When do proxy advisors improve corporate decisions?*

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Abstract

There is an ongoing debate about how proxy advisory firms affect corporate decisions. A major concern is that shareholders seeking to save costs use a proxy advisor's vote recommendation as substitute for own research, thereby reducing efficiency of shareholder decision-making. We show that the opposite effect -- complementarity between a proxy advisor's recommendation and shareholders' research effort -- occurs if two conditions are met: (i) the board of directors is sufficiently well informed; and (ii) shareholders can condition their investment in research on the proxy advisor's recommendation. In sum, a profit-maximizing proxy advisor can improve corporate decision making by stimulating shareholders' research through its own.

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1 Introduction

Shareholders vote on a variety of important issues, including director elections, executive compensation, and certain aspects of mergers and acquisitions. During the past two decades, shareholders' decision making has changed due to the rise of a new business: Proxy advisory firms (such as ISS and Glass Lewis) provide voting recommendations to shareholders. These recommendations have substantial impact on voting outcomes.[1] There is an ongoing public and scientific debate about the effects of proxy advisors (PAs in what follows) on the quality of decision making in shareholder meetings. The regulation of PAs is highly contentious.[2]

A key point of contention is that PAs may crowd out shareholders' incentives to invest in own research. Shareholders who rely on a PA's recommendation as a substitute for own research save costs individually, but negatively affect the collective by not contributing new information into the decision-making process. This intuition has been probed and developed in the influential analysis of Malenko and Malenko (2019).

In this paper, we instead show that under two arguably practically relevant assumptions the presence of a PA actually leads to either more shareholders' investment in research or at least not less. The presence of a PA hence (weakly) improves corporate decision quality.[3]

We theoretically analyze the strategic interaction surrounding shareholder decisions in a

[1] See, for example, Alexander et al. (2010); Choi, Fisch, and Kahan (2010); Ertimur, Ferri, and Oesch (2013); Iliev and Lowry (2015); Larcker, McCall, and Ormazabal (2013, 2015); Li (2018); Malenko and Shen (2016); McCahery, Sautner, and Starks (2016). For instance, Malenko and Shen (2016) show a causal effect on voting outcomes with a regression-discontinuity design.

[2] Spatt (2020) provides a recent survey of the literature with a focus on regulatory issues.

[3] Apart from the concern regarding crowding-out of shareholder incentives to conduct research, conflicts of interest may exist. Some PAs do not only sell recommendations to shareholders, but also effectively sell consultancy services to the firms. Thus, a concern is that a PA can be "captured" by a firm's management such that the recommendation to shareholders is biased in favor of acceptance of proposals. In this paper, we abstract from this problem because we wish to establish when a PA can potentially be value-increasing. This motivation is similar in spirit to Boot, Milbourn, and Schmeits (2006) who establish a raison d'être of credit rating agencies in the absence of conflicts of interest.

firm. There are three types of agents: Shareholders, the firm's board of directors, and a PA. Shareholders and the board both care about firm value, whereas the PA cares for its profit. Players are imperfectly informed about the correct decision on a given issue, i.e., about which decision will increase firm value most. The board and the PA receive a private imperfect and independently distributed signal about the correct decision. The board proposes a decision based on its own signal. Then, each shareholder individually decides whether to buy the PA's vote recommendation, i.e., the PA's signal, and whether to invest in own research, i.e., to obtain a private signal. The simple majority rule determines the outcome.

Two key assumptions are critical for our analysis. Our first key assumption, "BIB" (for better-informed board), is that the board is better-informed than any single shareholder alone. This is in line with a long tradition of studies in corporate governance arguing that insiders (the board and management) have information about the company, which may be superior to that of shareholders (Jensen and Meckling, 1976).[4]

The second key assumption, "PAF" (for proxy advice first), is that the PA moves first in the following sense: *after* receiving proxy advice, a shareholder can decide upon additional research about the issues at hand. This assumption is more likely to hold if shareholders are given sufficient time to conduct research (and if "robo-voting", that is, the practice of simply automatically converting the advice received by PAs into a vote, is forbidden). Below, we discuss explicitly how relaxing these two assumptions affects the results.

We solve this game-theoretic model for pure Perfect Bayesian Nash equilibria and mainly focus on symmetric equilibria that are not Pareto-dominated by other symmetric equilibria.[5]

[4]The focus of our model is the uncertainty that the company faces on how to improve firm value, not a potential conflict of interest between different stakeholders.

[5]In the Appendix, we provide a complete characterization of all symmetric equilibria for any combination of signal qualities of the board, the PA, and the shareholders. Pareto-dominated equilibria are based on

We find that the presence of a PA positively affects the shareholders' incentives to invest in own research or leaves these incentives unchanged, and we determine the corresponding parameter spaces. The underlying intuition for this finding is as follows: Begin with the situation without a PA. Consider a board's proposal and a shareholder who is interested in the firm value. This shareholder only needs to consider the case in which her vote decides whether the board's proposal passes or not, i.e., being pivotal. This shareholder realizes that in this case the other shareholders' votes in favor and against the proposal offset each other. Hence, the pivotal shareholder cannot infer from pivotality whether the other shareholders' signals tend to support or undermine the board's proposal in the aggregate. In this situation, only her own information and the board's information are usable. Now, even if this shareholder has invested into own research and this signal is not in favor of the board's proposal, it is still better to vote for the proposal since the board, by assumption, is better informed than the shareholder. Therefore, shareholders would not invest into own research in the first place and generally prefer to follow the board's proposal. Hence, our first leading assumption, BIB, leads to correlated votes and a lack of own investment incentives, even without any PA. In sum, PAs per se are not the root cause of insufficient incentives for shareholders.

This baseline result has a critical consequence for the potential value effects of a PA. Specifically, the presence of a PA leads to a different equilibrium behavior. If both the PA's recommendation and the shareholder's own research point against the board's proposal, it can be better for the shareholder to reject it. Therefore, investments into own research are beneficial *after* the PA's recommendation is against the proposal. Intuitively, a profit-maximizing PA can improve corporate decision making by pointing to the critical issues that

coordination failure. We extend the main results to asymmetric equilibria in Section 4.

deserve further investigation. Thus, PA and shareholder research efforts are complementary. At worst, the outcome with a PA is the same as without a PA; this obtains when the PA is either much less or much more informed than the board. However, when the board and the PA are similarly well informed, a PA strictly increases decision quality.

This novel positive result obtains under two assumptions: first, the board's proposals contain valuable information about the best decision (BIB) and second, a PA's advice arrives at the shareholders sufficiently early such that they can condition their own research investments on it (PAF). These two assumptions distinguish our approach from the existing literature. What are the implications of relaxing each of the two assumptions? Most importantly, when neither of them is satisfied, then proxy advice and own research are substitutes, such that the presence of a PA undermines the shareholders' research incentives. However, both BIB and PAF appear plausible in practice, and together they yield a simple reason for PAs to exist.

Although our model is necessarily stylized, it is helpful to consider how investors use the services of PAs in reality. Shu (2020) documents that in 2017, about a quarter of ISS customers and a very small fraction of Glass Lewis customers appear to have ‘‘robo-voted’’ (followed a PA ‘‘blindly’’). It is not clear why the other investors -- which constitute the large majority -- chose differently. Some may have different general policies. However, it appears quite plausible that at least some investors use the PA advice as a signal to conduct their own research on a contentious issue. Indeed, survey evidence indicates that at least sophisticated investors use PAs in the way our model suggests. For example, a 2018 survey (Swipra, 2018) containing responses of 44 asset managers including 24% of world-wide assets under management found that while almost all international asset managers purchase proxy advisory services, more than 70% of them mainly use the PA for obtaining vote-related governance materials and other information in comparable form; less than 30% state that they mainly look for the actual vote recommendation. By contrast, 90% of the 30 pension funds that participated in the survey primarily care for the binary recommendation and little for the underlying materials. These results suggests that international asset managers are able and willing to conduct the own research that enhances the ultimate decision quality, but that an asymmetric equilibrium as modeled in Section 4 is likely to obtain in practice.

Our paper is closely related to the literature considering common interest among shareholders who vote strategically (Malenko and Malenko, 2019; Bar-Isaac and Shapiro, 2020; Ma and Xiong, 2020). More broadly, our model also relates to the literature on strategic voting in a common interest setting that started with Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996).[6]

We contribute to the literature in four different respects: First, we show that muted shareholders' incentives to conduct their own research are not solely due to a PA acting as a substitute informer, but also occurs *without* a PA if the board's proposal is based on sufficiently valuable information. Second, we show that proxy advice that is given early enough can foster shareholders' own investments in research. This has the major policy implication that shareholders need to have sufficient time to conduct such research *after* receiving the advice from PAs. Under such conditions, PAs are likely to improve decision quality and social welfare, while maximizing their profits.[7] Third, we provide the conditions

[6]Recent contributions that consider public signals include Kawamura and Vlaseros (2017), Liu (2019), and Buechel and Mechtenberg (2019).

[7]In our analysis, the goal of the decision-making is to maximize firm value and the PA provides unbiased recommendations. Related papers explore deviations from these assumptions. As for the former deviation, Matsusaka and Shu (2020) show how a PA profitably caters to the preferences of investors with non-value-maximizing goals. As for the latter deviation, Malenko, Malenko, and Spatt (2020) analyze how a PA can

for the beneficial and detrimental effects of PAs by covering all qualities of information for board, PA and shareholders. Fourth, there may be applications to other areas. For example, Sangiorgi and Spatt (2017) argue that credit rating agencies can "crowd out" independent information production by investors. Future work might analyze whether credit rating agencies may also positively contribute to information production.

The paper proceeds as follows. Section 2 sets up the model. Section 3 provides the main result, considering symmetric equilibria. Section 4 generalizes to asymmetric equilibria. Section 5 discusses the results by considering deviations from the two core assumptions and presents policy implications. Section 6 concludes.

2 Model Setup

2.1 Basic Ingredients

We model voting on corporate decisions as strategic voting under uncertainty (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996, 1997, 1998). Thus, we follow frameworks such as Malenko and Malenko (2019), Bar-Isaac and Shapiro (2020) and Ma and Xiong (2020).

A firm is owned by $N > 1$ shareholders, where N is odd. The firm faces uncertainty with

enhance profits by biasing its recommendations against the more likely alternative a priori, thus increasing the value of its recommendations to shareholders. In Ma and Xiong (2020), a PA skews its recommendation either because of a conflict of interest or according to a bias on the side of the shareholders. Shareholders do not decide upon own research activities in their model. Levit and Tsoy (2020) show in a cheap talk communication game how an advisor may adopt one-size-fits-all recommendations in order to obscure its biases. While overall a theme of this line of research is that biasing recommendations may increase the fraction of shareholders who subscribe to the PA's service, we show that the presence of (unbiased) recommendations may incentivize subscribing shareholders to invest in own research and hence add valuable information into the decision-making process. Thus, we see our approach as complementary by establishing a basic reason why PAs can add value to corporate decision-making in the absence of these frictions.

respect to a binary decision.[8] Making the ex post correct decision will increase firm value by an amount normalized to 1, while the wrong decision leaves it unchanged.

More formally, there are two states of the world $\theta \in \{A, B\}$ with equal prior probability. The firm has to decide on a binary issue $\{A, B\}$ that yields value 1 if and only if the decision matches the true state.

The board of directors receives a binary signal regarding the issue to be voted on. The signal takes on values a or b. The signal quality is $q_B \in (\frac{1}{2}, 1)$, i.e., $Pr[s_B = a|\theta = A] = Pr[s_B = b|\theta = B] = q_B$. Slightly abusing notation, we assume that the board then proposes either action A or B.

A profit-maximizing proxy advisor (PA) offers advice to shareholders at fee $f > 0$. The PA receives a signal about the true state as well. The quality of that signal is $q_P \in (\frac{1}{2}, 1)$. The PA provides a vote recommendation *for* or *against* the board's proposal to subscribing shareholders.

Shareholders decide whether to subscribe to the PA's offer. If a shareholder subscribes, she receives the PA's recommendation. A shareholder *then* decides whether to invest $c > 0$ in own research about the issue at hand. If a shareholder expends own research costs, this leads to a private signal of quality $q_S \in (\frac{1}{2}, 1)$. When the shareholder meeting is held, each shareholder votes *yes* or *no*. Abstentions are excluded.[9] For simplicity each shareholder holds one share of the firm and each share provides one vote. The decision that receives a majority of votes is implemented. Conditional on state θ, all signals are independent, and precision levels q_B, q_P, and q_S are common knowledge.

[8]Examples vary by jurisdiction and include but are not restricted to director elections, payout and retention of earnings, the approval of a compensation report, compensation plans, or compensation amounts.

[9]Practically, shareholders may also abstain. However, according to most institutional settings abstentions are counted (either as *yes* or *no*) and hence shareholders' voting action is essentially binary.

Our first leading assumption is that the board knows best what is good for the company.

Assumption 1 (BIB). *The board is at least as well informed as a single shareholder, i.e.,*

$$q_S \leq q_B.$$

"BIB" stands for better-informed board. For the quality of the PA q_P we do not make an assumption that restricts it to be above or below the other agents' qualities.

In the course of the analysis it will come in handy to transform signal qualities $q \in (0.5, 1)$ into log-odds $\log(\frac{q}{1-q}) \in (0, \infty)$. We denote the log-odds of the board being correct as $\ell_B := \log(\frac{q_B}{1-q_B})$ and likewise $\ell_S := \log(\frac{q_S}{1-q_S})$ for the shareholders and $\ell_P := \log(\frac{q_P}{1-q_P})$ for the PA. Then Assumption BIB reads $\ell_S \leq \ell_B$.[10] This notation is convenient since it allows us to aggregate signal qualities by summation. To see this, consider the board's signal b and assume, for instance, that both the PA and one shareholder have received signals a and that there is no further information. Then, the board's signal is rather correct than not if and only if $q_B(1-q_P)(1-q_S) \geq (1-q_B)q_P q_S$, which is equivalent to $\ell_B \geq \ell_P + \ell_S$.

Our second leading assumption is that shareholders can condition their research investment on the PA's recommendation.

Assumption 2 (PAF). *Subscribing shareholders decide upon own research investment* after *they have received the PA's recommendation.*

"PAF" stands for "proxy advice arrives first". Practically, shareholders may make a bulk of their investment in research about a company independent of the proxy advice and

[10] Nitzan and Paroush (1982) show that among voters with idiosyncratic signal precision the optimal voting weights would be according to these log-odds.

also before receiving the PA's recommendation. Our assumption PAF is that the information relevant for deciding on a specific issue can be conditioned on the PA's recommendation.[11]

2.2 Simplification and Timeline

It turns out that we can substantially simplify the exposition without losing substance of the analysis by fixing the signal and behavior of the board and the behavior of the PA. The board receives a signal and then makes a proposal. We let the board's signal be always b (for board).[12] We fix the board's behavior by assuming it makes the proposal according to its signal, i.e., it has received signal b and now proposes action B. Likewise, we fix the PA's behavior to set fee $f > 0$ and recommend according to its signal, i.e., it recommends *for* if it has received signal b (for board) and it recommends *against* if it has received signal a (against board).[13]

The timeline, which is illustrated by Figure 1, summarizes the simplified setup. At $t = 0$ nature draws a state of the world and signals for all potential recipients of signals. At $t = 1$ each shareholder decides whether to pay the fee for the PA's report. Those who pay the fee receive the truthful vote recommendation which is equivalent to learning the PA's signal. At $t = 2$ each shareholder decides whether to invest costs c to receive an own independently and identically distributed signal of quality q_S. At $t = 3$ shareholders vote. At $t = 4$ the proposal passes if a majority approves it and payoffs are realized.

[11]Relaxing this assumption would change the timing of our model such that shareholders have to decide simultaneously about subscribing to the PA and about investing in own research. That is the assumption in Malenko and Malenko (2019). We discuss the consequences of making this assumption in our model in Section 5.

[12]This will exclude strategies that depend on the label of the alternative, such as always voting *yes* for alternative A and *no* for alternative B independent of which alternative the board has proposed.

[13]In an Online Appendix, we discuss how these simplifications affect the analysis.

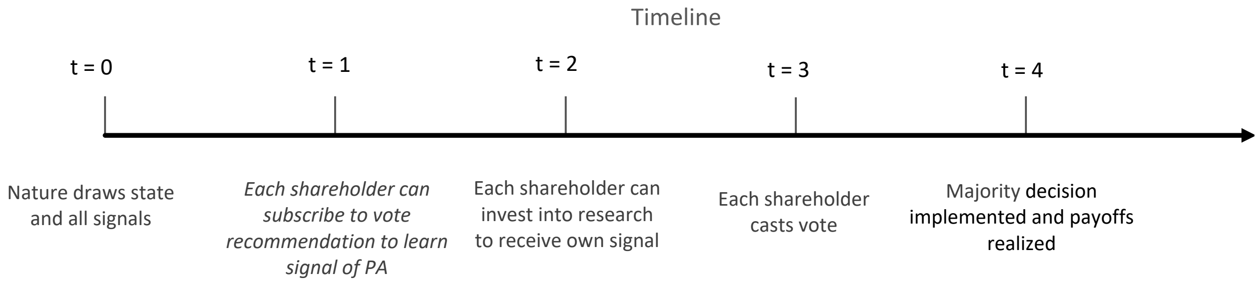

Figure 1: Simplified timeline. Simplification is that board's and PA's behavior is fixed. In particular, PA's recommendation strategy is fixed to be truthful such that subscribing shareholders learn the PA's signal. (Actions in italics only apply if there is a PA.)

2.3 Strategies

The most important strategic aspects concern the shareholders. They have several strategies both on the information acquisition stages ($t = 1$ and $t = 2$, respectively) and on the voting stage ($t = 3$). On the information acquisition stages, there are six strategies: A shareholder who does not subscribe may invest in own research (NotSubscribe-Invest) or not (NotSubscribe-NotInvest); a shareholder who does subscribe may unconditionally invest in research (Subscribe-Invest) or not (Subscribe-NotInvest) or, else, may invest in research only if the recommendation is *for* (Subscribe-InvestIFFfor) or only if the recommendation is textitagainst (Subscribe-InvestIFF$against$).

In the voting stage, any shareholder chooses *yes* or *no*. The set of voting strategies depends on the acquired information which may include the PA's signal and the own signal. For instance, for a shareholder who acquired both kinds of information (e.g., with Subscribe-Invest), a voting strategy is a mapping $v_i : \{for, against\} \times \{a, b\} \rightarrow \{yes, no\}$. Slightly abusing notation, we write σ_i for the information acquisition and voting strategy of a shareholder i, and we use $\sigma = (\sigma_1, ..., \sigma_N)$ to denote a strategy profile of shareholders.

We study Perfect Bayesian Nash equilibria in pure strategies, i.e., players best respond to their beliefs and update their beliefs according to Bayes' rule wherever possible. We focus on pure strategies.[14]

To analyze the model we take the perspective of a regulator. The regulator compares a market with PA, as in the game defined above, with a market in which no PA is admitted. The quality of corporate decisions is measured by $\Pi(\sigma)$, the ex ante probability that the decision will match the true state.[15]

3 Symmetric Equilibria

This section analyzes how the quality of corporate decisions depends on the presence of a PA. It develops the intuition by focusing on symmetric strategy profiles, whereas Section 4 generalizes to asymmetric equilibria.

3.1 Benchmark Setting: No Proxy Advisor

Consider first the benchmark case that no PA is admitted. Thus, posit that in the timeline of Figure 1 actions at $t = 1$ are suppressed. Then a shareholder's information acquisition decision reduces to whether to acquire an own signal or not in $t = 2$. Suppose for a moment that all shareholders do acquire such a signal and vote according to it. We call this strategy

[14]Technically this is a difference to Malenko and Malenko (2019) and Ma and Xiong (2020), who study equilibria in mixed strategies, but focus on symmetric equilibria in the sense that all shareholders must play the same strategy. In spirit, the difference is not so big: One interpretation for mixed strategies is that a fraction of the population plays a certain pure strategy, which we admit when studying asymmetric strategy profiles. When there are multiple equilibria in some area, we exclude those that are Pareto-dominated. This eliminates equilibria due to miscoordination, as we will explain.

[15]This is also called *informational efficiency*, which can be distinguished from *economic efficiency* (see, e.g., Buechel and Mechtenberg, 2019). Economic efficiency means welfare, which can be here defined as $\Pi(\sigma)$ net of the investment costs in own research since the prices paid to the PA are transfers. When investment costs c become arbitrarily small both concepts coincide.

profile *UNIS*, for "UNconditional Investment in own Signal."[16] In this strategy profile the decision quality amounts to $\Pi(\sigma^{UNIS}) = \pi(N)$, where $\pi(N) := \sum_{i=\frac{N+1}{2}}^{N} \binom{N}{i} q_S^i (1-q_S)^{N-i}$ is the probability that a majority decision of N shareholders is correct.

While the decision quality of such voting behavior is usually very high (De Caritat, 1785), it is unfortunately not an equilibrium under Assumption BIB. The intuition is straightforward once spelled out. A single shareholder i can improve by deviating to not acquire a signal and vote *yes*. When this shareholder i is pivotal, the signals of all $N-1$ other shareholders are split: there are as many a signals as there are b signals among them. Now, even if i's signal points against the board's proposal, Assumption BIB, i.e., the assumption that the board is at least as well informed as i, makes it beneficial to vote *yes*, i.e., for the board's proposal, and not to acquire own information in the first place.

More generally, the first proposition shows that in any symmetric equilibrium there will be no investment, given that the board is better informed than a single shareholder. Instead, in the Pareto-dominant symmetric equilibrium the shareholders do not acquire information and unconditionally vote *yes*. We call this strategy profile "*Rubber-stamping*" as all proposals of the board are accepted without further investigation. Rubber-stamping induces a decision quality $\Pi(\sigma) = q_B$ because it leads to the correct decision whenever the board's proposal is right.

Proposition 1 (SYM without PA)**.** *Suppose no PA is admitted. If Assumption BIB holds, then there does not exist a symmetric equilibrium in which shareholders invest in own research. Hence, decision quality in symmetric equilibria is bounded by:* $\Pi(\sigma) \leq q_B$*. The*

[16] The term "unconditional" will be justified later, when shareholders could potentially condition their investment in own research on the PA's vote recommendation.

Pareto-dominant symmetric equilibrium is Rubber-stamping and leads to decision quality $\Pi(\sigma^{Rubber}) = q_B$.

All proofs are collected in Appendix A. Proposition 1 shows that without a PA there is no incentive to invest in own research. Focusing on pivotality, shareholders prefer to ignore their own signal and follow the board. Assumption BIB, $q_S \leq q_B$, is in fact necessary and sufficient for this conclusion.[17]

3.2 Does Decision Quality Increase With a Proxy Advisor?

Consider now the situation when a PA is admitted and proxy advice arrives before the shareholders' decision to invest in own research (Assumption PAF holds). That is, all actions occur as illustrated in the timeline (Figure 1), including the actions in italics. The presence of a PA substantially increases a shareholder's set of information acquisition strategies. One of them, Subscribe-InvestIFF*against*, gives rise to the following symmetric strategy profile, which we denote by $\hat{\sigma}$ and call "*CAIS* (Conditional on Advice Invest in Signal):" All shareholders subscribe to proxy advice; if the recommendation is *for*, they vote *yes*; if the recommendation is *against*, they invest in own research and vote according to their own signal, i.e., vote *yes* if the signal is b and *no* if it is a. In this strategy profile shareholders use the PA's recommendation as a filter: *for* recommendations are followed without being challenged; *against* recommendations trigger further investigation of the issue. It turns out that based on this strategy profile the negative result of Proposition 1 can be mitigated by the presence of a PA, as Proposition 2 shows.

[17]Since we have Assumption BIB as a leading assumption, we only show sufficiency in the proof of Proposition 1.

Proposition 2 (SYM with PA)**.** *Let Assumptions BIB and PAF hold. Let costs c be arbitrarily small and let fee f be sufficiently smaller.*

i. Suppose there is a PA with $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$, then there exists a symmetric equilibrium in which shareholders conditionally invest in own research. The Pareto-dominant equilibrium is CAIS and leads to decision quality $\Pi(\sigma^{CAIS}) > q_B$.

ii. Otherwise, i.e., if $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$, there does not exist a symmetric equilibrium in which shareholders invest or conditionally invest in own research. The Pareto-dominant equilibrium is Rubber-stamping and leads to decision quality $\Pi(\sigma^{Rubber}) = q_B$.

Comparing Proposition 2 with Proposition 1 shows that the presence of a PA either strictly improves decision quality (part i.), or it leaves decision quality unchanged (part ii.), compared to the case without PA. The condition for the strict improvement can be rewritten as $\ell_S < |\ell_B - \ell_P|$, which has the following interpretation: the difference in quality of board and PA is not larger than the information quality of one shareholder. If this conditions is satisfied there is no equilibrium with information acquisition without a PA, while we have a new equilibrium (CAIS) in which all shareholders conditionally invest in own research. Hence, part i. of Proposition 2 shows that the presence of a PA can foster the shareholders' own research, and that this increases decision quality: $\Pi(\sigma^{CAIS}) > q_B$. For the strategy profile CAIS, each shareholder's utility is

$$u_i(\hat{\sigma}) = q_B q_P + [(1 - q_B)q_P + q_B(1 - q_P)](\pi(N) - c) - f, \tag{1}$$

where $\pi(N) = \sum_{i=\frac{N+1}{2}}^{N} \binom{N}{i} q_S^i (1-q_S)^{N-i}$. The utility consists of the probability that the board and the PA agree on the correct proposal times one, plus the probability of disagreement times payoff in that case ($\pi(N) - c$), minus the payment of the fee. Recall that $\pi(N)$ is the probability that the majority decision of N shareholders who vote in line with their own signal matches the true state. If the board and the PA agree on the wrong proposal, which happens with probability $(1 - q_B)(1 - q_P)$, then the payoff is 0.

The first intuition for the conditions of part i. of Proposition 2 can be seen from their violations. Consider the symmetric strategy profile CAIS. If $\ell_P \leq \ell_B - \ell_S$, we have $\ell_S + \ell_P \leq \ell_B$, i.e., the board is better informed than the PA and one shareholder together. Then there is a deviation from CAIS to Rubber-stamping. Intuitively, the board is sufficiently well informed that it does not pay off to acquire any information, even if it were costless. If $\ell_P \geq \ell_B + \ell_S$, i.e., the PA is better informed than the board and one shareholder together, then there is a deviation from CAIS to voting against the board's proposal. Indeed, the deviating shareholder's vote is only pivotal if board and PA disagree and voting *no* improves decision quality, given that the PA is so well informed. If costs c or f are not small enough, there is again a beneficial deviation, e.g., to Rubber-stamping, which saves costs. Finally, if the PA's fee f is not sufficiently smaller than the costs c, then deviating to UNIS saves costs without affecting the outcome.[18] Most importantly, the two key assumptions Assumption BIB and PAF are also necessary for the conclusion, as we will discuss below.

[18]The assumption c small enough assures that shareholder who can improve decision quality by investing in own research would not shy away due to the high costs. The assumption the costs are larger than zero matters when deviations that do not affect decision quality are considered. The assumption that fees f are sufficiently smaller than c means that the results answer the question whether there is a fee f such that a PA can profitably be active in the market.

3.3 Illustration and Discussion

Propositions 1 and 2 are illustrated in Example 1.

Example 1 (Symmetric Equilibria). *Let* $q_B = 0.75$, $q_P = 0.7$, *and* $q_S = 0.6$. *Then* $\ell_B = 0.477$, $\ell_P = 0.368$, *and* $\ell_S = 0.176$ *such that the condition* $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$ *of Proposition 2 part i. is satisfied, as* $0.368 \in (0.477 - 0.176, 0.477 + 0.176)$. *Table 1 illustrates the implications of Propositions 1 and 2 for decision quality. First, not admitting a PA leads to Rubber-stamping and hence to a decision quality of* $q_B = 0.75$, *independent of the number of shareholders* N *(Proposition 1). Second, when a PA is admitted,* $CAIS$ *is the Pareto-dominant symmetric equilibrium, which delivers a strictly higher decision quality (by Proposition 2). Its decision quality is further increasing in the number of shareholders* N *and converging to* $0.925 < 1$. *Finally, Table 1 shows the hypothetical case in which all shareholders play UNIS, i.e., invest in own research. This is a classic benchmark capturing the quality of majority decisions by* N *sincere voters, as already pointed out by the Marquis de Condorcet (De Caritat, 1785). It may start low, but converges to one as the number of voters grows. This is not an equilibrium.*

Setting	Decision quality	$N = 3$	$N = 5$	$N = 21$	$N = 101$	$N \to \infty$
No PA	$\Pi(\sigma^{Rubber}) = q_B$	0.75	0.75	0.75	0.75	0.75
With PA	$\Pi(\hat{\sigma}) = q_B q_P + p^{dis}\pi(N)$	0.784	0.798	0.855	0.917	0.925
Hypothetical	$\Pi(\sigma^{UNIS}) = \pi(N)$	0.648	0.683	0.826	0.979	1.0

Table 1: Decision quality in Example 1. The table considers the two Pareto-dominant symmetric equilibria, Rubber-stamping and CAIS, and strategy profile UNIS, which is not an equilibrium. Illustration of Propositions 1 and 2 for $q_B = 0.75$, $q_P = 0.7$, and $q_S = 0.6$. $p^{dis} := (1 - q_B)q_P + q_B(1 - q_P)$ is the probability that the board's and the PA's signal differ.

We now turn to illustrating Propositions 1 and 2 graphically. This will help explaining the relevance of the key assumption Assumption BIB and bridge our findings and those from the existing literature.

Figure 2 illustrates the parameter space. It uses the log-odds of the board's and PA's relative to the shareholders' signal qualities because this makes the conditions nicely linear. An entry (x, y) in this coordinate system has the simple interpretation that the board is equally well informed as x shareholders, while the PA is equally well informed as y shareholders.[19]

In the upper panel of Figure 2, no PA is admitted. By Proposition 1 Rubber-stamping is the Pareto-dominant equilibrium under Assumption BIB, i.e., $q_S \leq q_B$. This is illustrated in the area $\frac{\ell_B}{\ell_S} \geq 1$. Assumption BIB is necessary and sufficient for this conclusion as UNIS is the Pareto-dominant equilibrium for $\frac{\ell_B}{\ell_S} < 1$. Hence, when there is no PA, information acquisition would occur if and only if the board were less well informed than a single shareholder, which is precluded by Assumption BIB.

In the lower panel of Figure 2, there is a PA and Assumption PAF is satisfied. By Proposition 2 the parameter space in which CAIS is an equilibrium is given by the condition $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$ or $\frac{\ell_P}{\ell_S} \in (\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1)$, which defines a corridor around the 45-degree line.[20] On the 45-degree line the board and the PA are exactly equally well informed, i.e., $\frac{\ell_P}{\ell_S} = \frac{\ell_B}{\ell_S}$ (or $q_B = q_P$). Interestingly, this corridor is not bounded from the upper right. Hence, for arbitrarily well-informed board and PA, there is still an equilibrium with conditional information acquisition of all shareholders, as long as the board and the PA are roughly equally-well informed. Proposition 2 has shown that under Assumption BIB, i.e., for $\frac{\ell_B}{\ell_S} \geq 1$, we have either CAIS or Rubber-stamping as Pareto-dominant symmetric equilibrium.

The comparative static effects of changing information quality are easy to understand

[19] "Equally well informed" means here that if x shareholders have received a signal a (against the board) then both states A and B are equally likely. Hence, if more than x shareholders have received a signal a and there is no other information, then the board should be overruled.

[20] When studying asymmetric equilibria, we show that CAIS can be played by a majority of shareholders wide beyond this corridor. The corridor only restricts the area in which *all* shareholders play CAIS.

using this figure. Assume $\frac{\ell_B}{\ell_S} > 1$ (Assumption BIB) and start with an uninformed PA: $q_P \approx 0.5$ i.e., $\frac{\ell_P}{\ell_S} \approx 0$. Decision quality does not improve with the PA's information quality q_p (or $\frac{\ell_P}{\ell_S}$) first, then discontinuously increases from q_B to $\Pi(\hat{\sigma})$. Within the region where CAIS is an equilibrium, decision quality further improves as $\Pi(\hat{\sigma})$ is continuously increasing in q_P. Finally, it returns to the level q_B when Rubber-stamping is played again. Hence, there is a non-monotonic effect of a PA's information quality on the corporate decision quality with the latter being highest for a PA that is slightly better informed than the board. Comparative-static effects of the board's information quality are analogous if $\frac{\ell_P}{\ell_S} > 1$, i.e., the PA is better informed than a single shareholder. Finally, increasing signal quality of the shareholders, q_S, reduces $\frac{\ell_B}{\ell_S}$ and $\frac{\ell_P}{\ell_S} \approx 0$, which means graphically moving towards the origin. This improves decision quality of CAIS as shareholders base their decision on their own information when the PA's recommendation is *against*.

Assumption BIB, i.e., $\frac{\ell_B}{\ell_S} \geq 1$, rules out UNIS, the strategy profile in which all shareholders acquire information. Violating BIB, UNIS is the Pareto-dominant symmetric equilibrium in the lower left corner of the parameter space (in the lower panel of Figure 2), which is defined by the condition $\ell_S > \ell_B + \ell_P$. Hence, UNIS requires that one single shareholder must be better informed than board and PA together. Interestingly, this is an even stronger condition than the condition for UNIS when no PA is admitted: $\ell_S > \ell_B$.

Let us now compare the upper panel with the lower panel. Under Assumption BIB, i.e., for $\frac{\ell_B}{\ell_S} \geq 1$, the presence of a PA weakly improves decision quality, as it replaces Rubber-stamping with CAIS if anything. When Assumption BIB is violated, there can be a different effect. Suppose that the quality of the board is not much better than a coin flip, i.e., $q_B \approx 0.5$. Then $\frac{\ell_B}{\ell_S} \approx 0$ and there is the equilibrium with full information acquisition (UNIS)

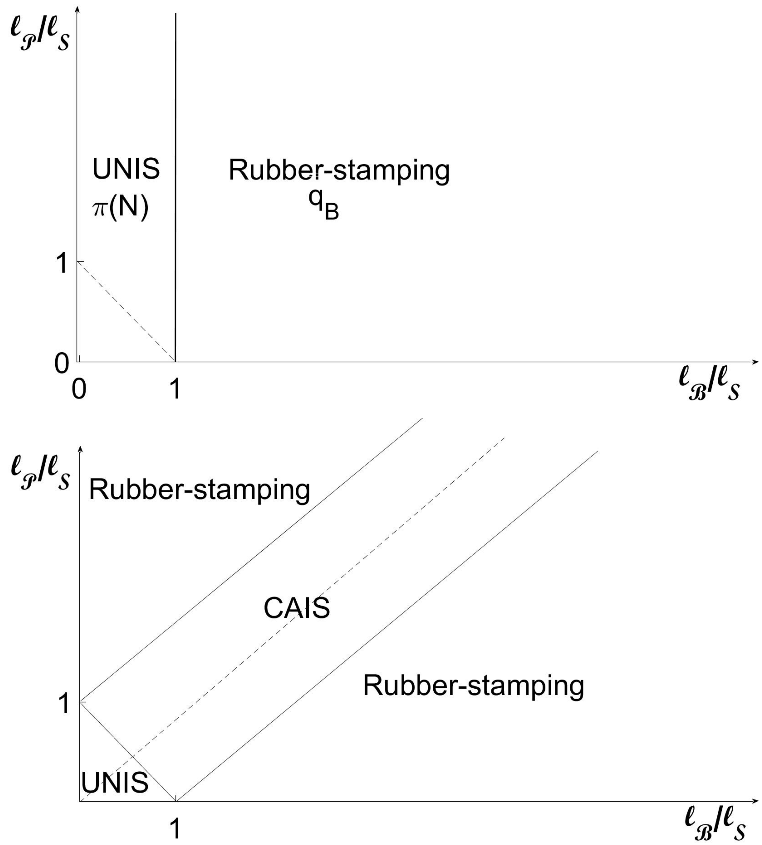

Figure 2: Parameter space with Pareto-dominant symmetric equilibria. Upper panel: without PA; lower panel: with PA

and high decision quality, as long as no PA is admitted. The presence of a PA who is better informed than a single shareholder ($\frac{\ell_P}{\ell_S} > 1$), i.e., being in a point that is on the y-axis above 1, destroys this equilibrium and reduces decision quality from $\pi(N)$ to $q_B \approx 0.5$. The reason is that conditional on pivotality a shareholder prefers to follow the PA's recommendation over acquiring and using the own signal. This is the substitution effect already established for well-informed PAs (Malenko and Malenko, 2019). Hence, Assumption BIB dramatically changes how admission of a PA affects decision quality.

3.4 All Symmetric Equilibria

We have so far presented the equilibria that are selected by the criterion of Pareto-dominance. Now, we briefly address the remaining symmetric equilibria.[21] The findings are summarized in Table 2.

When there is no PA there are three symmetric equilibria. UNIS, in which all shareholders invest in research, is restricted to an area of the parameter space where Assumption BIB is violated, as discussed above. Shareholders who do not invest in research can play Rubber-stamping or do the opposite: vote *no* unconditionally, which we call Protest. In fact, both these symmetric strategy profiles are trivial equilibria, as no shareholder is ever pivotal and they incur no costs. Protest induces a decision quality $\Pi(\sigma) = 1 - q_B$ because it leads to the correct decision whenever the board's proposal is wrong. Clearly, it is Pareto-dominated by Rubber-stamping, as $q_B > 0.5 > 1 - q_B$ and both induce the same costs (none).

These three equilibria also exist when there is a PA (last column of Table 2) and their discussion is analogous. Moreover, there are two additional equilibria, both based on information acquisition strategy Subscribe-InvestIFF$against$. One of them is CAIS. The other equilibrium, CAIS-2, only differs from CAIS in the voting behavior when the vote recommendation is for. In CAIS shareholders vote *yes*, while shareholders in CAIS-2 vote *no*, i.e., they do not approve the board's proposal when the PA recommends to. CAIS-2 is Pareto-dominated by CAIS since it induces the same costs, but a lower decision quality than CAIS. Violating Assumption BIB, we get two more equilibria, both based on information acquisition strategy Subscribe-InvestIFFfor, and both Pareto-dominated by UNIS.

[21]In Appendix A we provide a complete characterization of all pure-strategy equilibria under the symmetry assumption. There, we list all symmetric strategy profiles that can be equilibria and derive the corresponding parameter conditions (Lemmas A.1 and A.2).

Information acquisition strategy	No PA	With PA
NotSubscribe-NotInvest	Rubber*, Protest	Rubber*, Protest
NotSubscribe-Invest	(UNIS*)	(UNIS*)
Subscribe-NotInvest		--
Subscribe-Invest		--
Subscribe-InvestIFF*for*		(two further equilibria)
Subscribe-InvestIFF*against*		CAIS*, CAIS-2

Table 2: All symmetric equilibria arranged by information acquisition strategy. Equilibria in brackets are precluded by Assumption BIB. Equilibria marked by "*" are Pareto-dominant in some area of the parameter space.

More striking than the additional equilibria which are Pareto-dominated is the observation that there are no equilibria with information acquisition strategies Subscribe-NotInvest and Subscribe-Invest (again in Table 2, based on Lemmas A.1 and A.2 in Appendix A). To see why not, note that buying only the vote recommendation, i.e., information acquisition strategy Subscribe-NotInvest, is only worthwhile when using this information in an instance of pivotality. However, if all shareholders symmetrically use the vote recommendation, then no shareholder is ever pivotal. Similarly, acquiring both signals, i.e., information acquisition strategy Subscribe-Invest, is worthwhile only if shareholders condition their vote on both PA advice and own signal such that none is superfluous, e.g., by voting *yes* if and only if one of the latter is in favor of the proposal. When all shareholders adopt this strategy, pivotality already implies that the recommendation was *against*. Hence, saving the subscription fee by not subscribing to the PA is a profitable deviation.

4 Asymmetric Equilibria

Our main results (Propositions 1 and 2) were established based on selecting the Pareto-dominant symmetric equilibria. After having relaxed the Pareto-dominance selection criterion above, we now drop the symmetry assumption. We first show that without PA, the number of shareholders who invest in research is generally bounded from above. We then show that admitting a PA can alter this result, as there is an equilibrium in which a majority of shareholders conditionally invests in research for a large part of the parameter space.

4.1 Benchmark Setting: No Proxy Advisor

Consider again the benchmark setting in which no PA is admitted. While Proposition 1 stated that under Assumption BIB, there is no symmetric equilibrium in which *every* shareholder invests in own research, the next result extends this to asymmetric equilibria in some parameter range. It also states that generally, in equilibrium without PA there are always some shareholders not investing in research.

For the description of decision quality it is helpful to now define $\pi(l,k)$ as the probability that among l realizations with precision q_S at least k are correct, i.e., $\pi(l,k) := \sum_{i=k}^{l} \binom{l}{i} q_S^i (1-q_S)^{k-i}$.

Proposition 3 (ASYM without PA)**.** *Let Assumption BIB hold. Suppose no PA is admitted.*

i. If $\frac{\ell_B}{\ell_S} \geq \frac{N+1}{2}$, then there does not exist an equilibrium in which any shareholder invests in own research. Hence, decision quality in equilibrium is bounded by: $\Pi(\sigma) \leq q_B$. The Pareto-dominant equilibrium is Rubber-stamping and leads to decision quality $\Pi(\sigma^{Rubber}) = q_B$.

ii. If $\frac{\ell_B}{\ell_S} < \frac{N+1}{2}$, then the number of shareholders who invest in own research is at most z_1, with $z_1 := N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$.[22] In the Pareto-efficient equilibrium z_1 shareholders play UNIS

[22] The mathematical expression $\lfloor z \rfloor$ is defined as the largest integer that is lower or equal to z.

and $N - z_1$ shareholders play Rubber-stamping. Hence, decision quality in equilibrium is bounded by: $\Pi(\sigma) \leq q_B \cdot \pi(z_1, z_1 - \frac{N-1}{2}) + (1 - q_B) \cdot \pi(z_1, \frac{N+1}{2})$.

To understand Proposition 3, consider a shareholder who invested in own research. This investment can only be part of an equilibrium if this shareholder conditions on her own signal in some instance in which she is pivotal. In particular, this shareholder must vote *no* if the signal is a (against). This is a best response if pivotality implies that a sufficient number of other informed shareholders also have received information against the board's proposal. This, in turn, is possible in strategy profiles in which several uninformed shareholders rubber-stamp the board's proposal. When the number of shareholders who rubber-stamp is by roughly $\frac{\ell_B}{\ell_S}$ larger than the number of shareholders who vote unconditionally *no* (i.e., play Protest), then there might indeed be incentives to invest in own research and vote according to one's signal. If this difference, however, exceeds half of all shareholders, as considered in part i., then it is impossible to be pivotal in the first place. Otherwise, i.e., in the case addressed in part ii., it is possible to have informed shareholders, but their number is bounded from above by z_1. It turns out that the strategy profile with the highest decision quality is then $\sigma^{\mu,\nu}$, with $\mu = z_1 = N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ shareholders investing in own research and voting according to signal (UNIS), and $\nu = N - z_1 = \lfloor \frac{\ell_B}{\ell_S} \rfloor$ shareholders rubber-stamping. This strategy profile is Pareto-efficient and hence yields the upper bound for the decision quality.

Comparative statics imply that the maximal number z_1 of shareholders who invest is decreasing in the board's relative information quality $\frac{\ell_B}{\ell_S}$, starting with $N - 1$ for $\lfloor \frac{\ell_B}{\ell_S} \rfloor = 1$, continuously decreasing up to $\frac{N+1}{2}$ for $\lfloor \frac{\ell_B}{\ell_S} \rfloor = \frac{N-1}{2}$, and then discontinuously jumping to 0.[23] This validates the insight we gained from the symmetric equilibria. Without PA, well

[23]Only if $\frac{\ell_B}{\ell_S} < 1$, which is precluded by Assumption BIB, all N shareholders could be informed.

informed boards reduce shareholders' research incentives.

4.2 Does Decision Quality Increase With a Proxy Advisor?

Analogously to the analysis of symmetric equilibria in Section 3, the negative result obtained without PA can be mitigated when a PA is admitted. Again, the basic idea is that the PA's recommendation is used as a condition to invest in own research like in information acquisition strategy Subscribe-InvestIFF*against*, which constitutes CAIS. While this was true for all shareholders in Proposition 2, we now consider equilibria in which only some of the shareholders use this strategy.

Proposition 4 (ASYM with PA)**.** *Let Assumptions BIB and PAF hold. Let costs* $c > 0$ *be arbitrarily small and let fee* f *be sufficiently smaller. Suppose there is a PA with* $\frac{\ell_P}{\ell_S} \in (\frac{\ell_B}{\ell_S} - \frac{N+1}{2}, \frac{\ell_B}{\ell_S} + \frac{N+1}{2})$*. Then there exists an equilibrium in which the number of shareholders who invest or conditionally invest in own research is* z_2 $(\geq \frac{N+1}{2})$*, with* $z_2 := N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$*.*

The proposition states that there exists an equilibrium in which more than half of all shareholders invest or conditionally invest in own research. It is based, for instance, on the strategy profile $\hat{\sigma}^{\mu,\nu}$, in which $\mu = z_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play strategy CAIS, and the remaining $\nu = N - z_2$ shareholders play Rubber-stamping.

Let us sketch the proof of Proposition 4 for the case $q_P \leq q_B$. Consider the incentives of a shareholder i who subscribed to the PA and who has invested into an own signal after the PA recommended to refute the board's proposal. She is only pivotal when the other shareholders' votes constitute a tie. This only occurs when there are exactly ν more informed voters with signal a (against board) than with signal b, excluding i. Hence, conditioning on pivotality does provide her with additional information on how many signals were in favor of the board's proposal. In addition she knows her own signal, the PA's and the board's signals, and their respective qualities. For her investment in own research to be worthwhile, she must condition her vote on her signal. If her own signal is b, she must prefer to vote *yes*, i.e., $\nu\ell_S + \ell_P < \ell_B + \ell_S$. If her own signal is a, she must prefer to vote *no*, i.e., $\nu\ell_S + \ell_P + \ell_S > \ell_B$. Otherwise, she could beneficially deviate on the voting stage. These two conditions restrict the number of shareholders who play Rubber-stamping into an open interval around $\frac{\ell_B - \ell_P}{\ell_S}$, namely, $\nu \in (\frac{\ell_B - \ell_P}{\ell_S} - 1, \frac{\ell_B - \ell_P}{\ell_S} + 1)$. Conversely, a shareholder who plays Rubber-stamping must not benefit from deviating, e.g., to CAIS. This is assured if she does not even want to vote *no* when the PA recommends *against* and her own signal (that she would acquire in this deviation strategy) would be a: $(\nu - 1)\ell_S + \ell_P + \ell_S \leq \ell_B$. This condition further restricts the number of shareholders who play Rubber-stamping to $\nu \leq \frac{\ell_B - \ell_P}{\ell_S}$. Setting $\nu = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ satisfies both these constraints and in fact leaves no deviation incentive for any player.

Proposition 4 is illustrated in Figure 3. Suppose first that the PA is approximately equally well informed as the board: $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor = 0$, i.e., parameters are close to the 45-degree line. Then there is an equilibrium in which all $z_2 = N$ shareholders invest or conditionally invest. In fact, this case nests the symmetric equilibrium of Proposition 2, in which all shareholders conditionally invest. Next, let us go beyond this region. The term $\frac{|\ell_B - \ell_P|}{\ell_S}$ measures the difference between the information qualities of the board and the PA, respectively. Consider for instance the parameter setting $\frac{\ell_B}{\ell_S} = 10$ and $\frac{\ell_P}{\ell_S} = 7$. In this setting we have an equilibrium, in which $N - (10 - 7) = N - 3$ shareholders (conditionally) invest in own research and 3 play Rubber-stamping. Moving further away from the 45-degree line, if $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor = \frac{N+1}{2}$, then $z_2 = \frac{N+1}{2}$, i.e., only slightly more than half of the shareholders (conditionally) invest in own research, while almost half play Rubber-stamping.

Comparative statics of the strategy profile $\hat{\sigma}^{\mu,\nu}$ hence suggest that the number of conditionally investing shareholders is decreasing in the difference between information qualities of the PA and the board. The outer boundaries for equilibria with conditionally investing and Rubber-stamping shareholders are given by $\frac{\ell_P}{\ell_S} \in (\frac{\ell_B}{\ell_S} - \frac{N+1}{2}, \frac{\ell_B}{\ell_S} + \frac{N+1}{2})$, which has the following interpretation: The difference between the information quality of the PA and the information quality of the board must not exceed the aggregated information quality of about half of all shareholders together. Observe that the larger the number of shareholders N the less demanding this assumption is. Moreover, note that the number of conditionally investing shareholders in strategy profile $\hat{\sigma}^{\mu,\nu}$, is growing linearly with N, while the number of shareholders who rubber-stamp ν is constant.

Overall, the conditions for the existence of an equilibrium with information acquisition by a majority of shareholders are relaxed (Proposition 4), compared with those for a symmetric equilibrium in which all shareholders acquire information (Proposition 2). In fact, we move from the requirement that the normalized difference in expertise between board and PA equals at most one shareholder in the symmetric case to the corresponding requirement for the asymmetric case that this difference equals at most half of all shareholders, approximately. In sum, the novel type of equilibrium behavior that we find in this paper exists in a broad range of the parameter space.

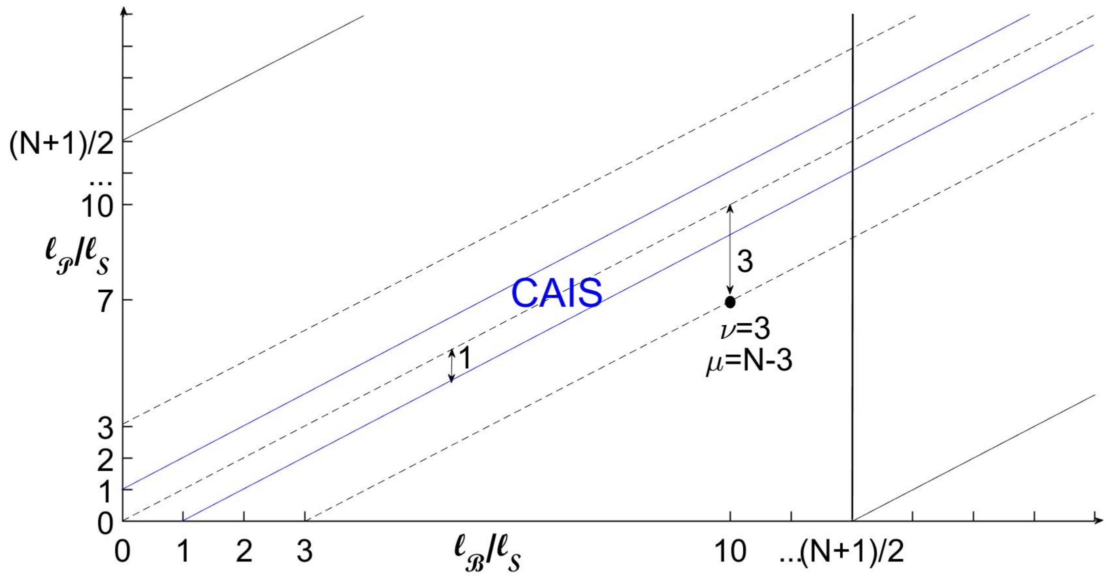

Figure 3: Parameter space. There are equilibria with conditional investment in own research in the whole area between the two outer lines: $\frac{\ell_P}{\ell_S} \in (\frac{\ell_B}{\ell_S} - \frac{N+1}{2}, \frac{\ell_B}{\ell_S} + \frac{N+1}{2})$. For instance, in the parameter setting $\frac{\ell_B}{\ell_S} = 10$ and $\frac{\ell_P}{\ell_S} = 7$ there is an equilibrium $\hat{\sigma}^{\mu,\nu}$, with $\mu = z_2 = N-(10-7) = N-3$ shareholders who conditionally invest, while $\nu = 3$ shareholders rubber-stamp (Proposition 4). In the corridor around the 45-degree line, indicated by blue lines, there is the equilibrium CAIS in which all N shareholders conditionally invest. Beyond the vertical line at $\frac{\ell_P}{\ell_S} = \frac{N+1}{2}$, there is no equilibrium with investment if no PA is admitted.

4.3 Illustration and Discussion

We now turn to assessing the effect of admitting the PA by comparing Proposition 3 with Proposition 4. Suppose first that $\frac{N+1}{2} \leq \frac{\ell_B}{\ell_S} < \frac{\ell_P}{\ell_S} + \frac{N+1}{2}$ holds. Then Proposition 3 part i. and Proposition 4 apply. Hence, the PA mitigates the issue that a comparatively well-informed board stifles shareholders' own research. It increases the number of conditionally investing shareholders from 0 to $z_2 = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ and it strictly increases decision quality above q_B. This is illustrated in Figure 3 in the (upper) right part.

Now, suppose that $\frac{\ell_B}{\ell_S} < \frac{N+1}{2}$, and hence $\frac{\ell_B}{\ell_S} < \frac{\ell_P}{\ell_S} + \frac{N+1}{2}$, holds. Then Proposition 3 part ii. and Proposition 4 apply. We compare the most informative equilibrium without PA $\sigma^{\mu,\nu}$

with the equilibrium with PA $\hat{\sigma}^{\mu,\nu}$. In the former, we have $\mu = z_1 = N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ who play UNIS and $\nu = \lfloor \frac{\ell_B}{\ell_S} \rfloor$ who play Rubber-stamping; in the latter we have $\mu = z_2 = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ who play CAIS and $\nu = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ who rubber-stamp. Hence, the number of shareholders who never invest has been reduced by roughly $\frac{\ell_P}{\ell_S}$, or, specifically, the number of shareholders who invest has increased from $z_1 = N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ who always invest to $z_2 = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ who conditionally invest, i.e., by roughly $\frac{\ell_P}{\ell_S}$. In this comparison the net effect on decision quality is ambiguous, as the following example illustrates. However, the equilibrium with PA that we consider here, $\hat{\sigma}^{\mu,\nu}$, is not necessarily Pareto-efficient.[24] Hence, when this equilibrium has a lower decision quality than the Pareto-dominant equilibrium without PA, then it is still possible that there is another equilibrium with PA that has a higher decision quality than both.

Example 2 (Asymmetric Equilibria). *Let $q_S = 0.6$, $q_B = 0.8$, and $q_P = 0.7$.*[25] *Then, $\ell_B/\ell_S = 3.4$ and $\ell_P/\ell_S = 2.1$, and the case distinction in Proposition 3 has a threshold at $N = 5.8$. Hence, for $N \leq 5$ part i. of Proposition 3 applies, while for $N \geq 6$ part ii. applies. The condition $\frac{\ell_P}{\ell_S} \geq \frac{\ell_B}{\ell_S} - \frac{N+1}{2}$ of Proposition 4 is satisfied. We get $z_2 = N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor = N - \lfloor 3.4 - 2.1 \rfloor = N - 1$. Hence, by Proposition 4 there is an equilibrium with $z_2 = N - 1$ shareholders invest or conditionally invest in an own signal. Without PA the highest decision quality is bounded by q_B as long as $N \leq 5$ and otherwise is determined by $\sigma^{\mu,\nu}$ with $\mu = z_1$ shareholders playing UNIS and $\nu = N - z_1$ Rubber-stamping. With PA the decision quality in a Pareto-efficient equilibrium is at least as high as in the strategy profile $\hat{\sigma}^{\mu,\nu}$, in which*

[24] The characterization of all Pareto-efficient equilibria with PA is work in progress.

[25] Example 2 differs from Example 1 in that the board is better informed: $q_B = 0.8$, instead of 0.75. As a consequence, the symmetric strategy profile CAIS is no longer an equilibrium, as $\ell_P \leq \ell_B - \ell_S$ or $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor \geq 1$.

$\mu = z_2$ shareholders play CAIS and all others Rubber-stamping. Table 3 illustrates some implications of Propositions 3 and 4 for decision quality with and without PA in this example.

Setting	Decision quality	$N = 3$	$N = 5$	$N = 21$	$N = 101$	$N \to \infty$
No PA	$\Pi(\sigma) \leq \Pi(\sigma^{Rubber-stamping}) = q_B$	0.8	0.8	--	--	--
No PA	$\Pi(\sigma) \leq \Pi(\sigma^{\mu,\nu})$	--	--	0.867	0.983	1
With PA	$\Pi(\sigma^*) \geq \Pi(\hat{\sigma}^{\mu,\nu})$	0.812	0.824	0.875	0.932	0.94

Table 3: Decision quality in asymmetric equilibria. Illustration of Propositions 3 and 4 for $q_B = 0.8$, $q_P = 0.7$, and $q_S = 0.6$, i.e., Example 2. The first row corresponds to Rubber-stamping, which is the best equilibrium without PA in part i. of Proposition 3. The second row corresponds to the strategy profile in which $\mu = z_1$ play UNIS and $\nu = N - z_1$ play Rubber-stamping, which is the best equilibrium without PA in part ii. of Proposition 3. The third row corresponds to the strategy profile with PA in which $\mu = z_2$ play CAIS and $\nu = N - z_2$ play Rubber-stamping, which is one equilibrium with PA that is used in Proposition 4.

5 Discussion and Regulatory Implications

This section sheds further light on the key results by considering deviations from the assumptions and discussing the implications. It also uses the model to interpret recent regulatory developments.

5.1 Different Timeline: Proxy Advice Arrives After Investment

What happens if Assumption PAF does not hold? That is, consider the situation when proxy advice arrives after the shareholders' decision to invest in own research. All actions occur as illustrated in the timeline (Figure 1), but proxy advice arrives at the end of period $t = 2$. We consider the cases where Assumption BIB holds and where it does not hold.

If Assumption BIB holds, the Pareto-dominant symmetric equilibrium is Rubber-stamping and hence decision quality is bounded by q_B. Hence, there is no positive effect of having a PA, as decision quality with or without PA is bounded by the quality of the board.

If Assumption BIB does not hold, that is, if the board does not have the best information regarding what is good for the company, we find that UNIS is an equilibrium and Pareto-dominant if and only if $\ell_S \geq \ell_B + \ell_P$. This condition is the same as in our model with early proxy advice (see bottom right corner of Figure 2 lower panel). It is more demanding than the condition in the setting without a PA, which was $\ell_S > \ell_B$. Specifically, the condition $\ell_S \geq \ell_B + \ell_P$ means that a single shareholder has to be better informed, not only than the board, but than both the board and the PA together. The reason is that there is an additional deviation possibility. A shareholder could invest in own signal and buy the vote recommendation (Subscribe-Invest) and then vote *no* only if both signals are against the board.

In sum, introducing a PA whose advice does not arrive sufficiently early does not induce equilibria with higher decision quality, but may even reduce decision quality. The positive effects of proxy advice in our model are hence indeed restricted to having both Assumption BIB and Assumption PAF satisfied.

These insights are important also in light of recent policy developments. In September 2019, the SEC issued guidance for investment advisors, stating that investment advisors satisfy their own fiduciary duties of care and loyalty and obligations to act in their clients' best interests, in part, through careful oversight of proxy advisory firms, such as by monitoring and analyzing the methodology and processes of proxy advisory firms, including their processes for engagement with companies and procedures to address errors. In other words, a simple

Rubber-stamping of proxy advice is seen to violate an investment advisor's fiduciary duties to its clients. The rule implies that indeed investors need to have (and take) enough time to conduct their own research. This is thus in line with the model's prediction that in such a case high decision-quality can arise. However, we also note that in the novel equilibrium behavior that we find (CAIS), for recommendations are rubber-stamped, while only *against* recommendations trigger further research. It remains to be seen whether such partial own research (partial non-rubberstamping) fulfills the fiduciary duties in the eyes of the SEC.

5.2 Regulation of Proxy Advisor Competence

In August 2019, the U.S. Securities and Exchange Commission (SEC) issued guidance on the role of PAs with the intention to enhance their accountability. The guidance includes recommendations on disclosure of the sources of information and methodology used by PAs and information regarding conflicts of interests. Similarly, the EU has also adopted disclosure rules for PAs in the new EU Shareholder Rights Directive (Directive (EU) 2017/828 amending Directive 2007/36/EC).[26]

While the August 2019 SEC rule focuses on disclosure requirements, in July 2020, the US Securities and Exchange Commission (SEC) adopted rules further enhancing pressure on PAs to produce high-quality reports. Specifically, PAs are now required to share rebuttals to their advice from executives. The SEC also defined instances where omitting information in a PA report could constitute fraud, and reaffirmed that it considers PAs to be under SEC

[26]PAs are required to publish a code of conduct which they apply and to report the application of the code (or explain why they do not have a code or deviate from it). Member states shall require PAs to publicly disclose certain information, such as, the main features of a PA's methodology, the main information sources used and the procedures put in place to ensure the quality of research, advice and voting recommendation. Finally, member states must ensure that PAs identify and disclose actual or potential conflicts to their clients. Disclosure itself is limited to the client, i.e., the institutional investors.

regulation governing solicitation.[27]

These regulations tend to increase signal quality for the PA. In our model, this implies the following effects: Better informed PAs up to a certain level may encourage information acquisition by the shareholders and improve decision quality; for even higher information quality of the PA, substantially beyond the board's, the effect is reversed. This holds if proxy advice is early enough for shareholders to condition their research investment on it. If instead proxy advice does not arrive sufficiently early, a competence-increasing regulation of the PA affects decision quality either negatively or not at all.

5.3 One Dominant Shareholder

We have thus far assumed that $N > 1$ and odd which means that we have at least three shareholders. Let us now consider the case of only one shareholder $N = 1$, which applies to any company with a shareholder who holds a decisive majority of shares. We can show that both main results carry over to this case. First, without PA, there is no incentive to invest in research under Assumption BIB, i.e., for $q_S \leq q_B$. Second, the presence of a PA with appropriate information quality improves decision quality, as it leads to a Pareto-dominant equilibrium in which the shareholder conditionally invests in research.

Interestingly, since one single shareholder is always pivotal, the Assumption PAF is not necessary for research investment in that special case. That is, even when the subscription decision and the information acquisition decision are made simultaneously, there is an equilibrium with investment in own research for $N = 1$. In this equilibrium strategy the shareholder subscribes to the vote recommendation and invests in own research (Subscribe-

[27]See Cooley (2020) for a non-technical summary of the background and the most recent ruling.

Invest) and votes *yes* if and only if at least one of the two supports the board's proposal. Hence, for the case of only one shareholder, there is a complementarity between proxy advice and own research, independently of the timing of the two decisions.

6 Conclusion

In this paper we analyze the effects of proxy advisory firms (PAs) on corporate decisions. As a benchmark PAs are not admitted. When the board's proposals are sufficiently informative, then shareholders do not have incentives to conduct their own research and simply rubber-stamp the board's proposals. Hence in the absence of PAs, decision quality is bound by the quality of the board. Introducing a PA whose information level is not too far from the board's alters this result and leads to a higher decision quality. This only holds if the vote recommendation of the PA arrives sufficiently early such that shareholders can respond to *against* recommendations with an own investigation of the issue. Otherwise, PAs may indeed only undermine corporate decision quality.

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Supplementary Appendix

A Proofs

A.1 Proof of Proposition 1

To prove Proposition 1 Lemma 1 is helpful.

Lemma A.1 (SYM without PA: All Equilibria)**.** *Let Assumption BIB hold. Suppose no PA is admitted.*

i. Protest, (i.e., no shareholder invests in research and all shareholders vote no) is a symmetric equilibrium for any $q_B, q_S \in (0.5, 1)$. Its decision quality is $1 - q_B$.

ii. Rubber-stamping (i.e., no shareholder invests in research and all shareholders vote yes) is a symmetric equilibrium for any $q_B, q_S \in (0.5, 1)$. Its decision quality is q_B.

iii. There are no other symmetric equilibria.

Proof. We address each part separately.

i. We have $N \geq 3$ shareholders (because $N > 1$ and odd). When all shareholders vote *no*, a single shareholder is never pivotal. Hence, there is no way to increase decision quality. Deviations can thus only affect costs. Since no information is acquired in this information acquisition strategy (NotSubscribe-NotInvest), costs are minimal. Hence, there is no unilateral improvement.

   Decisions always implement the opposite of the board's proposal. By assumption of the simplified model, the board's proposal corresponds to its signal (B). Hence, the ex ante probability that the true state matches the decision equals the probability that the board's signal does not match the true state, which is $1 - q_B$.

ii. The proof that Rubber-stamping is an equilibrium is fully analogous to part i. of Lemma 1. With Rubber-stamping, the decision quality equals the ex ante probability that the board's signal matches the true state, which is q_B.

iii. There are only two information acquisition strategies. For not investing own signal both strategies are symmetric equilibria (see part i. and ii.). Considering investment in own signal: Since agents pay c they must condition on own signal. Otherwise, they could improve by voting the same and not investing c. Conditioning on signal leaves two pure strategies: vote *yes* if b and *no* if a (i.e., UNIS) or the opposite (vote *yes* if a and *no* if b). If voting *yes* after a (against) was optimal, then voting *no* after a would also be. Hence, agents could improve to unconditionally voting A. Only UNIS remains for NotSubscribe-Invest.

We finally show that UNIS is not an equilibrium under Assumption BIB, i.e., $q_S \leq q_B$.[28] Consider shareholder i deviates to Rubber-stamping. The deviation changes the outcome only if i is pivotal and the own signal is a: Under UNIS i would vote *no*, under Rubber-stamping i would vote *yes*. Pivotality implies that among the $N-1$ other shareholders the signals are split in $\frac{N-1}{2}$ a signals and $\frac{N-1}{2}$ b signals. Conditional on that case, B is more likely to be true than A (such that Rubber-stamping weakly improves decision quality) if and only if

$$\begin{aligned}
q_B(1-q_S)\binom{N-1}{\frac{N-1}{2}}q_S^{\frac{N-1}{2}}(1-q_S)^{\frac{N-1}{2}} &\geq (1-q_B)q_S\binom{N-1}{\frac{N-1}{2}}(1-q_S)^{\frac{N-1}{2}}q_S^{\frac{N-1}{2}} \\
q_B(1-q_S) &\geq (1-q_B)q_S \\
\frac{q_B}{1-q_B} &\geq \frac{q_S}{1-q_S} \\
\ell_B &\geq \ell_S.
\end{aligned}$$

Hence, Rubber-stamping weakly improves decision quality for $q_S \leq q_B$, which is Assumption BIB. Moreover, Rubber-stamping saves costs c. Therefore, it strictly improves utility of the deviating shareholder i.

□

Now, we use Lemma A.1 to prove Proposition 1. Under Assumption BIB there are only two equilibria. Equilibrium Rubber-stamping leads to the same costs as the Protest equilibrium. Rubber-stamping Pareto-dominates because it leads to higher decision quality $\Pi(\sigma^{Rubber}) = q_B > 0.5 > 1 - q_B = \Pi(\sigma^{Protest})$.

A.2 Proof of Proposition 2

To prove Proposition 2 Lemma 2 is helpful.

Lemma A.2 (SYM with PA: All Equilibria)**.** *Let Assumptions BIB and PAF hold. Let costs c be arbitrarily small and let fee f be sufficiently smaller.*

i. Protest (i.e., no shareholder invests in research and all shareholders vote no) is a symmetric equilibrium for any $\ell_B, \ell_S \in (0, \infty)$. Its decision quality is $1 - q_B$.

ii. Rubber-stamping (i.e., no shareholder invests in research and all shareholders vote yes) is a symmetric equilibrium for any $\ell_B, \ell_S \in (0, \infty)$. Its decision quality is q_B.

iii. CAIS is a symmetric equilibrium if and only if $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$. Its decision quality is: $\Pi(\sigma^{CAIS}) = q_B q_P + [(1-q_B)q_P + q_B(1-q_P)]\pi(N)$.

iv. CAIS-2 is a symmetric equilibrium if and only if $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$. Its decision quality is: $\Pi(\sigma^{CAIS-2}) = (1-q_B)(1-q_P) + [(1-q_B)q_P + q_B(1-q_P)]\pi(N)$.

[28] In fact, UNIS is a symmetric equilibrium if and only if $q_S > q_B$.

v. There are no other symmetric equilibria. In particular, there is no equilibrium in which all shareholders subscribe to proxy advice and unconditionally invest in own signal (Subscribe-Invest).

Proof. We address each part of Lemma A.2 separately.

i. The proof is identical to the proof of Lemma A.1, part i.

ii. The proof is identical to the proof of Lemma A.1, part ii.

iii. CAIS is illustrated in Table A.1.

		Own Signal	
		b (for board)	a (against)
PA	*for*	yes	
	against	yes	no

Table A.1: CAIS: Invest in research iff vote recommendation is *against*; after *for* recommendation vote *yes*, after *against* recommendation vote *yes* iff b.

We show that CAIS is an equilibrium if and only if $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$.

Suppose first that $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$, i.e., either $\ell_P \leq \ell_B - \ell_S$ or $\ell_P \geq \ell_B + \ell_S$. We show that CAIS cannot be an equilibrium. In CAIS pivotality implies that the vote recommendation is *against* and that among the $N-1$ other shareholders the signals are split in $\frac{N-1}{2}$ a signals and $\frac{N-1}{2}$ b signals. (Indeed, after recommendation *for* no shareholder is pivotal.)

Let $\ell_P \leq \ell_B - \ell_S$. Consider a shareholder i who deviates to Rubber-stamping. This deviation alters the decision in comparison to CAIS if the vote recommendation is *against*, all other shareholder's signals are split, and i's signal is a: In CAIS, i would vote *no*, in the deviation i would vote *yes*. This deviation weakly improves decision quality if $\ell_B \geq \ell_P + \ell_S$, which holds by assumption. Since, the deviation saves costs c, it increases i's expected utility.

Let $\ell_P \geq \ell_B + \ell_S$. Consider a shareholder i who deviates to voting *no* without information acquisition (Protest). This deviation alters the decision in comparison to CAIS if the vote recommendation is *against*, all other shareholder's signals are split, and i's signal is b: In CAIS, i would vote *yes*, in the deviation i would vote *no*. This deviation weakly improves decision quality if $\ell_P \geq \ell_B + \ell_S$, which holds by assumption. Since, the deviation saves costs c, it increases i's expected utility.

Hence, if $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$, CAIS is not an equilibrium.

Now, suppose that $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$. In order to show that CAIS is an equilibrium, we show that there is no individual deviation that improves utility. We use that if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information acquisition strategy. Pivotality always implies that the vote recommendation is *against* and that among the $N-1$ other shareholders the signals are split in $\frac{N-1}{2}$ a signals and $\frac{N-1}{2}$ b signals.

(1) NotSubscribe-NotInvest. Rubber-stamping is not an improvement for low enough costs if $\ell_S + \ell_P > \ell_B$. Deviating to Rubber-stamping only changes the outcome if the PA has recommended *against*, i has received signal a (against), and all other shareholder's signals are split. It would weakly improve decision quality iff

$$\begin{aligned} q_B(1-q_P)(1-q_S)\binom{N-1}{\frac{N-1}{2}}q_S^{\frac{N-1}{2}}(1-q_S)^{\frac{N-1}{2}} &\geq (1-q_B)q_Pq_S\binom{N-1}{\frac{N-1}{2}}(1-q_S)^{\frac{N-1}{2}}q_S^{\frac{N-1}{2}} \\ q_B(1-q_P)(1-q_S) &\geq (1-q_B)q_Pq_S \\ \frac{q_B}{1-q_B} &\geq \frac{q_P}{1-q_P}+\frac{q_S}{1-q_S} \\ \ell_B &\geq \ell_P+\ell_S. \end{aligned}$$

By assumption $\ell_P > \ell_B - \ell_S$, Rubber-stamping strictly decreases decision quality. It does save costs f always and c with probability $q_B(1-q_P)+(1-q_B)q_P$. For low enough costs f and c, Rubber-stamping does not increase utility because of its lower decision quality.

Deviation to vote *no* without information acquisition (Protest) is not an improvement for low enough costs if $\ell_P < \ell_B + \ell_S$.

(2) NotSubscribe-Invest. Deviation to UNIS does not change the outcome. Indeed, after *for* recommendation i is not pivotal, after *against* recommendation i votes under UNIS as she does under CAIS. Hence, UNIS is an improvement only if it has lower costs. It is not if $f \leq c[q_Bq_P + (1-q_B)(1-q_P)]$, which is satisfied if f is sufficiently lower than c.

(3) Subscribe-NotInvest. Deviation to buy recommendation and follow it. Not an improvement if $\ell_P < \ell_B + \ell_S$ for low enough c.

(4) Subscribe-Invest. Deviation to buy both recommendation and signal. Case 1, illustrated in Table A.3, is outcome equivalent, but more costly. Case 2, illustrated in Table A.4, is not an improvement if $\ell_P < \ell_B + \ell_S$.

(5) Subscribe-InvestIFF*for*. Deviation to buy recommendation and invest iff recommendation is *for*. The case illustrated in Table A.5, is not an improvement if $\ell_P < \ell_B+\ell_S$. The alternative case, which differs by voting *yes* after *against* recommendation, is not an improvement if $\ell_S + \ell_P \geq \ell_B$.

(6) Subscribe-InvestIFF*against*. Deviation to same information acquisition strategy, but different voting strategy. Most attractive deviation votes *no* after *for* recommendation. This is outcome equivalent and equally costly. Not an improvement.

Hence, under the conditions assumed in part iii. of the Lemma CAIS is an equilibrium.

Finally, concerning decision quality, notice that if board and PA receive the same signal, this signal determines the decision, and if they receive a different signal, the signal that is received by a majority of shareholders determines the decision. Therefore, decision quality in CAIS is $(q_B q_P) * 1 + (1 - q_B)(1 - q_P) * 0 + [(1 - q_B)q_P + q_B(1 - q_P)]\pi(N) * 1$, as $q_B q_P$ is the probability that the board and the PA both receive the same and correct signal, and $[(1 - q_B)q_P + q_B(1 - q_P)]$ is the probability that the two receive a signal that is different from each other.

iv. CAIS-2 is illustrated in Table A.2.

		Own Signal	
		b (for board)	a (against)
PA	*for*	no	
	against	yes	no

Table A.2: CAIS-2: Invest in research iff vote recommendation is *against*; after *for* recommendation vote *no*, after *against* recommendation vote *yes* iff b.

The proof that CAIS-2 is an equilibrium if and only if $\ell_P \in (\ell_B - \ell_S, \ell_B + \ell_S)$ is identical to the proof that CAIS is an equilibrium under these conditions (cf. Proof of Lemma A.2, part iii.).[29]

Concerning decision quality, notice that if board and proxy advisor receive the same signal, the decision is opposite of this signal, and if they receive a different signal, the signal that is received by a majority of shareholders determines the decision. Therefore, decision quality in CAIS-2 is $(q_B q_P) * 0 + (1 - q_B)(1 - q_P) * 1 + [(1 - q_B)q_P + q_B(1 - q_P)]\pi(N) * 1$,

v. To show that there are no additional equilibria, we exhaustively discuss all pure strategies. Again, we organize the discussion by information acquisition strategy.

(1) NotSubscribe-NotInvest: not subscribe and not invest.

There are only voting strategies *yes* or *no*. Both lead to equilibria as shown in parts i and ii.

(2) NotSubscribe-Invest: not subscribe and invest.

Since agents pay c they must condition on own signal. Otherwise, they could improve by voting the same and not investing c. Conditioning on signal leaves two pure strategies: vote *yes* if b and *no* if a (i.e., UNIS) or the opposite (vote *yes* if a and *no* if b). If voting *yes* after a (against) was optimal, then voting *no* after a would also be. Hence, agents could improve to unconditionally voting A. Only UNIS remains. Under Assumption BIB, Rubber-stamping is an improving deviation from UNIS, as shown in Proof of Lemma A.1, part iii.

(3) Subscribe-NotInvest: subscribe and not invest (independent of recommendation).

Since agents pay f they must condition on recommendation. For instance, vote *yes* after *for* and *no* after *against*. Or the opposite. In either case, no shareholder is pivotal since all vote *yes* or *no* given one recommendation.

A shareholder can improve by not paying f and voting e.g., *yes*. Hence, there is no symmetric equilibrium with this information acquisition strategy.

[29] This is not surprising, as both strategies have the same information acquisition strategy, Subscribe-InvestIFF*against*, and they only differ in a voting action, where no player is pivotal.

(4) Subscribe-Invest: subscribe and invest (independent of recommendation).

Since shareholders pay both f and c they must condition their voting strategy on both vote recommendation and own signal. Otherwise, they could improve with the same voting behavior, but saving costs. This means that in fact only two voting strategies remain.

Case 1: vote *yes* except if both recommendation is *against* and signal is a, as in Table A.3. In this case no shareholder is pivotal if PA recommends for (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if recommendation is *against*. If so, i would vote according to signal. Hence, deviating to UNIS would not change the outcome because either i is not pivotal or i would also vote the signal. UNIS however saves fee f. Thus, strategy profile of case 1, illustrated in Table A.3, cannot be a symmetric equilibrium.

		Own Signal	
		b (for board)	a (against)
PA	for	yes	yes
	against	yes	no

Table A.3: A strategy based on acquiring both proxy advice and own signal, case 1: Subscribe-Invest and vote *yes*, except if PA's recommendation is *against* and the own signal is a.

Case 2: vote *no* except if both recommendation is for and signal is b (for board), as in Table A.4. The analogous argument as above for case 1 applies, as follows: In this case no shareholder is pivotal if PA recommends *against* (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if recommendation is for. If so, i would vote according to signal. Hence, deviating to UNIS would not change the outcome because either i is not pivotal or i would also vote the signal. UNIS however saves fee f. Thus, strategy profile of case 2 cannot be a symmetric equilibrium.

		Own Signal	
		b (for board)	a (against)
PA	for	yes	no
	against	no	no

Table A.4: A strategy based on acquiring both proxy advice and own signal, case 2: Subscribe-Invest and vote *no*, except if PA's recommendation is for and the own signal is b.

Therefore, there cannot be a symmetric equilibrium with this information acquisition strategy (Subscribe-Invest), in which shareholders unconditionally buy both PA's recommendation and own signal.

(5) Subscribe-InvestIFFfor: Subscribe and invest iff recommendation is for.

Since shareholders pay f and sometimes c they must condition their voting strategy on the recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation for shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated and not conditioning as well. This leaves two cases, which we address as Candidate 5a and Candidate 5b. We show that none of them is an equilibrium under Assumption 1.[30] Consider first Candidate 5a: shareholders vote *yes* except if vote recommendation is for and own signal is a (against) as in Table A.5.

		Own Signal	
		b (for board)	a (against)
PA	for	yes	no
	against	yes	

Table A.5: Candidate 5a. A strategy based on acquiring own signal iff the recommendation is for: Subscribe-InvestIFFfor and vote *yes*, except if PA's recommendation is for and the own signal is b.

Consider shareholder i who deviates to Rubber-stamping. This deviation only alters the outcome when the vote recommendation is for, all other shareholders' signals are split, and i's signal is a (against): Under Candidate 5a, i would vote *no*, but under Rubber-stamping she votes *yes*. Decision quality improves by this deviation if $\ell_B + \ell_P > \ell_S$. This condition is satisfied by Assumption 1. Moreover, costs of Rubber-stamping are lower than of Candidate 5a. Hence, Candidate 5a cannot be an equilibrium.

Now, consider Candidate 5b shareholders vote *no* except if vote recommendation is for and own signal is b (for board). Again, no shareholder is pivotal after recommendation *against*. Hence, Rubber-stamping is an improvement, identical to the case of Candidate 5a above.

(6) Subscribe-InvestIFF*against*: Subscribe and invest iff recommendation is *against*.

Since shareholders pay f and sometimes c they must condition their voting strategy on the recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation *against* shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated and not conditioning as well. This leaves two cases: CAIS and CAIS-2, which we have addressed. Hence, there are no further equilibria.

□

Now, we can turn to the proof of Proposition 2.

i. To show part i. of Proposition 2, we use Lemma A.2, which shows that besides CAIS there are three further equilibria in this paremeter space: Rubber-stamping, Protest, and CAIS-2. It remains to show that CAIS Pareto-dominates in this area.

[30] In fact, each of these strategy profiles is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$.

First, CAIS has the same costs as CAIS-2 and decision qualities are: $\Pi(\sigma^{CAIS}) = q_B q_P + [(1-q_B)q_P + q_B(1-q_P)]\pi(N)$. $\Pi(\sigma^{CAIS-2}) = (1-q_B)(1-q_P) + [(1-q_B)q_P + q_B(1-q_P)]\pi(N)$. CAIS has higher decision quality iff $q_B q_P > (1-q_B)(1-q_P)$, which always holds as $q_B, q_P > 0.5$. Hence, CAIS Pareto-dominates CAIS-2.

Second, decision quality of Rubber-stamping is q_B and of Protest is $1 - q_B < q_B$. CAIS has strictly higher decision quality than both iff

$$
\begin{aligned}
\Pi(\sigma^{CAIS}) &> q_B \\
q_B q_P + [q_B(1-q_P) + (1-q_B)q_P]\pi(N) &> q_B \\
q_B(1-q_P)\pi(N) + (1-q_B)q_P\pi(N) &> q_B(1-q_P) \\
(1-q_B)q_P\pi(N) &> q_B(1-q_P)[1-\pi(N)] \\
\frac{\pi(N)}{1-\pi(N)} \cdot \frac{q_P}{1-q_p} &> \frac{q_B}{1-q_B} \\
\log\left(\frac{\pi(N)}{1-\pi(N)}\right) + \log(\frac{q_P}{1-q_p}) &> \log(\frac{q_B}{1-q_B}) \\
\ell_N + \ell_P &> \ell_B. \qquad \text{(A.1)}
\end{aligned}
$$

Since $\ell_N > \ell_S$ and by assumption $\ell_P > \ell_B - \ell_S$, we have $\ell_N + \ell_P > \ell_S + \ell_P > \ell_B$. Hence, CAIS leads to strictly higher decision quality than both Rubber-stamping and Protest. It induces higher costs f and c. Thus, for low enough costs, CAIS Pareto-dominates them.

ii. To show part ii. of Proposition 2, we use again Lemma A.2. Under Assumption BIB and for $\ell_P \notin (\ell_B - \ell_S, \ell_B + \ell_S)$, only two equilibria remain: Rubber-stamping and Protest. Rubber-stamping Pareto-dominates because it leads to higher decision quality $\Pi(\sigma^{Rubber}) = q_B > 0.5 > 1 - q_B = \Pi(\sigma^{Protest})$, while it induces the same costs.

A.3 Proof of Proposition 3

Conditions to Invest in Own Signal. Consider all pure strategies. First, an agent that invests into the own signal must condition his voting behavior on the signal and be pivotal in at least one draw of nature. Otherwise, i.e., voting unconditionally of the signal or never being pivotal, there is an improvement by keeping the voting strategy and not investing into the signal. A shareholder that invests and conditions on the signal can either vote in line with his signal (i.e., vote *yes* if b (for board) and *no* if a (against board)), which we call UNIS; or do the opposite (*yes* iff a). The opposite (*yes* iff a) cannot be part of an equilibrium strategy. Indeed, if voting *no* after receiving signal b (for board) is a best response, then state A is more likely than B. Since the information technology of signals is monotonic, receiving signal a (against Board) makes true state A even more likely such that this voter also prefers to vote *no* if the signal is a (against). Hence, the opposite voting strategy can be ruled out and only UNIS remains for those who buy the signal.

Those who do not buy the signal have the following pure strategies: vote for Board *yes* (Rubber-stamping) and vote *no* (Protest).

For a given strategy profile σ and a realization of signals and a shareholder i let us define two numbers ν and δ_{-i}. For the voters who have not invested, let ν be the number of unconditional *yes* votes minus the unconditional *no* votes; for the voters who have invested, let δ_{-i} be the number of a (against) signals received minus the number of b (for board) signals received when excluding the focal shareholder i.

Suppose shareholder i invested in research and is pivotal. Pivotality implies that the number of total votes of others, $N-1$, is fifty-fifty split in *yes* and *no* votes. This implies, that $\nu = \delta_{-i}$.

The two conditions for voting according to the own signal to be optimal are: vote *yes* when signal is b, which requires $\ell_B + \ell_S \geq \delta_{-i}\ell_S$; and vote *no* after signal a, which requires $\delta_{-i}\ell_S + 1\ell_S \geq \ell_B$. This yields $\nu = \delta_{-i} \in (\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1)$. In fact, the interval is open. Suppose to the contrary, that $\nu = \frac{\ell_B}{\ell_S} - 1$. Then $\delta_{-i} = \nu$ implies $\delta_{-i}\ell_S + 1\ell_S = \ell_B$, i.e., an informed shareholder i is indifferent between voting *yes* and voting *no* after receiving signal a. After receiving signal b this shareholder prefers to vote *yes*. Hence, if this shareholder would vote unconditionally *yes*, she would induce the same decision quality. Therefore, this shareholder could unilaterally improve (upon her strategy UNIS) by Rubber-stamping, i.e., not invest in own signal and vote *yes* unconditionally, which saves costs $c > 0$. Analogously, it would be beneficial to switch to unconditionally voting *no* in case of $\nu = \frac{\ell_B}{\ell_S} + 1$.

Part i. For part i. of the proposition, we use that in particular we have $\nu = \delta_{-i} > \frac{\ell_B}{\ell_S} - 1$, while $\frac{\ell_B}{\ell_S} \geq \frac{N+1}{2}$. Thus, $\nu > \frac{N-1}{2}$ and hence $\nu \geq \frac{N+1}{2}$. Hence, the assumption that a shareholder invests in own research leads to the implication that at least $\nu \geq \frac{N+1}{2}$ more shareholders unconditionally vote *yes* than unconditionally vote *no*. The latter, however, implies that no voter is ever pivotal (since there is always a majority voting *yes*). This, in turn, contradicts the assumption that a shareholder invests in research. Thus, there cannot be an informed voter in equilibrium.

Part ii. In order to prove the upper bound for the number of informed shareholders, we use again that in particular we have $\nu > \frac{\ell_B}{\ell_S} - 1$. This implies $\nu \geq \lfloor \frac{\ell_B}{\ell_S} \rfloor$, as ν is a natural number. To create a vote difference of ν among those who did not invest, it takes at least ν voters, the two possibilities are ν vote *yes* while zero vote *no*, and zero vote *yes* while ν vote *no*. Hence, the number of informed voters is at most $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor = z_1$.

Equilibria. We first prove that $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ players voting always *yes* and $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ players voting according to their private signals is indeed an equilibrium. To this end, we show that neither of these two groups have an incentive to deviate from their strategy.

- A shareholder who invests in own signal: she has no incentive to deviate from investing in own information as she is sometimes pivotal (due to $\frac{\ell_B}{\ell_S}$ being small enough) and when being pivotal has no incentive to deviate (due to $\lfloor \frac{\ell_B}{\ell_S} \rfloor \in (\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1)$, see above.)

- A shareholder who always votes *yes*: even if she knew her signal in case of being pivotal, she would not want to vote *no*, as she is pivotal if and only if there are $\frac{N-1}{2} - (\lfloor \frac{\ell_B}{\ell_S} \rfloor - 1)$ players who obtained positive signals and $\frac{N-1}{2}$ players with negative ones. A negative signal would make her prefer to vote *no* if and only if $\ell_B + (\frac{N+1}{2} - \lfloor \frac{\ell_B}{\ell_S} \rfloor) \cdot \ell_S < \frac{N-1}{2} \cdot \ell_S + \ell_S$, which is equivalent to $\ell_B - \lfloor \frac{\ell_B}{\ell_S} \rfloor \cdot \ell_S < 0$ and thus to $\frac{\ell_B}{\ell_S} < \lfloor \frac{\ell_B}{\ell_S} \rfloor$, which cannot happen.

We will now prove that the equilibrium is Pareto-efficient. To this end, we first prove that for any equilibrium in which a positive number of players invests in own research, the difference ν between players always voting *yes* and those always voting *no* must equal $\lfloor \frac{\ell_B}{\ell_S} \rfloor$. Above, we have already derived that $\nu \in (\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1)$, which can only happen for $\nu = \lfloor \frac{\ell_B}{\ell_S} \rfloor$ or $\nu = \lfloor \frac{\ell_B}{\ell_S} \rfloor + 1$. In a fixed situation, let δ denote the difference between the number of informed shareholders voting *no* and those voting *yes*. A shareholder always voting *yes* will then be pivotal if and only if $\nu - 1 = \delta$ or, equivalently, $\nu = \delta + 1$. Now, let us assume that this shareholder still had the opportunity for own research and obtained a signal which happens to contradict the board's proposal. This shareholder would like to vote *no* if $\ell_B < \delta \ell_S + \ell_S$, which is equivalent to $\ell_B < \nu \ell_S$ or $\frac{\ell_B}{\ell_S} < \nu$. Thus, whenever $\nu > \frac{\ell_B}{\ell_S}$, we cannot have a rubber-stamping player in equilibrium because such a palyer would benefit from deviating to UNIS. As indeed $\lfloor \frac{\ell_B}{\ell_S} \rfloor + 1 > \frac{\ell_B}{\ell_S}$, $\nu = \lfloor \frac{\ell_B}{\ell_S} \rfloor + 1$ cannot result in an equilibrium. Thus, we overall must have $\nu = \frac{\ell_B}{\ell_S}$ in any equilibrium in which some shareholders invest in own research.

Overall, we now have established that apart from trivial equilibria in which no shareholder invests in private research, there can only be equilibria in which the difference between those always voting *yes* and those always voting *no* is exactly equal to $\lfloor \frac{\ell_B}{\ell_S} \rfloor$. Possible non-trivial equilibria are thus characterized by $\lfloor \frac{\ell_B}{\ell_S} \rfloor + \alpha$ shareholders always voting *yes* and α shareholders always voting *no*, with α being a non-negative integer. Regardless of α, the informed shareholders must overcome a vote deficit of $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ votes when trying to enforce a *no* decision. Moreover, observe that the decision quality is increasing in the number of informed shareholders (given that the informationally efficient strategy profile is played), thus $\alpha = 0$ delivers Pareto-efficiency.

Decision quality. Thus, the maximum decision quality is $\Pi(\sigma^{\mu,\nu}) = q_B \cdot \pi(N - \nu, \frac{N+1}{2} - \nu) + (1 - q_B) \cdot \pi(N - \nu, \frac{N+1}{2})$ for $\nu = \lfloor \frac{\ell_B}{\ell_S} \rfloor$. By $\nu = N - \mu$, that is $\Pi(\sigma^{\mu,\nu}) = q_B \cdot \pi(\mu, \mu - \frac{N-1}{2}) + (1 - q_B) \cdot \pi(\mu, \frac{N+1}{2})$ for $\mu = z_1$.

A.4 Proof of Proposition 4

Suppose there is a PA with $\frac{\ell_P}{\ell_S} \in (\frac{\ell_B}{\ell_S} - \frac{N+1}{2}, \frac{\ell_B}{\ell_S} + \frac{N+1}{2})$. We distinguish two cases: $q_P \leq q_B$ and $q_P > q_B$. In both cases we show that there is an equilibrium in which $\mu = z_2$ shareholders play CAIS and all other shareholders play Rubber-stamping in the former case, respectively Protest in the latter case.

First case: $q_P \leq q_B$. Suppose first that $q_P \leq q_B$. Then $\frac{\ell_P}{\ell_S} \in (\frac{\ell_B}{\ell_S} - \frac{N+1}{2}, \frac{\ell_B}{\ell_S}]$. Consider the strategy profile $\hat{\sigma}^{\mu,\nu}$ in which $\mu = z_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play strategy CAIS (i.e., invest after recommendation *against*, vote *yes* except if invested in own research and signal is a (against)); and the remaining $\nu = N - z_2$ shareholders play Rubber-stamping.

The decision quality in this profile is $\Pi(\sigma) = q_B q_P + q_B(1 - q_P)\pi(z_2, z_2 - \frac{N-1}{2}) + (1 - q_B) q_P \pi(z_2, \frac{N+1}{2}) > q_B$.

We have to show that no player has a deviation incentive. We begin with a shareholder who plays Rubber-stamping, then turn to a shareholder who plays CAIS.

A shareholder j who plays Rubber-stamping does not acquire any information and votes *yes*. In the strategy profile $\hat{\sigma}^{\mu,\nu}$, a Rubber-stamping player is pivotal iff the PA has recommended *against* and among the shareholders who play CAIS $\nu - 1$ more have received signal a than have received b.

In order to improve decision quality, there must be a difference in the vote, i.e., j must vote *no* in some instance of pivotality. The most attractive instance to vote *no* is the following: when the PA recommends *against* and the own signal is a (against). Suppose shareholder j would then vote *no*. This is a strict improvement of decision quality iff $(\nu - 1)\ell_S + \ell_P + \ell_S > \ell_B$, or equivalently iff $\nu > \frac{\ell_B - \ell_P}{\ell_S}$. However, setting $\nu = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ precludes this. Hence, a Rubber-stamping shareholder cannot improve decision quality by deviating when $\mu = z_2$. Considering that any deviation on the information acquisition stages moreover means that more costs are incurred, there is no beneficial deviation for any Rubber-stamping shareholder.

Let us now turn to shareholders who play CAIS. First, we show that there is no deviation which improves decision quality. We then proceed by showing that all deviations with identical decision quality come at the same or higher costs. Together, these two assertions then prove that no individual deviation can improve utility.

Concerning decision quality, it is obvious that improvements are impossible when the proxy advisor's recommendation is for, as in this case all shareholders vote *yes* and no shareholder is ever pivotal. Thus, an alternative strategy may only improve decision quality by changing the outcome when the PA recommends *against* and the shareholder is pivotal. Pivotality is possible when the number of Rubber-stamping shareholders ν is below $\frac{N+1}{2}$. As $\nu = \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$, this is assured by the condition $\frac{\ell_P}{\ell_S} \in (\frac{\ell_B}{\ell_S} - \frac{N+1}{2}, \frac{\ell_B}{\ell_S} + \frac{N+1}{2})$. Pivotality implies that among the $\mu - 1$ other shareholders who play CAIS, the difference between the numbers of a and b signals must equal ν. Those signals are thus split into $\frac{N-1}{2}$ a-signals and $\frac{N-1}{2} - \nu$ b-signals. Shareholder i may improve by always voting *yes* in these instances if and only if $l_B + (\frac{N-1}{2} - \nu) \cdot \ell_S > l_P + \frac{N-1}{2}\ell_S + \ell_S$, which is equivalent to $\ell_B - \ell_P > (\nu + 1)\ell_S$ or $\frac{\ell_B - \ell_P}{\ell_S} > \nu + 1$, and thus to $\nu < \frac{\ell_B - \ell_P}{\ell_S} - 1$. This, however, is precluded is by $\nu = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$. Similarly, shareholder i may improve by always voting *no* in these instances if and only if $l_B + (\frac{N-1}{2} - \nu) \cdot \ell_S + \ell_S < l_P + \frac{N-1}{2}\ell_S$, which turns out to be equivalent to $\nu > \frac{\ell_B - \ell_P}{\ell_S} + 1$, which is again at odds with $\nu = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$. Thus, it is impossible to improve decision quality by a deviating strategy. Even more, inspecting the above considerations shows that any strategy that attains the same decision quality as $\hat{\sigma}^{\mu,\nu}$ must be characterized by conditionally informed shareholders voting with their signals when the proxy advisor recommends *no* (to this end, simply replace the strong inequalities above by their weak counterparts).

The only possibility remaining in order to improve utility is thus to look for strategies that attain the same decision quality as $\hat{\sigma}^{\mu,\nu}$, but at lower costs. As the costs associated with $\hat{\sigma}^{\mu,\nu}$ are the fee f as well as costs c in case of the proxy advisor recommending *against*, there are two possibilities for reducing costs: the first one would be to get rid of conditional costs c, which however is infeasible as it is then no longer possible to vote according to private research when the proxy advisor recommends *against*, resulting in reduced decision quality, as explained already above. The second alternative is to get rid of the fee f, by always voting according to individual research. While this preserves the optimal decision quality and saves fee f, it comes with additional costs c when the proxy advisor recommends for. Thus, in expectation, costs decrease by f, but increase by c times the probability of a for recommendation (which is $q_B q_P + (1 - q_B)(1 - q_P)$. As we have assumed that f is sufficiently smaller than c, this is an overall increase of costs.

Taken together, we thus have shown that there is no utility improving alternative strategy for the conditionally investing players; after we had shown that there is no improving deviation for the Rubber-stamping players. Hence, for $q_P \leq q_B$, there is an equilibrium in which $\mu = z_2$ shareholders play CAIS and $\nu = N - z_2$ shareholders play Rubber-stamping.

Second case: $q_P > q_B$**.** Suppose now that $q_P > q_B$. Then $\frac{\ell_P}{\ell_S} \in (\frac{\ell_B}{\ell_S}, \frac{\ell_B}{\ell_S} \frac{\ell_B}{\ell_S} + \frac{N-1}{2})$. Consider the strategy profile $\hat{\sigma}^{\mu,\nu}$ in which $\mu = z_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play strategy CAIS (i.e., invest after recommendation *against*, vote *yes* except if invested in own research and signal is a (against)); and the remaining $\nu = N - z_2$ shareholders play Protest.

Analogous to the first case, there is no deviation incentive.