and distance. The first equation they test is a Griliches production function. The output of the firm $i$ is related to

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57 For example, Fingleton and López-Bazo (2006) assume that externalities across regions in long-run growth are mainly a substantive phenomenon due to technological diffusion and pecuniary externalities. More precisely, they “base the analysis on a structural growth model including externalities across economies, and apply the appropriate spatial econometrics tools to test for their presence and estimate their magnitude in the real world.” (Fingleton and López-Bazo, 2006, p. 179).

58 As an example, Varga (2000) examine “agglomerations effects on the intensity of local knowledge spillovers from universities to high technology innovations […] within the modified Griliches-Jaffe knowledge production function framework.” (Varga, 2000, abstract). His estimations concern the level of US metropolitan areas. He finds that the concentration in space of high technology workers is the major factor fostering local academic externalities. In other words, agglomeration has positive effects on local academic knowledge externalities (Varga, 2000, abstract p. 18).

59 We will only analyze the first two equations and not the third equation that Autant-Bernard and Massard (2004b) test, because this third equation and its implications are beyond the scope of our study.

60 To recall, knowledge externalities are a component of technological externalities.
capital ($K$), labor ($L$), internal R&D ($RD$), and spillovers arising from features of the local environment ($Z$). Thus, the equation is the following:

$$Q_t = \alpha + \beta_1 K_t + \beta_2 L_t + \beta_3 RD_t + \beta_4 Z + \varepsilon_t$$  \hspace{1cm} (5.1.2.1)

where $\alpha$ is the constant and $\varepsilon_t$ is a random disturbance. Features of local environment ($Z$) include factors that create pecuniary externalities, namely the number of firms that are locally present and the size of the employment market, and include also a measure of knowledge externalities through the intensity of private research from firms or organizations that are located close by. Estimations of this equation will allow an assessment of the relative weight of pecuniary and knowledge externalities. The second equation the authors test accounts for the impact of the distance on these externalities. To enable this, they measure the features of the local environment on distinct geographical levels, namely the knowledge produced in an agglomeration (French department) and the knowledge generated on the periphery of an agglomeration. The equation is the following:

$$Q_t = \alpha + \beta_1 K_t + \beta_2 L_t + \beta_3 RD_t + \beta_4 Z + \beta_5 WZ + \varepsilon_t$$  \hspace{1cm} (5.1.2.2)

where $W$ is a contiguity matrix of order $n$. They use techniques of spatial econometric, namely Anselin’s tests for spatial autocorrelation, to establish the geographical level that is relevant to define the periphery (Autant-Bernard and Massard, 2004b, pp. 4, 7-8).

In order to perform their study, Autant-Bernard and Massard (2004b) rely on the cross-mapping of two national surveys, namely the French Annual Company Survey (EAE) produced by the Ministry of Industry and the Institut National de la Statistique et des Etudes Economiques (INSEE), and the R&D survey carried out by the French Ministry of Research. These surveys are available at plant level.

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61 The model is specified in logarithms (Autant-Bernard and Massard, 2004b, p. 7).


63 Contiguity of order 1 or higher (Autant-Bernard and Massard, 2004b, p. 8).

64 Nevertheless, data from EAE used to estimate a Griliches production function are only available at firm level and not plant level, and the analysis of the geographical dimension necessitates a view at plant level. Therefore, estimations have been completed, as a first approach, on firms with one plant (1731 companies listed in the EAE carried out research between 1997 and 1998, and a sample of 822 companies was obtained after leaving aside multiplant firms). The impact of this restriction in terms of spatial distribution seems to be minimal since the sample’s regional proportion is the one observed for all establishments making research. However, the bias seems
This cross-mapping generates a sample of 822 firms with their workforce, sales, purchases of raw materials and intermediary consumptions, and internal R&D expenditures (IRDE). “The sales, workforce and purchases were taken from 1999. R&D is the combined IRDE figure for 1997 and 1998.” (Autant-Bernard and Massard, 2004b, p. 10). The features of the local environment are measured by “the number of companies present locally, the number of employees present locally, as a proxy of the labour market size but also of the final demand, [and] the local knowledge production intensity, measured by other companies’ R&D expenditure.” (Autant-Bernard and Massard, 2004b, pp. 10-11). To estimate the influence of the distance on knowledge and pecuniary externalities, the variables are measured for two geographic levels. The first level is NUTS 3 (the French department), and the second level is the bordering French departments. This gives individual data which allow to fine tune the results gotten at an aggregate level (Autant-Bernard and Massard, 2004b, pp. 11-12, 17).

Results of the estimations of Autant-Bernard and Massard (2004b) are given in Table 3 which provides information about location economies. Table 2 summarizes the set of variables that is used for the estimations (Autant-Bernard and Massard, 2004b, pp. 11-12).

### Table 2. List of variables

<table>
<thead>
<tr>
<th>Name</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>CA</td>
<td>Dependant variable: sales</td>
</tr>
<tr>
<td>LABOR</td>
<td>Employed workforce at the company</td>
</tr>
<tr>
<td>CAPITAL</td>
<td>Purchases of goods, raw materials and other supplies</td>
</tr>
<tr>
<td>DIRD</td>
<td>Firm’s internal R&amp;D expenditures</td>
</tr>
<tr>
<td>ETSDPTE</td>
<td>Number of firms in the same sector present in the department</td>
</tr>
<tr>
<td>ETSLIM</td>
<td>Number of firms from the same sector present in neighboring departments</td>
</tr>
</tbody>
</table>

larger in terms of size (the employed workforce for all establishments which make research is 1’179 compared to 412 in the sample). Moreover, this restriction would possibly conduct to an over-estimation of the role of the spatial dimension. This is confirmed by Henderson (2003; cited by Autant-Bernard and Massard (2004, p. 9)) who points out, in his study, that single plant enterprises depend more on local resources than establishments that belong to groups (Autant-Bernard and Massard, 2004b, pp. 8-9).
<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tbody>
<tr>
<td>Constant</td>
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<td>3.79***</td>
<td>3.99***</td>
<td>4.23***</td>
<td>4.11***</td>
<td>4.23***</td>
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<tr>
<td></td>
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<td>(0.22)</td>
<td>(0.22)</td>
<td>(0.25)</td>
<td>(0.22)</td>
<td>(0.25)</td>
</tr>
<tr>
<td>Ln(LABOR)</td>
<td>0.67***</td>
<td>0.68***</td>
<td>0.68***</td>
<td>0.67***</td>
<td>0.67***</td>
<td>0.67***</td>
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<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Ln(CAPITAL)</td>
<td>0.26***</td>
<td>0.26***</td>
<td>0.26***</td>
<td>0.26***</td>
<td>0.26***</td>
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<td>(0.04)</td>
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<tr>
<td>Ln(DIRD)</td>
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<td>0.12***</td>
<td>0.11***</td>
<td>0.12***</td>
<td>0.12***</td>
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<td>(0.02)</td>
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<tr>
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<td>-</td>
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<td>0.79E-01***</td>
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<tr>
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<td>-</td>
<td>(0.26E-01)</td>
<td>(0.20E-01)</td>
<td>(0.26E-01)</td>
</tr>
<tr>
<td>Ln(ETSLIM)</td>
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<td>-</td>
<td>-</td>
<td>0.82E-01**</td>
<td>0.47E-01**</td>
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<td>(0.17E-01)</td>
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<td>-</td>
<td>(0.41E-01)</td>
<td>(0.23E-01)</td>
<td>(0.41E-01)</td>
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<tr>
<td>Ln(SALDPT)</td>
<td>-</td>
<td>0.55E-01***</td>
<td>-</td>
<td>0.45E-02</td>
<td>-</td>
<td>0.68E-02</td>
</tr>
<tr>
<td></td>
<td>(0.13E-01)</td>
<td>-</td>
<td>-</td>
<td>(0.21E-01)</td>
<td>(0.22E-01)</td>
<td>(0.22E-01)</td>
</tr>
<tr>
<td>Ln(SALLIM)</td>
<td>-</td>
<td>0.26E-01*</td>
<td>-</td>
<td>-0.41E-01</td>
<td>-</td>
<td>-0.39E-01</td>
</tr>
<tr>
<td></td>
<td>(0.15E-01)</td>
<td>-</td>
<td>-</td>
<td>(0.35E-01)</td>
<td>(0.35E-01)</td>
<td>(0.38E-01)</td>
</tr>
<tr>
<td>Ln(RDDPT)</td>
<td>-</td>
<td>-</td>
<td>0.31E-01***</td>
<td>-</td>
<td>-0.20E-02</td>
<td>-0.28E-02</td>
</tr>
<tr>
<td></td>
<td>(0.09E-01)</td>
<td>-</td>
<td>-</td>
<td>(0.11E-01)</td>
<td>(0.12E-01)</td>
<td>(0.12E-01)</td>
</tr>
<tr>
<td>Ln(RDLM)</td>
<td>-</td>
<td>-</td>
<td>0.10E-01</td>
<td>-</td>
<td>-0.58E-02</td>
<td>-0.12E-02</td>
</tr>
<tr>
<td></td>
<td>(0.06E-01)</td>
<td>-</td>
<td>-</td>
<td>(0.91E-02)</td>
<td>(0.95E-02)</td>
<td>(0.95E-02)</td>
</tr>
<tr>
<td>Sectoral dummies</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.921</td>
<td>0.920</td>
<td>0.919</td>
<td>0.921</td>
<td>0.921</td>
<td>0.921</td>
</tr>
<tr>
<td>Obs.</td>
<td>822</td>
<td>822</td>
<td>822</td>
<td>822</td>
<td>822</td>
<td>822</td>
</tr>
</tbody>
</table>

* significant at 10% level; ** significant at 5% level; *** significant at 1% level. Numbers in bracket are standard errors.

Source: adaptations from Autant-Bernard and Massard (2004, p. 13)

The first estimations made by Autant-Bernard and Massard (2004b) are on the geographical dimension. Estimation (1) examines the effect of neighboring firms.
The results are close to Henderson’s estimations (2003) for the United States. The productive capacity of a firm is increased, if it is located in a region with many firms of the same industry. This is confirmed by the elasticity of production to local number of plants which is 0.08 (and significant at 1 percent level) for the department scale, and 0.04 (but only significant at 5 percent level) for bordering departments. Thus, pecuniary externalities are spatially bounded as the elasticity is decreasing with distance. Estimation (2) evaluates the “home market effect” which is approximated by the number of workers. This “home market effect” is significant at 1 per cent level at the department scale. However, when distance increases, the estimated parameter is only significant at 10 per cent level. Therefore, it is not possible to conclude with certitude about the effect of workers when distance increases. However, this tends to confirm the assumption of a local spread of pecuniary externalities. Estimation (3) introduces knowledge externalities. R&D made in a region increases the production of local plant (inside the region, the elasticity is of 0.03, and the estimated parameter is significant at 1 per cent level and thus is highly reliable), but when distance increases, R&D expenditures are not significant anymore. This result supports the assumption under which knowledge externalities spread only locally. Estimations (4) to (6) bring together the forces mentioned above (neighboring firms, number of workers and R&D expenditures) in order to assess their relative weight. The results emphasize strong agglomeration effects from the proximity between plants. This is the main attraction force since local R&D and the number of workers become non-significant and, in some cases, negative. The absence of local R&D as agglomeration effect is quite confusing. It means that the main attraction force depends on the presence of other plants, independent from the level of R&D expenditures. Nevertheless, this does not mean that there are no knowledge spillovers. Knowledge spillovers result from productive activities, but not from specific knowledge productions (Autant-Bernard and Massard, 2004b, pp. 12-14).

We can conclude that pecuniary externalities, through the presence of other plants are, according to the study of Autant-Bernard and Massard (2004b), the main agglomeration force, and that they are geographically bounded. Knowledge
externalities are only a by-product of the determinants of pecuniary externalities, but they are also spatially bounded. In other words, pecuniary externalities have a stronger role in the productive capability of a firm, and, consequently, in the creation and diffusion of knowledge than knowledge externalities have. Since the diffusion of pecuniary externalities is spatially bounded, this tends to restrict the spatial spread of knowledge. Therefore, in the study of Autant-Bernard and Massard (2004b), the agglomeration or concentration of firms is an innovation-explaining variable (Autant-Bernard and Massard, 2001b, p. 4). However, these conclusions have to be considered carefully since other potential attraction forces can have been neglected (Autant-Bernard and Massard, 2004b, p. 14).

The conclusions of the study of Autant-Bernard and Massard (2004b) can approximately be confirmed by the study of Audretsch and Keilbach (2007). They examine the creation of new firms at the county level in Germany to know whether entrepreneurial capital is situated in one region. Their analysis shows a positive spatial auto-correlation which suggests that knowledge and capability for entrepreneurship, and thus for creating new activities and innovative firms, diffuse only at a local level and does not spill over to neighboring regions. In other words, the diffusion of knowledge externalities is spatially bounded. Nevertheless, in this analysis, also pecuniary externalities arise through the presence of other firms, and the diffusion of this kind of externalities is also spatially bounded (Autant-Bernard et al., 2007b, pp. 344, 347 and Audretsch and Keilbach, 2007, pp. 1, 12).

Social networks have increasing importance in the process of knowledge creation and diffusion, and this raises challenges for the analysis of the spatial dimension

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65 It is worth noting that the study of Autant-Bernard and Massard (2004b) focuses on static externalities, i.e. they focus on the influence of existing local industrial environment, and not on dynamic externalities. Their study should include dynamic externalities in order to analyze the influence of transformations in the economic structures on production and agglomeration. A second problem arises in the study of Autant-Bernard and Massard (2004b). The results from the sample bring out multicollinearity, and have potential endogeneity problems. Actually, the number of employees, research laboratories or plants in a given region can vary according to local wages, tax levels, etc. These local specificities can also explain the distinct levels of production and productivity between plants. Such mechanisms can generate endogeneity (Autant-Bernard and Massard, 2004b, pp. 18-19).
of externalities. In the next section, we will test the influence of social networks on the spatial diffusion of knowledge.

5.2 Measure of the spatial diffusion of externalities and influence of social networks

In this section, we will analyze two econometric studies which present different methods (patent citations and co-authored publications) of measuring externalities. We will examine the impact of social networks on the spatial diffusion of externalities. Even though we concluded in the previous section that pecuniary externalities have a stronger role than knowledge externalities in the problematic of spatial diffusion of innovation, in this section we will focus on the measure of knowledge externalities. This kind of externalities is more complicated to measure than pecuniary externalities, and thus raises more challenges. Pecuniary externalities are based on market interactions such as price and, consequently, are more easily quantifiable than knowledge externalities which arise from non-market interactions. It is also important to focus on knowledge externalities as they have a non-negligible role in the diffusion of innovation. However, even if we focus on knowledge externalities, it is possible that we have indeed to do with pecuniary externalities as the border between these both types of externalities is not so easily discernable.

5.2.1 Introduction

Trying to define and measure knowledge externalities and to estimate their geographical dimension is a hard task as it is difficult to find (Autant-Bernard et al., 2007b, pp. 341-42) “localised data on innovation and knowledge processes and [to address] the methodological issues due to the spatial aggregation of the available data.” (Autant-Bernard et al., 2007b, p. 342). Moreover, “[the] ways and conditions of transmission considerably depend on the origin of transmitted
externalities, of modes taken by the individual interactions and at last of the determinants of absorptive capacity.” (Autant-Bernard and Massard, 2001b, p. 19). The origin of externalities is related to the nature of the knowledge that is transmitted. Thus, it can be public or private, fundamental or applied, or tacit or codified. Finally, there are also problems in the externalities’ measure itself, and in the consideration of the dimension of the space (Autant-Bernard and Massard, 2001b, pp. 17, 19-20). Consequently, trying to measure knowledge externalities raises challenges.

A method that tries to measure knowledge externalities consists of searching indicators of technological externalities. This has been done by many studies which are based on patent citations.66 These studies use patent citations as a “paper trail” of technological spillovers. However, it is not so easy to find a relevant indicator of technological externalities, because the latter cannot be directly measured (Autant-Bernard and Massard, 2001b, p. 4). We will analyze now the empirical study of Breschi and Lissoni (2003) which belongs to the kind of studies just described. Breschi and Lissoni (2003) examine patent citations in order to take into account the influence of social networks on the spatial diffusion of knowledge externalities.

5.2.2 Empirical study of patent citations

Breschi and Lissoni (2003) test the existence and magnitude of localized knowledge externalities. They exploit information in patents to control for the mobility of researchers and for network links that this mobility establishes. More precisely, Breschi and Lissoni (2003) select three samples of “originating” patents, consisting of 1987, 1988 and 1989 patent applications. In each cohort, they incorporate all patent applications by Italian firms and institutions to the European Patent Office (EPO) that obtained at least one subsequent citation by the

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end of 1996. For the construction of the citing sample, Breschi and Lissoni (2003) take into account only patent applications made before 1996 inclusive. Finally, they take the primary classification code at the 4-IPC-digit level from each citing patent, and create with this a sample of “control” patents (Breschi and Lissoni, 2003, abstract pp. 10-11).

Breschi and Lissoni (2003) measure the frequency of matching by geographic area between cited and citing (control) patents. They state that two patents match geographically, if they share at least one inventor’s location (Breschi and Lissoni, 2003, p. 11). Moreover, they evaluate the relative importance of pre-existing linkages among patents on the probability of a spatial match. Thanks to names, addresses, company affiliation etc. of each inventor, patents enable to measure one type of linkage arising from the participation in the same group of inventors. “As the composition of teams changes among patents and over time, exploring the composition of teams and their evolution permits to reconstruct the network of collaborative relationships linking inventors (and, through them, patents).” (Breschi and Lissoni, 2003, p. 12). Using a graph that describes relations between patents and inventors allow measures of connectedness among pairs of patents to be derived. The challenge is to use information from the network of inventors to determine the existence of a linkage between citing (control) and cited patents, besides the link that comes from the citation or non-citation itself.

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67 “The 1987 originating cohort contains 699 patents that had received a total of 1’631 citations by the end of 1996. The 1988 originating cohort contains 843 patents that had received a total of 1’784 citations by the end of 1996. The 1989 originating cohort contains 779 patents that had received a total of 1’615 citations by the end of 1996.” (Breschi and Lissoni, 2003, p. 10). Breschi and Lissoni (2003) eliminate patent applications made by individual inventors. For each cohort of originating patents, they also exclude applications which either obtained citations only from foreign organizations, or whose applicant was an Italian organization, but did report an inventor who is not Italian. This implies that their study analyzes the extent of intra-national localization of patent citations and not international localization (Breschi and Lissoni, 2003, p. 10).

68 Breschi and Lissoni (2003) exclude all observations in which the applicant of originating and citing patents is the same (Breschi and Lissoni, 2003, p. 11).

69 “Specifically, for each citing patent [they identify] all patents in the same patent class with the same application year. [They] then [choose] from that set a control patent whose application date [is] as close as possible to that of the citing patent, and that [does] not cite the same originating patent. The resulting data set therefore consists of all “originating” patents, for which there is a matching of citing and control patents. In turn, each citing patent is paired with a specific control patent within the same technological class and […] with (approximately) the same application date. The final sample consists of 366 originating patents, which have received 483 citations from other Italian organisations.” (Breschi and Lissoni, 2003, p. 11).
done analyzing a pre-existing network of collaboration among inventors. It is possible to derive a measure of the extent to which citing (control) and cited patents are joined by links (other than the citation itself). If there is a linkage (perfect, direct or indirect) between two patents (citing (control) and cited patents), they are connected. Otherwise, they are not connected (Breschi and Lissoni, 2003, pp. 12, 14, 16).

To implement the ideas just described, Breschi and Lissoni (2003) create a biographical dataset based on every patent application at the EPO from 1978 to 1999 which included at least one Italian inventor. The database holds information on 30’170 inventors (surname, name and address), and on 38’868 patent applications (name and address of the applicant, application date and year, and technology classification code). Thanks to this dataset, Breschi and Lissoni (2003) construct the affiliation network of patents, inventors and applicants, and also the one-mode projection of the same network onto just inventors for years from 1986 to 1995. Breschi and Lissoni (2003) conclude that the size of the network increases when new inventors begin to patent. The average number of inventors in each component of the network increases as well since teams which were previously disconnected are joined by inventors that are mobile. This affiliation network helps derive measures of linkage between cited and citing (control) patents. For each pair of cited-citing patents at time $T$ (e.g. a patent that comes out in 1987 is cited by a patent issued in 1995), they create the network of inventors at time $T - 1$ (e.g. in 1994), and determine measures of linkage (connectedness and distance) (Breschi and Lissoni, 2003, pp. 16-17).

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70 “An affiliation network is a network in which actors (e.g. inventors) are joined together by common membership to groups of some kind (e.g. patents). Affiliation networks can be represented as a graph consisting of two kinds of vertices, one representing the actors (e.g. inventors) and the other the groups (e.g. patents). In order to analyse the patterns of relations among actors, however, affiliation networks are often represented simply as unipartite (or one-mode) graphs of actors joined by undirected edges - two inventors who participated in the same patent, in our case, being connected by an edge” (Breschi and Lissoni, 2003, p. 12). An illustration of an affiliation network is provided in the annex 1.

71 The variable “connectedness” is the following: the variable has value 1, if some inventors of cited and citing patents are situated in the same component at time $T - 1$. If inventors are located in disconnected components, then the variable takes value 0 (Breschi and Lissoni, 2003, p. 17). The variable “distance” is the following: “the variable measures the shortest distance between the team of inventors of the citing and the cited patent.” (Breschi and Lissoni, 2003, p. 18). The
The following table depicts co-location percentages for control and citing patents.

Table 4. Co-location percentages for citing and control patents

<table>
<thead>
<tr>
<th>Co-location level</th>
<th>Citing (n. of patents)</th>
<th>Control (n. of patents)</th>
<th>Odds Ratio (chi-sq.; df)</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
<td>25.1 (121)</td>
<td>17.4 (84)</td>
<td>1.6* (8.477; 1)</td>
</tr>
<tr>
<td>Province</td>
<td>38.7 (187)</td>
<td>29.8 (144)</td>
<td>1.5* (8.498; 1)</td>
</tr>
<tr>
<td>Region</td>
<td>53.8 (260)</td>
<td>40.6 (196)</td>
<td>1.7* (17.014; 1)</td>
</tr>
</tbody>
</table>

*99% significant; **95% significant; ***90% significant.

Source: adaptations from Breschi and Lissoni (2003, p. 19)

The first column of Table 4 details the percentage of citing patents which are co-located with cited patents, at the city, province and regional level. The spatial unit of analysis is defined by the Nomenclature des Unités Territoriales Statistiques (NUTS). There are 8’100 cities (NUTS4), 95 provinces (NUTS3) and 20 regions (NUTS2) (Breschi and Lissoni, 2003, pp. 18-19). The second column describes the same percentage for the control sample, and the third column reports the “odds ratio”. “Odds ratio greater than one [signals] that the differences between the values in the first and second columns are significant” (Breschi and Lissoni, 2003, p. 19). Equivalently, “odds ratios greater than one suggest a positive association between two probabilities, [...] the probability for a patent to come from the citing sample, and the probability of being co-located to the patent it cites.” (Breschi and Lissoni, 2003, p. 19). Therefore, if the odds ratio is greater than one, localized knowledge externalities exist. As can be seen in Table 4, odds ratios for every variable can vary between 0 and infinity. It takes value 0, if at least one inventor is reported in both citing and cited patents. The variable has a positive and finite value, if citing and cited patents have no inventor in common, but some inventors of citing and cited patents were in the same component at time \( T - 1 \). Finally, the variable takes an infinite value, if no inventor from citing and cited patents belonged to the same component at time \( T - 1 \) (Breschi and Lissoni, 2003, p. 18).
geographical scale are greater than 1, and are 99 percent significant. Thus, this confirms the hypothesis of localized knowledge externalities. Moreover, as shown in Table 4, the percentage of co-located citing-cited patents is greater than the same percentage for control patents. “In the [odds ratio] terminology, the probability of co-location between a cited and a citing patent is much higher \((OR > 1)\) than the probability of co-location between the same cited patent and the [control] one.” (Breschi and Lissoni, 2003, p. 19).

In Tables 5 and 6, Breschi and Lissoni (2003) made the same kind of calculations as in the previous table, but here they control for ties among inventors. In Table 5, they check the connectedness value of each pair of patents (citing-cited or control-cited), and calculate the co-location percentages for citing-cited and control-cited patents (Breschi and Lissoni, 2003, p. 20).

**Table 5. Co-location percentages, citing vs. control patents, adjusted for connectedness**

<table>
<thead>
<tr>
<th>Co-location level</th>
<th>Connected Citing (n. of patents)</th>
<th>Connected Control (n. of patents)</th>
<th>Non-connected Citing (n. of patents)</th>
<th>Non-connected Control (n. of patents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>City</td>
<td>68.2 (90)</td>
<td>44.4 (44)</td>
<td>10.4 (40)</td>
<td>8.8 (31)</td>
</tr>
<tr>
<td>Province</td>
<td>82.6 (109)</td>
<td>58.6 (58)</td>
<td>22.4 (86)</td>
<td>22.2 (78)</td>
</tr>
<tr>
<td>Region</td>
<td>87.9 (116)</td>
<td>68.7 (68)</td>
<td>33.3 (128)</td>
<td>41.0 (144)</td>
</tr>
</tbody>
</table>

Size of samples after adjusting for connectedness: Non-connected: 351 (citing) + 384 (control) Connected: 132 (citing) + 99 (control)

Source: Breschi and Lissoni (2003, p. 20)

As can be seen in Table 5, co-location percentages for non-connected patents (from control or citing sample) are lower than those for connected patents and those from Table 4. This implies that connectedness has great influence on the
results from a spatial analysis of patent citations. The results from Table 5 also suggest that when control and citing patents have no social connections with cited patents, they bear almost no differences in co-location at the province and city level. However, they bear differences at the regional level since citing-cited patents are more co-located than control-cited patents. On the other hand, when control and citing patents have social connections, they bear differences at every spatial level and citing-cited patents are more co-located than control-cited patents at every geographical scale (Breschi and Lissoni, 2003, pp. 20-21).

Table 6. Odds ratios for “citation and co-location”, for connected vs. non-connected patents

<table>
<thead>
<tr>
<th>Co-location level</th>
<th>Odds Ratio (chi-sq.; df)</th>
<th>Breslow-Day: Non-zero corr: Common OR 95%: confidence limit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-connected</td>
<td>Connected</td>
</tr>
<tr>
<td>City</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>2.7*</td>
</tr>
<tr>
<td></td>
<td>(0.528;1)</td>
<td>(13.086;1)</td>
</tr>
<tr>
<td>Province</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>3.3*</td>
</tr>
<tr>
<td></td>
<td>(0.003;1)</td>
<td>(16.255;1)</td>
</tr>
<tr>
<td>Region</td>
<td>1.4**</td>
<td>3.3*</td>
</tr>
<tr>
<td></td>
<td>(4.655;1)</td>
<td>(12.857;1)</td>
</tr>
</tbody>
</table>

*99% significant; **95% significant; ***90% significant.

Source: adaptations from Breschi and Lissoni (2003, p. 21)

Table 6 confirms these results by an odds ratio analysis. Breschi and Lissoni (2003) calculate distinct sets of “citation and co-location” odds ratios for pairs of patents which are connected and non-connected. As can be seen in Table 6, “citation and co-location” odds ratios are always greater for connected patents than for non-connected patents. This result is further validated by the Breslow-Day test as differences between odds ratios are always non homogenous. However, according to the Breslow-Day test, these differences are a little less

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72 They also test homogeneity between the two sets of odds ratios thanks to Breslow-Day statistics. Finally, they utilize Mantel-Haenszel methods to make a non-zero correlation test that analyzes if some association between citation and co-location resists to the adjustment for connectedness, and to calculate “the lower-bound 95 [percent] confidence limit of so-called “common odds ratios”: for any positive association between citation and co-location to be significant this value must be higher than one.” (Breschi and Lissoni, 2003, p. 20).
significant at the regional level, compared to other geographical levels. Moreover, odds ratios for connected patents are greater than odds ratios for all patents, while the opposite is true for odds ratios for non-connected patents (see Table 4). Thus, here again, it can be seen that connectedness has a great influence on the results from a spatial analysis of patent citations. Non-zero correlation tests imply that the overall association between citation and co-location is weakened by the introduction of controls for connectedness (with the exception of regional level) (Breschi and Lissoni, 2003, pp. 20-21). “Similarly, the 95 [percent] lower bound limit for [odds ratios includes] value one for analysis at the city level, and barely excludes it for the province level.” (Breschi and Lissoni, 2003, p. 21).

The variable “connectedness” holds two kinds of social links, namely those created directly by the mobility of inventors (the geodesic distance between two connected patents decreases to zero), and those which arise from indirect links (e.g. knowledge flows) between teams of inventors (“such that the distance [between two connected patents] is finite, but never less than one”) (Breschi and Lissoni, 2003, p. 21). To test the implications of these two kinds of social links, Breschi and Lissoni (2003) run a series of logit regressions. They consider only patents from the citing sample. The dependent variable is the binary variable “co-location”. There are three explanatory variables. The first one is related to connectedness and makes a distinction between the origin of connectedness as arising from “mobility” (more precisely, mobility of inventors) and “know-who” (more precisely, knowledge flows between teams of inventors) connectedness. There is no overlapping, and this variable never takes the value 0. The second explanatory variable controls for co-location of technological activities. “For each citing patent, it takes value one if the related control patent is co-located with the cited one, and zero otherwise” (Breschi and Lissoni, 2003, p. 22). Finally, the third explanatory variable controls for likelihood of patent connections by technological field. Breschi and Lissoni (2003) control, if the control and cited patents are joined by “know-who” or “mobility” connections, or if they are not connected at all. Breschi and Lissoni (2003) run backward-inclusion regressions only for major effects, beginning with the insertion of every explanatory variable.
A regression is run for every geographical level. Table 7 summarizes the results of the regressions in terms of parameter estimation. Table 8 determines odds ratios that can be derived from the parameters (Breschi and Lissoni, 2003, p. 22).

### Table 7. Co-location probability for citing patents: logit estimates

(Chi-sq. in bracket)

<table>
<thead>
<tr>
<th></th>
<th>City</th>
<th>Province</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-2.55</td>
<td>-1.62</td>
<td>-0.92</td>
</tr>
<tr>
<td></td>
<td>(149.43)</td>
<td>(101.99)</td>
<td>(31.48)</td>
</tr>
<tr>
<td>Connection:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know-who</td>
<td>1.61</td>
<td>1.61</td>
<td>1.42</td>
</tr>
<tr>
<td></td>
<td>(21.71)</td>
<td>(26.43)</td>
<td>(18.21)</td>
</tr>
<tr>
<td>Mobility</td>
<td>4.70</td>
<td>5.58</td>
<td>4.75</td>
</tr>
<tr>
<td></td>
<td>(100.27)</td>
<td>(30.00)</td>
<td>(21.86)</td>
</tr>
<tr>
<td>Control co-location</td>
<td>1.17</td>
<td>1.19</td>
<td>0.61</td>
</tr>
<tr>
<td></td>
<td>(11.65)</td>
<td>(22.87)</td>
<td>(8.08)</td>
</tr>
<tr>
<td>Control know-who</td>
<td>b. e.</td>
<td>b. e.</td>
<td>b. e.</td>
</tr>
<tr>
<td>Control mobility</td>
<td>b. e.</td>
<td>b. e.</td>
<td>b. e.</td>
</tr>
<tr>
<td>Lombardy</td>
<td>-</td>
<td>-</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(13.44)</td>
</tr>
</tbody>
</table>

b.e.= backward eliminated.

*all reported parameters are 99% significant.

Source: adaptations from Breschi and Lissoni (2003, p. 23)

### Table 8. Co-location probability for citing patents: odds ratios

(95% confidence lower bound in bracket)

<table>
<thead>
<tr>
<th></th>
<th>City</th>
<th>Province</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Know-who vs. No connection</td>
<td>5.02</td>
<td>5.01</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>(2.55)</td>
<td>(2.71)</td>
<td>(2.15)</td>
</tr>
</tbody>
</table>

For regional analysis, they add a dummy variable for Lombardy: “that is for the possibility that at least one of the inventor of the cited patent points to Lombardy” (Breschi and Lissoni, 2003, p. 22) as this region is the most innovative one of Italy. This dummy variable improves the estimates and also clarifies the results from tables 5 and 6 for regional level (Breschi and Lissoni, 2003, p. 22).
As can be seen in Table 7, the control for the localization of technological activities is confirmed. This variable is significant and has a positive influence on the dependent variable at every geographical scale. This can be further validated by the odds ratios that citations are co-localized along with control patents, because they are greater than one at every geographical scale (see Table 8). However, the control for social connections of technological activities is not necessary. Social connections of control patents do not influence the co-location probability for citations, and are excluded in backward exclusion regressions (see Table 7) (Breschi and Lissoni, 2003, pp. 22-23). As shown in Table 7 and Table 8, social connections between citing and cited patent are the major determinant for co-location between the two. In Table 7, the parameters of “know-who” and “mobility” variables are significant and positive for every geographical scale. In Table 8, “citing patents linked to cited ones by at least one inventor (i.e. mobility) are more than hundred times more likely to be co-located than non-connected ones” (Breschi and Lissoni, 2003, p. 23). This means that Italian inventors move across firms, but they prefer not to re-settle in distinct regions, provinces or cities. Indirect social connections (due to “know-who”) between teams of inventors are also important (see Table 8), because citing patents connected to cited patents are about five times more likely to be co-located than non-connected patents to be co-located. Nevertheless, as can be seen in Table 8, indirect social connections (due to knowledge flows) have a smaller influence on the co-location probability of citing and cited patents than the mobility of inventors. The latter kind of connections induces co-location probability to be at least twenty times more likely.

<table>
<thead>
<tr>
<th>Mobility vs. No connection</th>
<th>110.38</th>
<th>264.14</th>
<th>115.20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(43.96)</td>
<td>(35.93)</td>
<td>(15.75)</td>
</tr>
<tr>
<td>Mobility vs. Know-who</td>
<td>21.99</td>
<td>52.73</td>
<td>27.94</td>
</tr>
<tr>
<td></td>
<td>(7.95)</td>
<td>(6.76)</td>
<td>(3.54)</td>
</tr>
<tr>
<td>Control co-location</td>
<td>3.24</td>
<td>3.28</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>(1.65)</td>
<td>(2.02)</td>
<td>(1.21)</td>
</tr>
<tr>
<td>Lombardy</td>
<td>-</td>
<td>-</td>
<td>2.22</td>
</tr>
</tbody>
</table>

(1.45)

*Source: Breschi and Lissoni (2003, p. 23)*
than the former kind of connections. Finally, Breschi and Lissoni (2003) remark that the regional effect that appears in Tables 5 and 6 is mostly due to the fact that the Italian innovation system is principally concentrated in the region of Lombardy (Breschi and Lissoni, 2003, pp. 23-24). “The control variable for the location of cited patents in this region is positive and significant, so that citations directed to patents from Lombardy are two times more likely than others to be located in the same region.” (Breschi and Lissoni, 2003, p. 24).

The empirical analysis of Breschi and Lissoni (2003) on Italian data confirms the intuition of many economists who assert that knowledge externalities are tacit, and they travel along with individuals who master them. More than indirect social connections due to knowledge flows, individuals’ mobility allows knowledge to diffuse in space. On the contrary, if agents are not mobile, then access to this knowledge will be constrained in bounded locations. Therefore, knowledge flows as highlighted by patent citations are localized, if labor mobility and networks ties are as well localized. In other words, localization effects tend to disappear when citing and cited patents are not linked by a network relationship. Therefore, spatial closeness is not per se a sufficient condition to have access to local knowledge. An active participation in a network where knowledge is exchanged is also required. We noticed in the previous section, thanks to the study of Autant-Bernard and Massard (2004b), that knowledge and pecuniary externalities are spatially bounded. However, it does not mean, according to Breschi and Lissoni (2003), that simply by being closed to, agents can benefits from these externalities. The adherence to social networks, and thus interacting with neighboring individuals seem also crucial as member of a social network know each other, and thus can exchange information and knowledge (Breschi and Lissoni, 2003, abstract pp. 7, 24). Therefore, we can notice the important influence of social networks (through connectedness) on the spatial spread of externalities and knowledge.

According to Breschi and Lissoni (2003), members of social networks that exchange knowledge are usually spatially bounded “since spatial proximity may
help the network members to communicate more effectively and patrol each other’s behaviour (compliance with the social norms of inward openness and outward secrecy).” (Breschi and Lissoni, 2003, p. 7). Moreover, externalities from an active member of a social network will reach distant members with some delay and imprecision, but will never reach outsiders. Therefore, we can conclude that social networks are geographically concentrated, and thus the importance of geography as an explanatory variable of externalities decreases in favor of social proximity. Table 8 confirms this conclusion. As can be seen in this table, although there is a certain mobility of inventors across firms, they do not move across distinct regions, provinces or cities. This confirms that inventors do not move far in space, and social networks tend to concentrate geographically. This conclusion can be further corroborated with the results from Table 5. In this table, co-location percentages for both citing-cited and control-cited patents are higher when patents are connected than when they are not connected. This is validated by an odds ratio analysis in Table 6. In this table, odds ratios are higher at every geographical scale for connected patents than for non-connected patents. This means that the positive association between the probability that a patent comes from the citing sample and the probability that this patent is being co-located to the patent it cites is higher for connected patents than for non-connected ones (Breschi and Lissoni, 2003, p. 7).

Therefore, we have here several proofs to the fact that social networks concentrate in space and that, consequently, the diffusion of knowledge externalities is also bounded in space.

However, in the study of Breschi and Lissoni (2003) a problem persists. They analyze the diffusion of localized knowledge externalities, but it is possible that pecuniary externalities arise instead of knowledge externalities. In fact, to the extent that this knowledge can be appropriated, knowledge externalities disappear and only pecuniary externalities are present (e.g. availability of skilled labor and specialized inputs, infrastructure of cities and regions, etc.) (Breschi and Lissoni, 2003, pp. 4, 8). More precisely, “if the inventor is able to fully appropriate the value of […] knowledge and social capital, the externality is fully internalised. Moreover, even if the inventor cannot fully appropriate the value of […]
knowledge, only a pecuniary externality is likely to arise, i.e. the new employer gets access to a fundamental knowledge input at a price lower than its full quality price.” (Breschi and Lissoni, 2003, pp. 8-9). The intuition of Breschi and Lissoni (2003) is that “mobile inventors are most likely to be scientific or technological “stars” whose wages or fees pay for the access they can provide to a valuable club good such as the knowledge stock mastered by them, and the inventors in their network. If it is so, either externalities may be fully internalised or just pecuniary externalities arise (stars can accept lower wages and fees to the extent that their employees and customers do not ask them to relocate), but no pure [knowledge] spillovers.” (Breschi and Lissoni, 2003, pp. 24-25). These statements are in accordance with the results of the study of Autant-Bernard and Massard (2004b) which states that, in the problematic of knowledge diffusion, externalities are more of pecuniary kind than of knowledge kind. However, as Breschi and Lissoni (2003) do not test their intuition, we must not take it as a general result. Further research needs to be done to confirm their intuition about knowledge and pecuniary externalities.

The studies of Maggioni et al. (2007) and of Hussler and Ronde (2006) about co-patenting confirm the conclusions of Breschi and Lissoni (2003). Maggioni et al. (2007) confront spatial effects with effects that result from relational proximity. “They investigate whether relationships in networks of geographically distant clusters prevail or not over diffusive patterns based on spatial contiguity.” (Autant-Bernard and Massard, 2007b, p. 345). Their analysis concerns 109 European NUTS 2 regions from United Kingdom, France, Spain, Germany and Italy. Their study is based on the participation in the same research networks (within the EU Fifth Framework Programme), and on EPO co-patent applications for the period 1998-2002. Maggioni et al. (2007) employ two methodologies in their study. First, they use a gravity model (OLS) to analyze to which extent co-patenting between two regions is affected by their social and physical distance. Their conclusion is that both relational and spatial proximity

---

74 “The 5FP is a five-year [program] that finances research projects within a time intervals of 5 years (1998-2002)” (Maggioni et al. 2007, p. 475).
75 The EPO was created in Munich in 1977 (Maggioni et al. 2007, p. 478).
matter. Secondly, they use a spatial knowledge production function to evaluate whether relational and/or spatial autocorrelation happen for co-patenting. Their conclusion is that physical distance-based spatial autocorrelation is more important than relational distance-based spatial autocorrelation. This confirms that the regional propensity to patent is more closely related to local knowledge externalities than to externalities which result from distant collaborations (Autant-Bernard and Massard, 2007b, pp. 345, 349 and Maggioni et al. 2007, pp. 471-73, 478).

Hussler and Ronde (2006) analyze the determinants of the spatial diffusion of technological knowledge generated by academic scientists. They use patent co-invention as indicator of knowledge flows, and co-publication data to proxy social networks (Hussler and Ronde, 2006, pp. 1, 4). The assumption they test is the following: “academic technological knowledge diffuses in a larger geographic area when the scientist who takes part to this technology transfer has a more far-reaching social network.” (Hussler and Ronde, 2006, p. 1). In order to test their assumption, they analyze “the networks of a French science university (University Louis Pasteur in Strasbourg), using micro-data on (the location of) all the researchers which co-invented at least one European patent with one university member.” (Hussler and Ronde, 2006, p. 2). More precisely, Hussler and Ronde (2006) utilize European patent applications from 1978 to 1996 to construct their database on the University Louis Pasteur (ULP) (Hussler and Ronde, 2006, p. 6). They “concentrate on the patents with at least one French inventor (an inventor with a French address) and combine [this] data set with the list of ULP scientists in the year 1996.” (Hussler and Ronde, 2006, p. 6). They find a total of 147 patents. They construct as well an indicator to evaluate the spatial dispersion of the publishing network of each ULP member involved in patenting activities. To build this indicator, they select the papers authored by each member of ULP until the end of 2002, thanks to the Science Citation Index (SCI), and they determine the proportion of co-publications of each member of ULP with regional, national and international co-authors. They find a total of 6’390 co-authored papers. Hussler and Ronde (2006) propose an econometric model (OLS regression) which
examines the determinants of the localization of co-inventors. The results of their empirical analysis show that academic knowledge externalities are not automatically spatially bounded (Hussler and Ronde, 2006, pp. 7, 10-11, 14). However, “higher relational capacities within the academic community do not systematically lead to more widespread technological knowledge flows” (Hussler and Ronde, 2006, p. 12). Hence, Hussler and Ronde (2006) conclude that social networks are generally spatially bounded, and thus do not explain spatial diffusion of technological knowledge which is, consequently, geographically localized (Hussler and Ronde, 2006, p. 1).

To try to further confirm the conclusion of the above studies, we will analyze, in the next section, another study on this subject, namely the study of Autant-Bernard and Massard (2000) on the French case.

### 5.2.3 Empirical study of co-authored publications

Autant-Bernard and Massard (2000) test empirically the following assumption: “[knowledge] spillovers are mediated by interactions and those interactions are themselves facilitated by geographic proximity.” (Autant-Bernard and Massard, 2000, p. 2). Autant-Bernard and Massard (2000) measure social interactions by co-authoring. The latter can be defined as published articles which are signed by authors from distinct institutions. Co-authored articles create a paper trail that can be utilized to quantify “connectedness”. In their study, Autant-Bernard and Massard (2000) use two methods to quantify connectedness. First, they compare the co-authoring structure to the geographic structure thanks to a pretopologic approach. More precisely, they analyze whether interactions are favored by geographic proximity. Then, they determine the relation between social interactions and local externalities by means of an econometric analysis (Autant-Bernard and Massard, 2000, pp. 2-3).

A department that wants to have access to research conducted in other departments must accumulate knowledge, research means, etc. to enhance its
absorptive capacity, but it must also have connections with other departments in order to benefit from outside source of knowledge. Therefore, interactions between users and producers of knowledge are necessary. There are many communication channels that allow interactions between agents, but scientific collaboration, which has increased during the last twenty years, is a very important one (Autant-Bernard and Massard, 2000, pp. 3-4). Co-authorship is a very good indicator of scientific interactions. It brings out a qualitatively distinct type of interactions than do for example patent citations. “Joint co-authorship reflects joint research, which is an important opportunity for the exchange of tacit knowledge. By contrast, [patent] citations may be seen as an acknowledgement of the exchange of codified knowledge [...] and do not necessarily imply real relations between scientists.” (Autant-Bernard and Massard, 2000, p. 4).

Therefore, Autant-Bernard and Massard (2000) analyze co-authorship of scientific articles between French departments in order to assess the geographical diffusion of externalities and the influence of social interactions on the diffusion of externalities (Autant-Bernard and Massard, 2000, p. 4).

Autant-Bernard and Massard (2000) use data that come from the Observatoire des Sciences et des Techniques (OST), and that are extracted from the SCI. For each year between 1992 and 1997, Autant-Bernard and Massard (2000) construct a matrix $C = [c_{xy}]_{x,y \in \{1, \ldots, n\}}$, where $c_{xy}$ gives the number of co-authored articles that are written at least by one author who belongs to department $x$, and at least by one author who belongs to department $y$. First, they do a simple statistical analysis that makes it possible to distinguish the central co-authorship behaviors of the departments and some characteristics of the two by two relations. Then, by dint of a pretopologic approach, they extract structural information from the data (Autant-Bernard and Massard, 2000, p. 4).

The statistical analysis gives the following results. First, between 1992 and 1997, the number of co-authored articles between French departments has augmented.76

76 Co-authored articles increased from 10’280 in 1992 to 14’335 in 1997 (Autant-Bernard and Massard, 2000, p. 5).
Moreover, co-authorship between French departments shows a high spatial concentration;\textsuperscript{77} the majority of co-authoring appears between scientists who are situated in the same department\textsuperscript{78} (Autant-Bernard and Massard, 2000, p. 5).

Secondly, Autant-Bernard and Massard (2000) calculate an externalization index which is defined for each department as the share of co-authored publications of a department implying other departments compared to the totality of its co-authoring.\textsuperscript{79} The externalization index enables to estimate the tendency of each department to publish with scientists from the same department or from other departments. According to the externalization index, Autant-Bernard and Massard (2000) find two groups of departments. In the first one, there are departments that have a high co-authoring activity. These departments are usually major university centers. Externalization indexes are below the average rate. This means that the share of co-authored papers of these departments implying other departments compared to their total co-authoring is relatively low. In the second group, there are departments with low scientific activities. The externalization indexes are usually high. In other words, there is a negative correlation between the number of co-authored papers and the externalization index of the departments. An illustration of these statements is provided by Figure 12 (Autant-Bernard and Massard, 2000, p. 5).

\textsuperscript{77} "In 1997, the first 14 (on a total of 101 departements) represent 90 [percent] of the co-authoring in France while, at the opposite, 51 [percent] of the departements represent less than 1 [percent] of the total co-authoring." (Autant-Bernard and Massard, 2000, p. 5).

\textsuperscript{78} In 1997, 9'618 papers have been written by scientists located in the same department, whereas only 4'717 papers have been written by scientists whom institutions are situated in distinct departments (Autant-Bernard and Massard, 2000, p. 5).

\textsuperscript{79} The externalization index is $e_{x} / e_{x}$, where $e_{x} = c_{x} - c_{xx}$; $c_{x}$ is the total number of co-authored papers of the department $x$ (Autant-Bernard and Massard, 2000, p. 5).
This figure can be justified by the fact that departments that benefit from few scientific activities inside their boundaries need to compensate this lack with scientific activities from outside. Therefore, this suggests that departments choose to make internal co-authorship and only afterwards search for co-authorship outside their boundaries, if it is missing inside. This is a primary indication about the role of geographic proximity (Autant-Bernard and Massard, 2000, p. 5).

In the third place, Autant-Bernard and Massard (2000) analyze the role of spatial dimension in the case of external co-authoring as it is important to know whether an author who wants to choose an external partner will prefer to work with an author from a neighboring department. In order to do this analysis, it is necessary to calculate the following ratio:

\[
\frac{\sum_{x=1}^{n} CD_{xy}}{\sum_{x=1}^{n} C_x}.
\]

\(CD_{xy}\) is the number of co-authored papers between a scientist from a department \(x\) and a scientist from a department \(y\) situated at a distance \(D\) from \(x\). \(C_x\) is the total number of outside co-authored papers of the department \(x\). The distance is measured by the number of departments it is necessary to cross in order to go from the department \(x\) to the department \(y\). In France, \(D_{xy}\) takes values between 1
(for bordering departments) and 12. Autant-Bernard and Massard (2000) construct for each distance $D = 1, ..., 12$ the above indicator, and normalize it by the share of the cells of the matrix which corresponds to the distance concerned in the total of the possible cells, namely

\[
\frac{\sum_{x=1}^{D_x}}{TOT}
\]  

(5.2.3.2)

$D_x$ is the number of cells where $D = 1$ (respectively $2, ..., 12$) and $TOT$ is the total number of cells in the matrix ($TOT = 8'742$) (Autant-Bernard and Massard, 2000, pp. 5-6) “An index higher than 1 reveals a tendency to privilege the relations with departements situated to the distance analysed compared to what we may expect in regarding the part of these departements in the total number of possible partners.” (Autant-Bernard and Massard, 2000, p. 6). Results are described in Table 9.

Table 9. Collaboration as a function of the distance between departments

<table>
<thead>
<tr>
<th>$D_{xy}$</th>
<th>$\frac{\sum CD_{xy}}{\sum C_x}$</th>
<th>$\frac{\sum D_x}{8'742}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{xy} = 1$</td>
<td>0.1793</td>
<td>3.2668594</td>
</tr>
<tr>
<td>$D_{xy} = 2$</td>
<td>0.1261</td>
<td>1.25045159</td>
</tr>
<tr>
<td>$D_{xy} = 3$</td>
<td>0.0795</td>
<td>0.59253701</td>
</tr>
<tr>
<td>$D_{xy} = 4$</td>
<td>0.1156</td>
<td>0.7670199</td>
</tr>
<tr>
<td>$D_{xy} = 5$</td>
<td>0.1335</td>
<td>0.85975526</td>
</tr>
<tr>
<td>$D_{xy} = 6$</td>
<td>0.1245</td>
<td>0.87543207</td>
</tr>
<tr>
<td>$D_{xy} = 7$</td>
<td>0.0750</td>
<td>0.669181</td>
</tr>
<tr>
<td>$D_{xy} = 8$</td>
<td>0.0775</td>
<td>0.99057964</td>
</tr>
<tr>
<td>$D_{xy} = 9$</td>
<td>0.0683</td>
<td>1.4786376</td>
</tr>
<tr>
<td>$D_{xy} = 10$</td>
<td>0.0176</td>
<td>0.87903932</td>
</tr>
<tr>
<td>$D_{xy} = 11$</td>
<td>0.0025</td>
<td>0.5549315</td>
</tr>
<tr>
<td>$D_{xy} = 12$</td>
<td>2.5366$^{E-05}$</td>
<td>0.1108731</td>
</tr>
</tbody>
</table>

Total 1

*Source*: Autant-Bernard and Massard (2000, p. 6)
As shown in this table, the number of co-authorship declines rapidly as the distance that separates departments increases. It is worth noting that departments co-publish a lot with departments situated at distance $D = 1$ and $D = 2$ as ratios are higher than 1. However, an unexpected result arises at distance $D = 9$ since the ratio is superior than 1 (normally, when the distance increases, every ratio becomes inferior than 1). This is mainly due to the role of Paris as an attraction force for university departments which are usually situated at distance $D = 9$ (Autant-Bernard and Massard, 2000, p. 7).

The results presented above are due to a simple statistical analysis which makes it possible to differentiate the central co-authorship behaviors of the departments and some characteristics of the two by two relations. Nevertheless, there is no global view of the structure of co-authorship relations in France, and it is not possible to locate each department from each others in that sense. To achieve this, Autant-Bernard and Massard (2000) extract structural information of the scientific collaborations between departments thanks to a pretopologic approach (Autant-Bernard and Massard, 2000, pp. 4, 7).

The results achieved for the matrix of co-authored papers in 1997 with this pretopologic methodology are presented in Figure 13. A list with the names and related numbers of the French administrative departments is given in annex 2 (Autant-Bernard and Massard, 2000, p. 8).

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80 If you would like more information on the pretopologic approach used by Autant-Bernard and Massard (2000) to achieve these results, you can consult pages 7 to 9 from Autant-Bernard and Massard (2000).
As can be seen in this figure, a department can have connections to another department either directly or indirectly through other departments. In this figure, the numbers of departments that are not surrounded by a circle represent departments that apply an attraction force on each department which is member of the group. Every member of each group on this figure mainly publishes with the single attractor of its related group. The attractors correspond to the main French University centers. They usually attract small departments that are situated closed by. An interesting point to notice on this figure is that the department 75 (Paris) and the department 91 (Essonne) are strong attractors for all other departments on the figure. This means that every department has directly or indirectly published with these two departments. Autant-Bernard and Massard (2000) conclude that large University departments do not favor co-publication between them; they
mainly publish in-house, and select frequently Paris as partner. They are also often chosen as main partners by small neighboring departments, because the latter have little publishing activities, and thus need to compensate for this lack with co-authoring with the closest university center (Autant-Bernard and Massard, 2000, p. 9).

It can be concluded from these statements that in the French case, globally, scientific interactions are improved by geographic proximity. The role of Paris and other important University centers seems to be determined by a proper attractive effect, and not by a spatial effect. However, the role of this local dimension of individual’s interactions on the diffusion of knowledge externalities is not yet clarified. Therefore, in order to analyze it, Autant-Bernard and Massard (2000) construct a model that tests the existence of local knowledge externalities, and measure to which extent these are enhanced by individual’s interaction. More precisely, they use a knowledge production function to model knowledge externalities and to test the impact of spatial dimension by confronting distinct spatial levels. They introduce knowledge externalities in the production function as an external stock of knowledge (Autant-Bernard and Massard, 2000, pp. 9-10). “The local characteristic of externalities is studied by taking into account not only R&D conducted within a geographic area but also R&D carried out nearby and finally R&D conducted in a more distant neighbourhood.” (Autant-Bernard and Massard, 2000, p. 10). This enables to demonstrate the influence of spatial distance by taking into account the fact that an agent can be more affected by her neighbors’ actions than by the actions of physically distant individuals. Therefore, the localization of externalities can be tested by comparing the effect of close neighborhood with the effect of distant neighborhood. If knowledge externalities or spillovers are geographically bounded, then local innovation will be more influenced by neighboring R&D than by R&D carried out at a greater distance (Autant-Bernard and Massard, 2000, p. 10).

The general equation is the following:
\[ I_{gi} = \alpha + \beta_2 RD_{gi} + \beta_2 RD_{vi} + \beta_3 RD_{vri} + \beta_4 \sum RD_{gj} + \beta_5 \sum RD_{vj} + \beta_6 \sum RD_{vij} + u_g + v_{gi} \]  

(5.2.3.3)

with \( j = 1, \ldots, J \) and \( i \neq j \). \( I \) is an indicator of innovation output, \( RD \) is a measure of the stock of knowledge, \( g \) is the geographic area considered (it is a French administrative department), \( v \) is the close neighborhood of the geographic area (it represents bordering departments of \( g \)), \( v' \) is a farther neighborhood (it is the bordering departments of \( v \)), \( i \) and \( j \) are sector indexes, \( \alpha \) is a constant, \( u_g \) is the geographic effect, and \( v_{gi} \) is the random disturbance. Autant-Bernard and Massard (2000) test the existence of knowledge externalities by analyzing the relation between the innovative output of area \( g \), and the R&D carried out in the neighborhood. There will be local knowledge externalities, if \( \beta_2 > \beta_3 \) for infra-sectoral analysis, and \( \beta_4 > \beta_5 > \beta_6 \) for inter-sectoral analysis. Autant-Bernard and Massard (2000) introduce as well an indicator of human capital. This indicator tests, if externalities are more supported by individuals, or if knowledge flows freely. “The indicator used here is the ratio between the number of researchers and the

The innovative output is approximated by the number of patents. Concerning the inputs, the stock of knowledge (\( RD \)) is measured by R&D expenditures. However, as the interest is in the role of human interactions, Autant-Bernard and Massard (2000) introduce as well an indicator of human capital. This indicator tests, if externalities are more supported by individuals, or if knowledge flows freely. “The indicator used here is the ratio between the number of researchers and the

---

81 This geographic level (French departments) is perhaps not the most appropriated to take into account for measuring local externalities. They occur perhaps at a subtler geographic level. Nevertheless, departments are a globally satisfactory geographic level as it is the smallest administrative division for which data are available, and it is a coherent geographic level in the sense that they represent a large town with its urban agglomeration. Therefore, French departments present certain homogeneity (Autant-Bernard and Massard, 2000, p. 12).

82 The model is expressed in logarithms (Autant-Bernard and Massard, 2000, p. 11).
total research staff. So, the human capital variable (noted $KH$) represents the proportion of researchers relatively to the total research staff.”83 (Autant-Bernard and Massard, 2000, p. 11).

Data are available from 1991 to 1996.84 The triple dimension of the data (spatial, temporal and sectoral) controls for individual heterogeneity. Spatial effects $u_g$ are introduced. The average temporal effect on innovation is introduced by the variable $TREND$. This variable takes the value 0 for year 1991 etc., up to value 5 for year 1996. The sectoral dimension is taken into account by the introduction of sectoral dummies85 (Autant-Bernard and Massard, 2000, p. 12).

Results for the function of production of innovation with knowledge externalities and local interactions are summarized in Table 10 (Autant-Bernard and Massard, 2000, p. 21).

### Table 10. Function of production of innovation with knowledge spillovers and local interactions

6'204 observations, Tobit estimation with random effects

<table>
<thead>
<tr>
<th>Variables</th>
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<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>−10.53***</td>
<td>−10.19***</td>
<td>−14.85***</td>
<td>−12.09***</td>
</tr>
<tr>
<td></td>
<td>(0.35)</td>
<td>(0.40)</td>
<td>(0.16)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>$RD_{gl}$</td>
<td>0.19***</td>
<td>0.19***</td>
<td>0.05***</td>
<td>0.07***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$RD_{gj}$</td>
<td>0.18***</td>
<td>0.14***</td>
<td>0.05***</td>
<td>0.05***</td>
</tr>
<tr>
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83 Data stem from the R&D inquiry of the French Ministry of National Education, Research and Technology, and from the OST (Autant-Bernard and Massard, 2000, p. 11).


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* significant at 10% level; ** significant at 5% level; *** significant at 1% level. Numbers in bracket are p-values.

Source: adaptations from Autant-Bernard and Massard (2000, p. 21)
The first column depicts results achieved without local interactions. Columns 2, 3 and 4 give results when interactions are introduced in the model. From the first column, it can be seen that innovation is primarily influenced by internal activity; the positive and significant coefficients are basically those of variables which are internal to the area \( g \) \( (RD_{gi}, RD_{gj}, KH_{gj}, VA_g) \) (Autant-Bernard and Massard, 2000, pp. 12-13). Nevertheless, \( KH_{gi} \) has a negative sign, and this is unexpected. “It may come from the particular construction of the human capital variable and from the small level of observation that [results] from a sectoral analysis.” (Autant-Bernard and Massard, 2000, p. 13). Therefore, when a spatial area has a low level of activity, the number of scientists in a given sector is usually identical to the total scientists as there is just one individual. Thus, the proportion of scientists will equal 100 percent of the staff. Consequently, the human capital in one sector and one spatial area will be very high, if the area has a low level of innovative activity. Nevertheless, human capital is not high in the reality. Hence, the negative coefficient of the variable \( KH_{gi} \) can be better understood, and has a different meaning as the one expected (Autant-Bernard and Massard, 2000, p. 13).

The fact that innovation of a given spatial area is basically related to the R&D carried out inside this area can result from company’s own R&D, but also from externalities phenomena. This is true, because the number of patents from sector \( i \) and area \( g \) also highly depends on R&D carried out in the same area, but in different sectors. Nevertheless, positive interregional externalities arise as well as local innovation is positively affected by research carried out at distant areas. These externalities arise from both human capital and R&D expenditures since the coefficients of \( RD_{vi}, KH_{vj}, \) and \( KH_{vij} \) are positive and significant. However, it can be seen from the coefficients of human capital variables that the majority of externalities diffuse through human capital as the coefficients of \( KH_{vj} \) and \( KH_{vij} \) are higher than the one of \( RD_{vi} \). This confirms the assumption that knowledge is embodied into individuals, and needs face-to-face contacts to diffuse (Autant-Bernard and Massard, 2000, p. 13).
Autant-Bernard and Massard (2000) analyze also the sectoral origin of externalities. They remark that both infra- and inter-sectoral externalities arise, and that these externalities are localized in space. Externalities from R&D expenditures depend on sectoral origin as \( RD_{ui} \) has a significant and positive coefficient. Thus, local innovation depends positively on R&D carried out in the same sector by neighboring areas. This is further confirmed as \( RD_{uj} \) has a negative and significant effect on the dependent variable. On the contrary, inter-sectoral externalities arise only inside the department as \( RD_{gj} \) has a positive and significant coefficient and \( RD_{uj} \) has a negative coefficient. Thus, it can be seen that positive R&D’s externalities between regions are basically infra-sectoral. At a distance, infra-sectoral externalities are more likely to arise, whereas spatial proximity increases knowledge flows between sectors. Therefore, infra-sectoral externalities diffuse over a greater distance than inter-sectoral spillovers. Nevertheless, the former diffuse not totally over space since \( RD_{ui} \) is not

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86 Other studies exist on the influence of infra- and inter-sectoral externalities on the spatial diffusion of knowledge. As an example, Autant-Bernard (2003) use panel data, namely patent and R&D data for the period 1992-2000, to estimate a knowledge production function which takes into account externalities arising inside the same industry and outside. The sample includes 94 metropolitan French departments and 11 sectors. Her conclusion is that inter-sectoral externalities are favored by spatial proximity, whereas infra-sectoral externalities are favored by distance (Autant-Bernard, 2003, pp. 1, 3, 6-7). The result of Autant-Bernard (2003) on the French case confirms the one of Autant-Bernard and Massard (2000). As a second example, consider the study of Paci and Usai (1999). They evaluate to which extent the degree of specialization or diversity of externalities has an influence on the innovative output of a specific local industry. Data on innovation activity in the European regions come from a database created by the Centro Ricerche Economiche Nord Sud (CRENoS). Innovative activity is measured by patent applications to EPO for the period 1978-1995, classified by inventors’ residence. The study of Paci and Usai (1999) is based on 85 industrial sectors at the three-digit level and on 784 Italian local labor system (Paci and Usai, 1999, pp. 381-83, 387) to “assess the extent to which technological activity in a local industry is affected by the degree of production specialisation in the same local industry […] and by the degree of industrial diversity in the local system” (Paci and Usai, 1999, p. 385), and to evaluate the process of geographical agglomeration of production activities and innovation. Their study is based on a spatial autoregressive model (OLS estimates). The first result of their empirical analysis is that both infra-sectoral (specialization) and inter-sectoral (diversity) externalities influence positively innovative activities in a local industry. This result contrasts with some recent studies (e.g. Audretsch and Feldman (1999)) on the United States where specialization externalities are not present. The second result of the study of Paci and Usai (1999) is that both specialization and diversity externalities are spatially bounded (Paci and Usai, 1999, pp. 381, 387, 389). This last conclusion does not totally confirm the one of Autant-Bernard and Massard (2000) who state that infra-sectoral externalities are favored by physical distance. Therefore, the study of Paci and Usai (1999) relativizes the results of Autant-Bernard and Massard (2000). For further information on the influence of infra- and inter-sectoral externalities on the spatial spread of innovation, see the study of Paci, R. and Usai, S. (1999) “The role of specialisation and diversity externalities in the agglomeration of innovative activities”, Working Paper, University of Cagliari, University of Sassari and CRENoS.
significant (see column 1). Considering externalities stemming from human capital, it can be seen that $KH_{gj}$, $KH_{ej}$, and $KH_{vij}$ create positive effects on the innovation of area $g$. Inter-sectoral externalities from human capital appear to be more diffused than R&D’s externalities since their coefficients are higher. Nevertheless, externalities from human capital also have a spatial dimension as their coefficients decrease with spatial distance (Autant-Bernard and Massard, 2000, pp. 13-14).\footnote{Up to here, data are always taken from column 1 of Table 10.}

The analysis from the above paragraph confirms globally that externalities are higher when they come from close geographic areas. Therefore, Autant-Bernard and Massard (2000) give a proof to the local dimension of spillovers. However, it is also important to analyze why externalities are spatially localized. The great influence of human capital variables indicates that individuals have an important role in the diffusion of knowledge. This can be the reason why location is relevant. The following statements will be based on the columns 2, 3, and 4 from Table 10 (Autant-Bernard and Massard, 2000, p. 14).

The analysis on page 94 already proved that knowledge is embodied into people, and thus requires face-to-face contacts to diffuse, but individual’s interactions were not yet taken into account. Autant-Bernard and Massard (2000) give further proof to this statement by rewriting the model presented above. They let the coefficients of $RD_{vl}$, $KH_{ej}$, $KH_{vij}$ fluctuate with the level of co-authorship between regions (Autant-Bernard and Massard, 2000, pp. 14-15). They estimate the following equation:

$$I_{gi} = \delta + \gamma_1 RD_{gi} + F_1(REL_{gvi})RD_{vi} + \gamma_2 RD_{vi} + \gamma_3 RD_{gj} + \gamma_4 RD_{vj} + \gamma_5 RD_{vij} + \gamma_6 KH_{gi} + \gamma_7 KH_{gj} + \gamma_8 KH_{vi} + F_2(REL_{gvi})KH_{vj} + \gamma_9 KH_{vij} + F_3(REL_{gvi'})KH_{v'j} + u'_g + v'_gi$$

They test whether the influence of knowledge externalities is a function of individuals’ interactions between the area $g$ and its neighbors $v$. “$RD_{vl}$, is related
to the co-authoring publications between areas $g$ and $v$ in sector $i$. $KH_{vj}$ is related to the co-authoring between $g$ and $v$ in all technological fields. And $KH_{vri}$ is related to co-authoring between $g$ and $[v']$ in all technological fields.” (Autant-Bernard and Massard, 2000, p. 15). Autant-Bernard and Massard (2000) assume that these functions take the following form:

$$F = a \log(REL_g) + b$$

(5.2.3.5)

It consists in introducing three join variables to the model: $RD_{vi} \times REL_{gvi}$, $KH_{vj} \times REL_{gv}$ and $KH_{vri} \times REL_{gvr}$ (Autant-Bernard and Massard, 2000, p. 15). Results are described in the columns 2, 3 and 4 from Table 10.

Concerning R&D’s externalities, the coefficients of both $RD_{vi}$ and $RD_{vi} \times REL_{gvi}$ are significant and positive. This means that externalities from R&D carried out in area $v$ and in sector $i$, which benefit to innovation of area $g$, depend on co-authoring between the two regions. Nevertheless, the link between externalities and local interaction is not strong for the variable $RD$ in the sense that a doubling of co-authoring will only result in an increase in externalities of 0.2 percent. The link between externalities and local interactions is stronger for the human capital variable as the coefficient of $KH_{vj} \times REL_{gvi}$ is significant and positive, and is higher than the coefficient of $RD_{vi} \times REL_{gvi}$. This means that, if more and more individuals interact, then a higher level of knowledge will spread. However, this result is less clear for interactions happening at more distant regions. The co-authoring level between area $g$ and area $v'$ has a low influence on knowledge flows between $g$ and $v'$ as the coefficient of $KH_{vri} \times REL_{gvr}$ is 0.01 in regression (3) and is not significantly distinct from zero in regression (4). This definitively confirms that externalities depend more on individuals’ interactions when they are local. Nevertheless, some problems persist in this model. First of all, when interactions are taken into account, some coefficients are disturbed. For example, $RD_{vi}$ and $RD_{vri}$ become significant and positive when co-authoring is introduced. These results were not expected. Therefore, further analysis is necessary. Secondly, the data utilized to account for individuals’ relations are not entirely satisfying as publications basically reflect public research. Thus, only a special
kind of interactions between public scientists is taken into account. Hence, using co-authored publications as an indicator of global interactions is a simplifying hypothesis. Therefore, further investigations including other measures of social interactions are required (Autant-Bernard and Massard, 2000, pp. 15-16). In spite of these problems, Autant-Bernard and Massard (2000) provide a precise and significant analysis of the role of the local dimension of individual’s interactions on the spatial diffusion of knowledge externalities which is geographically bounded.

The general conclusion of Autant-Bernard and Massard (2000) is the following. “[…] spillovers seem to have a local dimension because they need [face-to-face] contacts and because these interactions are enhanced by geographic proximity. But geography matters also because [face-to-face] contacts are more incline to generate spillovers if interactions are local.” (Autant-Bernard and Massard, 2000, p. 16). We can conclude that the study of Autant-Bernard and Massard (2000) confirms that social networks (through individual’s interactions taken into account by co-authored publications) have an influence on the diffusion of knowledge externalities which are mostly spatially bounded. However, in the study of Autant-Bernard and Massard (2000), only knowledge externalities and not pecuniary externalities are analyzed. As it is stated in the studies of Autant-Bernard and Massard (2004b) and Breschi and Lissoni (2003), pecuniary externalities are more likely to arise than pure knowledge externalities in the problematic of knowledge diffusion. Nevertheless, the boundary between these two kinds of externalities is not perfectly clear. As pecuniary externalities are easily measurable, it is possible that their role in the diffusion of knowledge is overestimated, and that knowledge externalities have a stronger role in the diffusion of innovation than highlighted in the studies of Autant-Bernard and Massard (2004b) and Breschi and Lissoni (2003). Nevertheless, the study of Autant-Bernard and Massard (2000) does not focus on the question of the nature of externalities, and this must be further analyzed.
There are of course many other studies which analyze the influence of social networks on the spatial diffusion of innovation. For example, Autant-Bernard et al. (2007a) estimate the existence of network effects relative to spatial effects using a binary choice model (logit). They introduce physical and social distance to explain the incentives to cooperate in R&D. Their study is based on data on collaborative projects presented to the European 6th Framework Program. More precisely, their study is on individual firm data from 139 European firms and 75 French firms in micro and nanotechnologies (Autant-Bernard et al., 2007a, p. 1 and Autant-Bernard et al., 2007b, pp. 346, 349). They conclude that, at the European level, “the firms’ position within a network (measured by their number of links with other organisations and by their geodesic distance in the previous 5th Framework Program) matters more than their geographical location. At the European level, only co-location in core countries or in peripheral countries increases the probability to cooperate.” (Autant-Bernard et al., 2007b, p. 346). Hence, at the European level, it can be seen that network effects are present, and thus the position in the network from each individual influence the probability of collaboration between them. In other words, firms which have a high number of partners and a small social distance will collaborate on a research project with a higher probability. Therefore, network effects and social distance prevails over spatial effects and physical distance. However, at the French level, both spatial distance and social networks effects affect positively the probability of R&D cooperation. To conclude, the analysis of Autant-Bernard et al. (2007a) proves that spatial proximity as such is not the main determinant of agglomeration effects. Geographic proximity is only the outcome of the existence of other phenomena, such as local networks relations. Hence, networks effects contribute to spatial concentration (Autant-Bernard et al., 2007a, pp. 1, 3, 19-20).

As a second example, consider the study of Ponds et al. (2007). They analyze the relation between spatial proximity and collaboration, i.e. knowledge exchange or diffusion. Their study is based on co-publication data\textsuperscript{88} in scientific subfields as a

\textsuperscript{88} The data Ponds et al. (2007) use come from “Web of Science” which is a product proposed by the Institute of Scientific Information (ISI). The “Web of Science” brings together information on publications in every main journal in the world from 1988 (Ponds et al., 2007, pp. 427-28).
proxy for research collaboration, for the period 1988-2004, from 40 NUTS 3 regions in Netherlands (Autant-Bernard et al., 2007b, pp. 345, 348 and Ponds et al., 2007, pp. 423, 428, 441). In their study, they employ two methodologies “to assess the extent to which institutional distance increases the need for geographical proximity in scientific collaboration.” (Autant-Bernard et al., 2007b, p. 345). They first use a censored tobit regression to analyze to which extent the distance between partners is a function of the kind of collaboration considered. They differentiate three types of organizations, namely firms, academic institutions and governmental and non-profit-making organizations, for different technologies, and determine the addresses of each organization involved. The result is that, when cooperating institutions are of distinct nature, spatial distance between them is lower. Secondly, they use a negative binomial gravity model to examine whether spatial proximity between two organizations has an influence on the probability of collaboration. Their analysis confirms the presence of spatial effects which become more significant when institutions are not of the same nature (e.g. the influence of spatial distance is higher in the case of academic and non-academic relations). To sum up, Ponds et al. (2007) conclude that cooperative organizations of distinct nature are more spatially localized than similar collaborative organizations. Therefore, spatial proximity appears to be an indirect mean to overcome institutional differences, but not a direct mean to stimulate interactions as it is assumed in other studies (Autant-Bernard et al., 2007b, pp. 345, 348 and Ponds et al., 2007, pp. 423-24, 430, 437, 441-42).

To clarify and conclude the analyses from chapters 1 and 2, we will provide a synthesis of these both chapters in the final chapter of this study.
6 Concluding comments on the influence of social networks on the spatial diffusion of innovation

In chapter 1, we clarified the notion of innovation and distance. We stated that two kinds of distance exist in the problematic of spatial diffusion of knowledge, namely physical distance and social distance. It is now possible to affirm that physical distance overcomes social distance since the analysis from chapter 2 proved that social networks are spatially bounded. As a result, face-to-face contacts are more adequate to transmit knowledge than communities of practice. We stated in chapter 1 that the type of knowledge which is mostly enhanced through face-to-face contacts is tacit knowledge. Therefore, since social networks are localized and face-to-face contacts are important, tacit knowledge has a more significant role in the problematic of spatial diffusion of innovation than codified knowledge. However, as tacit and codified knowledge are present at every stage of the “production” of an innovation, from the very beginning to its completion, it is possible that codified knowledge is also present in the problematic of the spatial diffusion of innovation. Therefore, it is not possible to affirm with certainty which type of knowledge mostly arise in this problematic. Nevertheless, it seems at first sight that tacit knowledge has a greater influence on the spatial diffusion of innovation than codified knowledge.

In chapter 2, we analyzed the diffusion of externalities and wondered whether these are mainly of a pecuniary or of a knowledge kind. The study of Autant-Bernard and Massard (2004b) concludes that pecuniary externalities have a stronger influence on the spatial diffusion of knowledge than knowledge externalities as these are only a by-product of the determinants of pecuniary externalities. Pecuniary externalities arise mainly through the presence of other plants. Hence, the concentration of firms is an important innovation-explaining variable. Autant-Bernard and Massard (2004b) conclude that pecuniary externalities, but also knowledge externalities are spatially bounded, and this tends to limit the spatial spread of knowledge. The other two empirical studies,
namely the one of Breschi and Lissoni (2003) and the one of Autant-Bernard and Massard (2000) do not test whether pecuniary or knowledge externalities mostly arise in the problematic of spatial diffusion of knowledge. They strictly test the influence of knowledge externalities on the spatial diffusion of knowledge. However, Breschi and Lissoni (2003) assert in their study that it can indeed be pecuniary externalities which are present instead of knowledge ones. Therefore, as we stated in chapter 2, it is not possible to differentiate clearly these two kinds of externalities and to affirm with certainty which one mostly arise in the problematic of the spatial diffusion of knowledge. As pecuniary externalities are easily measurable, it is possible that their role in the spatial diffusion of knowledge is overvalued, and that knowledge externalities have a more important role in the spatial diffusion of innovation than highlighted in the studies of Autant-Bernard and Massard (2004b) and Breschi and Lissoni (2003). This question is not yet completely answered and further research on this subject needs to be done.

In their study, Breschi and Lissoni (2003) and Autant-Bernard and Massard (2000) take into account the influence of social networks on the spatial spread of knowledge externalities. Both conclude that the diffusion of knowledge externalities is spatially bounded as social networks (through patent citations or co-authored publications) are localized in space, and this restricts the spatial diffusion of innovation. In other words, the diffusion of knowledge is dependent on individual’s interactions which are enhanced by geographic proximity.

It can definitely be concluded that social networks have an influence on the diffusion of innovation which is geographically bounded. This conclusion is also confirmed by other studies briefly analyzed in the second part of this study. Therefore, we can affirm that innovation is concentrated in space as well as social networks are.

The next chapter introduces a relatively new challenge concerning the problematic of the spatial diffusion of innovation, namely NICT. We will examine whether the
introduction of NICT can modify the results obtained in our study of the spatial spread of innovation.

7 Further challenge concerning the spatial diffusion of innovation: NICT

This chapter analyzes briefly the influence of NICT on the spatial diffusion of innovation. At first sight, it seems clear that NICT, more precisely, internet, e-mails, mobile phones, etc. would liberate innovative activities and knowledge externalities from any spatial constraint by decreasing the transmission costs of information and knowledge. Hence, with the introduction of NICT, the diffusion of knowledge appears to be globalized (Autant-Bernard et al., 2003, p. 1 and Autant-Bernard and Massard, 2001a, p. 20).

However, a disappearance of physical distance between communicating actors, with the introduction of NICT, has not yet been confirmed by theoretical and empirical studies. Although it is true that NICT facilitate the access to knowledge (e.g. scientific literature available on the web and not only in a few libraries), they do not replace totally face-to-face contacts since the latter has many advantages that NICT do not have (e.g. informal discussions in a conference or comprehension of the information transmitted). It follows that electronic contacts and face-to-face contacts are rather complements than substitutes as some contacts are too subtle to be realized electronically and as electronic contacts are often followed by face-to-face contacts which need spatial proximity to be realized (more precisely, electronic contacts enhance face-to-face collaborations by facilitating first contacts). Moreover, we stated in our study that knowledge (especially tacit knowledge) is highly context dependent. Therefore, the capacity of NICT to diffuse knowledge globally in space is not so evident. To conclude, we can assert that agglomeration forces exceed dispersion forces when we take NICT into account. This conclusion is confirmed by the empirical studies of Gaspar and

Nevertheless, this result must be considered cautiously as data on NICT are only available relatively recently (mid-1990s). This raises problems for the evaluation of the geographical implications of NICT as they can only be analyzed on a short period and as there exists few of studies on this problematic (Autant-Bernard et al., 2003, p. 14 and Autant-Bernard and Massard, 2001a, pp. 20-21).

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Conclusion

This study analyzed the influence of social networks on the spatial diffusion of innovation. In the first part of this study, we examined theoretically what exactly social networks are, how they are constructed, etc. To achieve this, we analyzed a model of networks formation, based on a game theoretic approach, that determines which kinds of networks arise in equilibrium (are stable) and efficient. We found that empty, star and complete networks are stable and efficient. In this first part, we examined as well three particular networks structures (the regular graphs, the small worlds and the random graphs) and determined which one fosters more knowledge creation and/or diffusion. The result is that the small world is the network structure that promotes the most knowledge creation and diffusion, and thus maximizes the average long run knowledge level.

In the second part of this study, we did an empirical analysis of the influence of social networks on the spatial diffusion of innovation. More precisely, we defined the concepts of distance and innovation while focusing on the notions of tacit and codified knowledge. We analyzed as well three empirical studies on spatial diffusion of externalities to know whether knowledge is spatially concentrated. The first one examines whether knowledge or pecuniary externalities arise in the problematic of the spatial diffusion of knowledge. It finds out that mostly pecuniary externalities arise in this problematic. The other two studies examine the influence of social networks on the spatial diffusion of knowledge externalities. They only focus on knowledge externalities, but it is possible that, in fact, pecuniary externalities arise instead of knowledge externalities as it is difficult to define and distinguish them clearly. These two studies conclude that knowledge externalities are bounded in space as social networks are. This conclusion is confirmed by many other studies briefly described in the second part of this study. Hence, as social networks are localized and thus face-to-face contacts are important, we can conclude as well that it is tacit knowledge, and not codified knowledge, which has an important role in the problematic of spatial diffusion of innovation. Finally, we also briefly analyzed the influence of NICT
on the spatial spread of innovation. We found that the diffusion of innovation remains spatially bounded.

We can definitively conclude that knowledge and social networks are concentrated in space. Nevertheless, it must not be forgotten that the empirical models from the studies analyzed in the second part of this study suffer from some shortcomings, and thus their results must be considered cautiously. Furthermore, the studies analyzed in our paper concentrate only on some specific countries and time periods. Further empirical research on other countries and time periods needs to be done in order to corroborate the conclusion that knowledge and social networks are concentrated in space.

We can conclude as well that the concentration of knowledge and social networks in space raises challenges for governments and other institutions that want to foster innovation as it can generate divergences in the level of development of distinct regions of a country (or other geographical areas). In other words, regions in which innovation is concentrated will develop better and faster than regions in which innovation is hardly present. As a result, inequalities in development between distinct regions of a country can generate further difficulties to governments that want to establish policies in order to promote economic development of the whole country. We can now well understand the importance of the study of the spatial diffusion of knowledge and the role of social networks for governments’ decisions to promote innovation.
Bibliography


Malerba (eds.) *Clusters, Networks and Innovation*, Oxford University Press, pp. 343-78.


Annexes

Annex 1. Affiliation network of applicants, patents and inventors

Source: Breschi and Lissoni (2003, p. 13)
Annex 2. French administrative departments

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Source: Autant-Bernard and Massard (2000, p. 19)