Supplementary Online Material (SOM) for "When do proxy advisors improve corporate decisions?"

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1 All Symmetric Equilibria

In the main text, the equilibrium analysis was restricted to the Assumptions BIB and PAF, i.e., that the board is better informed than a single shareholder and that proxy advice arrives before shareholders decide on own research. In this section we characterize all symmetric equilibria with and without these two assumptions. An overview of all symmetric equilibria is provided in SOM Table 1.1.

Information-acquisition strategy	No PA Benchmark	With early PA Under PAF	With late PA Violating PAF
NotSubscribe-NotInvest	Rubber*, Protest	Rubber*, Protest	Rubber*, Protest
NotSubscribe-Invest	(UNIS*)	(UNIS*)	(UNIS*)
Subscribe-NotInvest		--	--
Subscribe-Invest		--	--
Subscribe-InvestIFF*for*		(Cand. 5a), (Cand. 5b)	
Subscribe-InvestIFF*against*		CAIS*, CAIS-2	

Table 1.1: All symmetric equilibria arranged by information-acquisition strategy. Equilibria in brackets are precluded by Assumption BIB. Equilibria marked by "*" are Pareto-efficient in some area of the parameter space. The "--" indicates that there are symmetric strategy profiles with this information-acquisition strategy, but none of them is an equilibrium; in contrast to the empty cells which indicate that these information-acquisition strategies cannot be played due to the setting.

When there is no PA there are three symmetric equilibria. UNIS, in which all shareholders invest in research, is restricted to an area of the parameter space where Assumption BIB is violated. Shareholders who do not invest in research can play Rubber-stamping or do the opposite: vote *no* unconditionally, which we call Protest. Both these symmetric strategy profiles are trivial equilibria, as no shareholder is ever pivotal and they incur no costs. Protest induces a decision quality $\Pi(\sigma) = 1 - q_B$ because it leads to the correct decision whenever the board's proposal is wrong. Clearly, it is Pareto-dominated by Rubber-stamping, as $q_B > 0.5 > 1 - q_B$ and both induce the same costs (none).

These three equilibria also exist when there is a PA (last two columns of SOM Table 1.1) and their discussion is analogous. With information-acquisition strategy Subscribe-InvestIFFfor, there are two additional symmetric equilibria,labelled Cand. 5a and Cand. 5b. However, they are both precluded by Assumption BIB and moreover Pareto-dominated (in fact by UNIS). Moreover, there are two additional equilibria, based on information-acquisition strategy Subscribe-InvestIFF$against$. One of them is CAIS. The other equilibrium, CAIS-2, only differs from CAIS in the voting behavior when the vote recommendation is for. In CAIS shareholders vote *yes*, while shareholders in CAIS-2 vote *no*, i.e., they do not approve the board's proposal when the PA recommends to. CAIS-2 is Pareto-dominated by CAIS since it induces the same costs, but a lower decision quality than CAIS.

More striking than the additional equilibria which are Pareto-dominated is the observation that there are no equilibria with information-acquisition strategies Subscribe-NotInvest and Subscribe-Invest (independent of Assumptions BIB and PAF). To see why not, note that buying only the vote recommendation, i.e., information-acquisition strategy Subscribe-NotInvest, is only worthwhile when using this information in an instance of pivotality. However, if all shareholders symmetrically use the vote recommendation, then no shareholder is ever pivotal. Similarly, acquiring both signals, i.e., information-acquisition strategy Subscribe-Invest, is worthwhile only if shareholders condition their vote on both PA advice and own signal such that none is superfluous, e.g., by voting *yes* if and only if one of the latter is in favor of the proposal. When all shareholders adopt this strategy, pivotality already implies that the recommendation was *against*. Hence, saving the subscription fee by not subscribing to the PA is a profitable deviation.

Remember that Lemmas A.1 and A.2 in the Appendix of the main text provide all symmetric strategy profiles that can be equilibria, together with their parameter conditions and decision quality under the Assumptions BIB and PAF. We now provide the corresponding results for all remaining equilibria that were excluded by Assumptions BIB and PAF -- first, for the benchmark case that there is no PA, then for a PA whose advice arrives early, i.e., under Assumption PAF, finally for the case that there is a PA whose advice arrives later, i.e., violating Assumption PAF. In SOM Table 1.1, this means that the main text has already addressed the first two columns with the equilibria that are not in brackets and we now complete the analysis by addressing the equilibria in brackets of the first column in SOM Section 1.1, then the equilibria in brackets of the second column in SOM Section 1.2; and finally all equilibria of the last column in SOM Section 1.3.

1.1 Remaining Symmetric Equilibria in Benchmark Setting without a Proxy Advisor

Lemma 1.1 (SYM without PA: All Remaining Equilibria)**.** *Suppose no PA is admitted. In addition to the equilibria provided in Lemma A.1, there is the following symmetric equilibrium when Assumption BIB is relaxed:*

i. UNIS, i.e., all shareholders invest in own research and vote according to their signal, is a symmetric equilibrium if and only if $q_S > q_B$. Its decision quality is $\Pi(\sigma^{UNIS}) = \pi(N)$.

Proof. Suppose first that $q_S \leq q_B$ (i.e., Assumption BIB holds). Then UNIS is not an equilibrium as shown by Lemma A.1 (in the Appendix A of the main text).

Now, suppose $q_S > q_B$. In order to show that UNIS is an equilibrium, we show that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. As we consider the setting without a PA, there are only two information-acquisition strategies: acquiring an own signal or not. Pivotality always implies that among the $N-1$ other shareholders the signals are split in $\frac{N-1}{2}$ a-signals and $\frac{N-1}{2}$ b-signals, which occurs with positive probability.

1. When no own signal is acquired, strategies are Rubber-stamping and Protest.

   Consider first the deviation from UNIS to Rubber-stamping. When pivotal, voting *yes* would weakly increase decision quality if $q_B \geq q_S$. Rubber-stamping decreases decision quality as $q_S > q_B$ holds by assumption. Rubber-stamping saves costs c. For small enough c (as it is assumed in the lemma), this deviation does not increase utility.

   The deviation to always vote *no* without information acquisition (Protest) changes the outcome to *no* in case of pivotality. It would then induce A despite the fact that, conditional on pivotality, all other shareholder's signals balance each other and that the board's signal is b. Hence, decision quality is affected in a worse way than with Rubber-stamping, while cost savings are equal. Therefore this latter deviation is not as attractive as the deviation to Rubber-stamping.

2. When a signal is acquired, the information-acquisition strategy is unchanged. Deviation to a different voting strategy is not an improvement. First, not conditioning on the acquired signal is less attractive than the deviation to Rubber-stamping or to unconditionally voting *no*, as it involves higher costs. Conditioning on the own signal leaves one deviation on the voting stage: vote *yes* if a and *no* if b, which is the opposite of UNIS. However, if voting *yes* after a (against) was optimal in case of pivotality, then voting *no* after a would also be. Hence, shareholders could improve by unconditionally voting *no*.

We have established that Rubber-stamping is the most attractive deviation. For $q_B > q_S$, the deviation to Rubber-stamping strictly decreases decision quality, and hence does not increase utility for low enough costs c.

Finally, the decision quality of UNIS equals $\pi(N)$ because the signal that has been received by a majority of the N (odd) voters determines the decision. Hence, the ex ante probability that the decision matches the true state equals the probability that among N independent signals of quality q_S the majority is correct, which is the definition of $\pi(N)$.

Let us now show that there are no further symmetric equilibria. There are only two information-acquisition strategies. For not investing in an own signal both strategy profiles are symmetric equilibria, as already addressed in parts i. and ii. of Lemma A.1). Consider now investment in an own signal. Since shareholders pay c they must condition their vote on their own signal. Otherwise, they could improve by voting in the same way and not investing c. Conditioning on their signal leaves two pure strategies: vote *yes* if b and *no* if a (i.e., UNIS) or the opposite (vote *yes* if a and *no* if b). If voting *yes* after a (against) was optimal, then voting *no* after a would also be. Hence, shareholders could improve by unconditionally voting A. Only UNIS remains when shareholders acquire an own signal.

□

1.2 Remaining Symmetric Equilibria with an Early Proxy Advisor

Lemma 1.2 (SYM with PA under Assumption PAF: All Remaining Equilibria)**.** *Let Assumption PAF hold. Let costs c be arbitrarily small and let fee f be sufficiently smaller. In addition to the equilibria provided in Lemma A.2, there are the following symmetric equilibria when Assumption BIB is relaxed:*

i. UNIS is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$. Its decision quality is $\Pi(\sigma^{UNIS}) = \pi(N)$.

ii. Cand. 5a (i.e., shareholders subscribe to PA and invest in own research iff the vote recommendation is for, i.e., Subscribe-InvestIFFfor; when the recommendation is for, they vote yes iff their own signal is b; when the recommendation is against, they vote yes), as illustrated in Table A.4, is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$. Its decision quality: $\Pi(\sigma^{Cand.\ 5a}) = q_B(1-q_P) + [q_Bq_P + (1-q_B)(1-q_P)](\pi(N))$.

iii. Cand. 5b (i.e., shareholders subscribe to PA and invest in own research iff the vote recommendation is for, i.e., Subscribe-InvestIFFfor; when the recommendation is for, they vote yes iff their own signal is b; when the recommendation is against, they vote no), as illustrated in SOM Table 1.2, is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$. Its decision quality is $\Pi(\sigma^{Cand.\ 5b}) = (1-q_B)q_P + [q_Bq_P + (1-q_B)(1-q_P)](\pi(N))$.

Proof. We address each part of SOM Lemma 1.2 separately.

i. Suppose first that $\ell_S \leq \ell_B + \ell_P$. We show that there is a utility improving deviation.

Consider shareholder i who deviates to strategy CAIS. (CAIS is illustrated in Table 1.) A deviation to CAIS differs from UNIS only when the vote recommendation is for and the own signal is a: with UNIS she would vote *no*, with CAIS she votes *yes*. It weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which we assumed at the beginning of this argument. It saves costs if $f < c[q_B q_P + (1 - q_B)(1 - q_P)]$, which is satisfied by assumption that f is sufficiently lower than c.[1] Hence, i improves utility by deviating to CAIS.

Suppose now that $\ell_S > \ell_B + \ell_P$. In order to show that UNIS is an equilibrium, we show that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. Pivotality always implies that among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ a-signals and $\frac{N-1}{2}$ b-signals, which occurs with positive probability.

(1) NotSubscribe-NotInvest. Deviating to Rubber-stamping decreases decision quality, as $\ell_S > \ell_B + \ell_P > \ell_B$ implies that $q_S > q_B$. It saves costs c. Hence, the deviation to Rubber-stamping is not beneficial for low enough c. Deviation to unconditionally voting *no* is even less attractive than deviating to Rubber-stamping (see also Proof of SOM Lemma 1.1).

(2) NotSubscribe-Invest. A deviation to using the same information-acquisition strategy as in UNIS (NotSubscribe-Invest) but a different voting strategy is not an improvement (see Proof SOM Lemma 1.1).

(3) Subscribe-NotInvest. Deviating to subscribe to the PA without investing into an own signal (Subscribe-NotInvest) is most attractive when voting according to the PA's recommendation. (When not conditioning on the vote recommendation, the shareholder could better deviate to no information acquisition, i.e., NotSubscribe-NotInvest.) Voting according to the PA's recommendation weakly improves decision quality iff $q_P \geq q_S$, which is precluded by $\ell_S > \ell_B + \ell_P$. Hence, this deviation strictly decreases decision quality. Consider now the costs. The deviation costs an additional f but saves c. If costs c are sufficiently low (as it is assumed in SOM Lemma 1.2), the decision quality difference dominates any cost difference, and hence, this deviation does not increase i's utility.

(4) Subscribe-Invest. Deviating to subscribe to the PA and invest into an own signal requires conditioning on both the vote recommendation and the own signal. Otherwise, there are other deviations which are more attractive because of the costs. Two cases are possible.

Case 1 is the deviation illustrated in Table A.2, i.e., voting *yes* except when both the own signal and the PA's recommendation contradict the board's proposal, then the deviating shareholder votes *no*. This deviation cannot alter the outcome after recommendation *against* because voting according to one's signal is like in UNIS. The difference to UNIS occurs if the vote recommendation is for and i's signal is a (in UNIS i would vote *no*, in this deviation she would vote *yes*). Conditional on that case and on pivotality, the deviation weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which is precluded by assumption $\ell_S > \ell_B + \ell_P$. Considering that the deviation is more costly than UNIS, it is not beneficial.

The second case is illustrated in Table A.3, i.e., voting *no* except when both the own signal and the PA's recommendation are aligned with the board's proposal. This deviation cannot alter the outcome after recommendation for because voting according to one's

[1] We use this assumption here only for the special case $\ell_B + \ell_P = \ell_S$.

signal is like in UNIS. The difference to UNIS hence only occurs if the vote recommendation is *against* and the signal is b: in UNIS i would vote *yes*; in this deviation she would vote *no*. Conditional on that case and conditional on pivotality, the deviation weakly improves decision quality iff $\ell_P \geq \ell_S + \ell_B$, which is precluded by assumption $\ell_S > \ell_B + \ell_P$. Hence, this deviation is not beneficial, considering that it is also more costly than UNIS.

(5) Subscribe-InvestIFFfor. Consider the deviation to subscribing to the PA and investing into an own signal iff the PA's recommendation is for.

Since shareholders pay f and sometimes c they must condition their voting strategy on the information they have acquired. (Otherwise, there would be a more attractive deviation without these costs.) In particular, they must vote according to their signal after the for recommendation. There are two cases. They correspond to the strategies Cand. 5a and Cand. 5b.

Cand. 5a: shareholders vote *yes* except if the vote recommendation is for and the own signal is a (against) as in Table A.4. The deviation to Cand. 5a cannot alter the outcome after recommendation for because voting according to one's signal is like in UNIS. The difference to UNIS hence only occurs if the vote recommendation is *against* and the signal is a (in UNIS i would vote *no*, in this deviation she would vote *yes*). Conditional on that case and on pivotality, the deviation would weakly improve decision quality iff $\ell_B \geq \ell_S + \ell_P$. Since $\ell_S > \ell_B + \ell_P$ by assumption, we have $\ell_B < \ell_S + \ell_P$. Hence, this deviation decreases decision quality. The deviation costs f with certainty and c with probability $q_B q_P + (1 - q_B)(1 - q_P)$. UNIS costs c with certainty. The cost difference (costs of UNIS minus costs of Cand. 5b) is hence $c[q_B(1 - q_P) + (1 - q_B)q_P] - f$ which is small for c arbitrarily small (as it is assumed in the lemma). Hence, the deviation does not increase utility of a deviating shareholder.

Cand. 5b: shareholders vote *no* except if the vote recommendation is for and the own signal is b (for board) as in SOM Table 1.2.

		Own Signal	
		b (for board)	a (against)
PA	for	yes	no
	against	no	

Table 1.2: Cand. 5b: A strategy based on acquiring an own signal iff the PA's recommendation is for: Subscribe-InvestIFFfor and vote *no*, except if the PA's recommendation is for and the own signal is b.

The deviation to Cand. 5b cannot alter the outcome after recommendation for because voting according to one's signal is like in UNIS. The difference to UNIS hence only occurs if the vote recommendation is *against* and the signal is b (in UNIS i would vote *yes*, in this deviation she would vote *no*). Conditional on that case and on pivotality, the deviation would weakly improve decision quality iff $\ell_P \geq \ell_S + \ell_B$. By assumption $\ell_S > \ell_B + \ell_P$, we have $\ell_P < \ell_S + \ell_B$. Hence, this deviation reduces decision quality. The cost difference (costs of UNIS minus costs of Cand. 5b) is again $c[q_B(1 - q_P) + (1 - q_B)q_P] - f$, which is small for c arbitrarily small (as it is assumed in the lemma). Hence, the deviation does not increase utility of a deviating shareholder.

(6) Subscribe-InvestIFF*against*. Consider a deviation to subscribing to the PA and investing into an own signal iff the vote recommendation is *against*.

Since shareholders pay f and sometimes c they must condition their voting strategy on the information they have acquired. (Otherwise, there would be a more attractive deviation, saving the costs.) In particular, they must vote according to their signal after the *against* recommendation. Two cases remain: CAIS as illustrated in Table 1 and CAIS-2 as illustrated in Table A.1. A deviation to CAIS differs from UNIS only when the vote recommendation is for and the own signal is a (with UNIS she would vote *no*, with CAIS she votes *yes*). It would weakly improve decision quality iff $\ell_B + \ell_P \geq \ell_S$. Since $\ell_S > \ell_B + \ell_P$, it strictly decreases decision quality. The cost difference (costs of UNIS minus costs of CAIS) is $c[q_B q_P + (1 - q_B)(1 - q_P)] - f$ and decreases in c. Hence, the difference in decision quality dominates, and the deviation to CAIS lowers the deviating shareholder's utility lower.

Consider now a deviation to strategy CAIS-2 (which only differs from CAIS by voting *no* when the vote recommendation is for). It differs from UNIS only when the vote recommendation is for and the own signal is b (indeed, with UNIS she votes *yes*, with CAIS-2 she would vote *no*). The deviation to CAIS-2 would weakly improve decision quality iff $\ell_B + \ell_S + \ell_P \leq 0$, which is never satisfied. It decreases decision quality. Since we assume sufficiently low costs, this deviation is not profitable.

Now, we have covered all deviations that can occur. If $\ell_S > \ell_B + \ell_P$, UNIS is an equilibrium for low enough costs c.

ii. We have to show that Cand. 5a is a symmetric equilibrium if and only if $\ell_S > \ell_B + \ell_P$. The voting strategy is illustrated in Table A.4. A shareholder is not pivotal after the *against* recommendation, but after the for recommendation when all the other $N - 1$ shareholder's signals are exactly split.

Suppose first that $\ell_S \leq \ell_B + \ell_P$. We show that there is a beneficial deviation. Consider shareholder i who deviates to Rubber-stamping. This deviation only alters the outcome when the vote recommendation is for, all other shareholders' signals are split, and i's signal is a (against): Under Cand. 5a, i would vote *no*, but under Rubber-stamping she votes *yes*. Decision quality weakly improves by this deviation given $\ell_B + \ell_P \geq \ell_S$. Moreover, costs are lower. Hence, Cand. 5a cannot be an equilibrium.

Suppose now that $\ell_S > \ell_B + \ell_P$. In order to show that Cand. 5a is an equilibrium, we show that there is no individual deviation that improves utility. We use the following principle: if a deviation is more attractive than an other deviation in terms of utility, then excluding the former is sufficient to exclude the latter. We organize the potential deviations by information-acquisition strategy. Pivotality always implies that the vote recommendation is for and among the $N - 1$ other shareholders the signals are split in $\frac{N-1}{2}$ a-signals and $\frac{N-1}{2}$ b-signals, which occurs with positive probability.

(1) NotSubscribe-NotInvest. Deviating to Rubber-stamping, while saving costs, decreases decision quality as $\ell_B + \ell_P < \ell_S$. Hence, the deviation to Rubber-stamping is not beneficial for low enough c and f.

Deviating to unconditionally voting *no* (Protest) lowers decision quality in any case and is therefore less attractive than deviating to Rubber-stamping.

(2) NotSubscribe-Invest. Consider deviating to information-acquisition strategy NotSubscribe-Invest. The most attractive combination of this strategy with a voting strategy is UNIS:

UNIS conditions the vote on the acquired signal in a way that maximizes decision quality. However, a deviation to UNIS does not change the outcome, since in case of pivotality (when the vote recommendation is for) the voting strategies are identical. Cand. 5a is less expensive than UNIS if $f + [q_B q_P + (1 - q_B)(1 - q_P)]c \leq c$, which is $f \leq c(1 - [q_B q_P + (1 - q_B)(1 - q_P)]$. This is satisfied for f sufficiently lower than c. Hence, UNIS is not a profitable deviation.

(3) Subscribe-NotInvest. A deviation to subscribing to the PA is most attractive when voting according to the recommendation. (Voting the opposite of the recommendation reduces decision quality. When not conditioning on the vote recommendation, the shareholder could save costs and deviate to no information acquisition, NotSubscribe-NotInvest.) Voting according to the recommendation alters the outcome if vote recommendation is for, other shareholder's signals are split, and i's signal is a (against): With Cand. 5a, i would vote *no*, with this deviation i would vote *yes*. The deviation weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which is precluded by $\ell_S > \ell_B + \ell_P$. It decreases decision quality. The deviation saves costs c, but for low enough c it is not beneficial to deviate.

(4) Subscribe-Invest. A deviation to subscribing to the PA and investing into an own signal requires to conditioning on both vote recommendation and own signal. Otherwise, there are other deviations which are more attractive, producing the same decision quality and saving the costs. Two cases are possible.

Case 1 is the deviation illustrated in Table A.2. The difference to UNIS only occurs if the vote recommendation is for and the own signal is a (in UNIS i would vote *no*, in this deviation she would vote *yes*). Conditional on that case and conditional on pivotality, the deviation weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which is precluded by assumption. Considering that the deviation is more costly than Cand. 5a, this deviation is never beneficial.

The second case is illustrated in Table A.3. This deviation does not alter the outcome (after vote recommendation for, voting is according to signal like in Cand. 5a, and after recommendation *against*, the deviating shareholder is not pivotal). Considering that the deviation is also more costly than Cand. 5a, this deviation is never beneficial.

(5) Subscribe-InvestIFFfor. Consider a deviation that keeps information-acquisition strategy Subscribe-InvestIFFfor but changes the voting strategy. Changes after the recommendation *against* are ineffective, as no shareholder is pivotal. Since shareholders pay c after recommendation for they must condition their voting strategy on the information they have acquired. (Otherwise, there would be a more attractive deviation that produces the same decision quality but saves c.) In particular, they must condition their vote on their signal after the for recommendation. This means that the deviation is voting *yes* when the own signal is a (*against*) and voting *no* when the own signal is b (for board). This deviation leads to the same costs as Cand. 5a but clearly reduces decision quality.

(6) Subscribe-InvestIFF*against*. Consider a deviating shareholder who subscribes to the PA and invests into an own signal iff the PA's recommendation is *against*. Since the shareholder pays f and sometimes c, she must condition her voting strategy on the information she has acquired. (Otherwise, there would be a more attractive deviation that produces the same decision quality but saves costs.) In particular, she must vote according to her signal after the *against* recommendation. Two cases remain.

Case 1 is CAIS as illustrated in Table 1. It alters the outcome of Cand. 5a only when the vote recommendation is for and the own signal is a: Indeed, with Cand. 5a she would vote *no*, with CAIS she votes *yes*. The deviation weakly improves decision quality iff $\ell_B + \ell_P \geq \ell_S$, which is precluded by assumption. It reduces decision quality. It does save some cost: Cand. 5a costs $f + [q_B q_P + (1 - q_B)(1 - q_P)]c$, CAIS costs $f + [q_B(1 - q_P) + (1 - q_B)q_P]c$. For low enough costs c, CAIS is not an improvement.

Case 2 is called CAIS-2 and illustrated in Table A.1 (it only differs from CAIS by prescribing to vote *no* when the vote recommendation is for). CAIS-2 differs from Cand. 5a only when the vote recommendation is for and the own signal is b (with Cand. 5a the shareholder would vote *yes*, with CAIS-2 she votes *no*). The deviation would weakly improve decision quality iff $\ell_B + \ell_S + \ell_P \leq 0$, which is precluded by assumption. It reduces decision quality. Hence, even for small enough costs c, this is not a beneficial deviation.

Now, we have covered all deviations that can occur. If $\ell_S > \ell_B + \ell_P$, Cand. 5a is an equilibrium for low enough costs c and sufficiently lower f.

The decision quality of the equilibrium Cand. 5a amounts to $q_B q_P * \pi(N) + q_B(1 - q_P) * 1 + (1 - q_B)q_P * 0 + (1 - q_B)(1 - q_P) * \pi(N)$.

iii. The proof that Cand. 5b is an equilibrium under the same conditions as Cand. 5a is identical to the proof for Cand. 5a (immediately above).

The decision quality of the equilibrium Cand. 5b amounts to $q_B q_P * \pi(N) + q_B(1 - q_P) * 0 + (1 - q_B)q_P * 1 + (1 - q_B)(1 - q_P) * \pi(N)$.

To show that there are no additional equilibria, we exhaustively discuss all pure strategies. Again, we organize the discussion by information-acquisition strategy.

(1) NotSubscribe-NotInvest. There are only voting strategies *yes* or *no*. Both lead to equilibria as shown in parts i. and ii. of Lemma A.1. (Assumption BIB does not matter for these results.)

(2) NotSubscribe-Invest. Since shareholders pay c they must condition on their own signal. Otherwise, they could improve by voting in the same way and not investing c. Conditioning on the own signal leaves two pure strategies: voting *yes* if the signal is b and voting *no* if the signal is a (i.e., UNIS) or the opposite (voting *yes* if the signal is a and voting *no* if the signal is b). If voting *yes* after the own signal is a (against) was optimal, then voting *no* after signal a would also be so. Hence, shareholders could improve by unconditionally voting *no*. Only UNIS remains. When UNIS is an equilibrium has been already addressed in this lemma, SOM Lemma 1.2, part i.

(3) Subscribe-NotInvest. Since shareholders pay f, they must condition on the PA's recommendation. Hence, they either vote *yes* after for and *no* after *against*, or they do the opposite. In either case, no shareholder is pivotal since all votes are the same given one particular vote recommendation. A shareholder can improve by not paying f and voting unconditionally, e.g., *yes*. Hence, there is no symmetric equilibrium with this information-acquisition strategy.

(4) Subscribe-Invest. Since shareholders pay both f and c, they must condition their voting strategy on both the vote recommendation and the own signal. Otherwise, they could improve their utility by exhibiting the same voting behavior, but saving costs. This means that only two voting strategies remain.

Case 1: Consider the strategy to vote *yes* except if the vote recommendation is *against* and the signal is a, as in Table A.2. In this case no shareholder is pivotal if the PA recommends for (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if the vote recommendation is *against*. If so, i votes according to her own signal. Hence, deviating to UNIS would not change the outcome because either i is not pivotal or i would also vote according to the own signal. However, UNIS saves fee f and is hence a profitable deviation. Thus, the strategy profile of case 1, illustrated in Table A.2, cannot be a symmetric equilibrium.

Case 2: Consider the strategy to vote *no* except if the vote recommendation is for and the own signal is b (for board), as in Table A.3. The analogous argument as above for case 1 applies, as follows: In this case no shareholder is pivotal if the PA recommends *against* (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if the recommendation is for. If so, i votes according to the own signal. Hence, deviating to UNIS would not change the outcome because either i is not pivotal or i would also vote according to the own signal. However, UNIS saves fee f and is hence a profitable deviation. Thus, the strategy profile of case 2 cannot be a symmetric equilibrium.

Therefore, there cannot be a symmetric equilibrium with this information-acquisition strategy (Subscribe-Invest), in which shareholders unconditionally buy both PA's recommendation and own signal.

(5) Subscribe-InvestIFFfor. Since shareholders pay f and sometimes c, they must condition their voting strategy on the vote recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation for, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated and not conditioning as well. This leaves two cases, which we have already addressed as Cand. 5a and Cand. 5b in this lemma (SOM Lemma 1.2) in parts ii. and iii.

(6) Subscribe-InvestIFF*against*. Since shareholders pay f and sometimes c, they must condition their voting strategy on the vote recommendation and the own signal when they acquire them. In particular, after having bought the own signal on top of the recommendation *against*, shareholders must vote according to their signal in equilibrium. Voting the opposite is dominated and not conditioning as well. This leaves two cases: CAIS and CAIS-2, which we have already addressed in Lemma A.2. (Assumption 1 does not affect these equilibria.)

Hence, there are no further symmetric equilibria.

□

1.3 All Symmetric Equilibria with a Late Proxy Advisor

Consider now the situation when a PA is admitted and proxy advice arrives after the shareholders' decision to invest in own research. That is, all actions occur as illustrated in the timeline (Figure 1), but proxy advice arrives at the end of period $t = 2$. This timeline admits information-acquisition strategies NotSubscribe-NotInvest, NotSubscribe-Invest, Subscribe-NotInvest, and Subscribe-Invest, which are all also playable when Assumption PAF is satisfied. However, the violation of Assumption PAF precludes the two information-acquisition strategies Subscribe-InvestIFFfor and Subscribe-InvestIFF*against*, the latter of which was crucial for our main results. SOM Table 1.1 illustrates this and already indicates that in this setting, the equilibria are very similar to the setting where there is no PA.

Comparing these two settings (the benchmark setting of no PA with a PA whose recommendation arrives late) we note that the presence of a PA who violates Assumption PAF admits two additional strategies: Subscribe-NotInvest, and Subscribe-Invest. It turns out that none of these two additional information-acquisition strategies is part of an equilibrium. Intuitively, buying only the vote recommendation, i.e., information-acquisition strategy Subscribe-NotInvest, is only worthwhile when using this information in an instance of pivotality. However, if all shareholders symmetrically use the vote recommendation, then no shareholder is ever pivotal. If they do not use it, they could save costs by not paying the fee f. Similarly, and more importantly, acquiring both signals, i.e., information-acquisition strategy Subscribe-Invest, is never part of an equilibrium. The reason is that shareholders who acquire both the PA's signal and an own signal must condition their vote on both such that none is superfluous, e.g., by voting *yes* if and only if one of them is in favor of the proposal. When all shareholders adopt such a strategy, pivotality already implies the content of the vote recommendation. For instance, if shareholders vote *yes* if and only if either the vote recommendation or the own signal is in favor of the proposal, pivotality implies that the recommendation was *against*. Hence, there is a deviation to not paying the subscription fee. Analogous arguments exist for every other possible voting strategy that conditions on combinations of PA advice and the own signal. Therefore, admitting a late PA does not lead to additional equilibria. In contrast, it may destroy equilibria that existed without a PA because it offers additional deviation possibilities. This in fact occurs for some parameter range, as illustrated in SOM Figure 1.1 in the left area above the triangle, where UNIS ceases to be an equilibrium, while Rubber-stamping becomes the Pareto-efficient equilibrium. SOM Proposition 1.1 provides the general result.

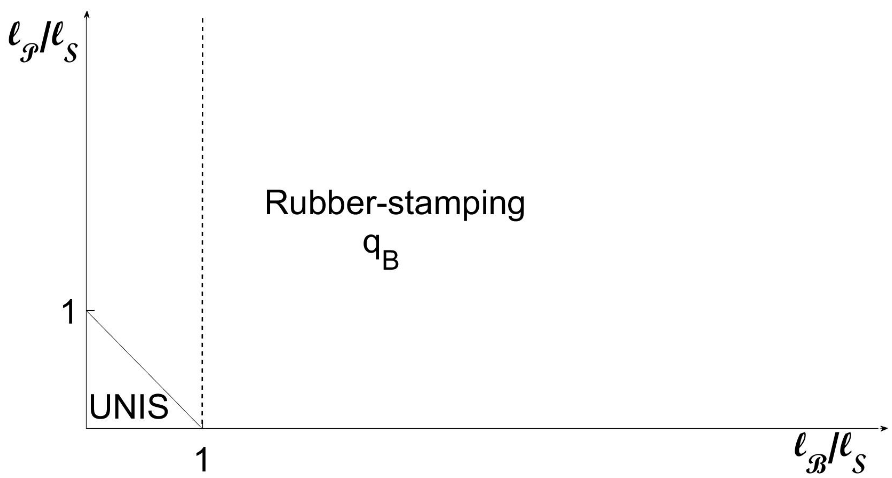

Figure 1.1: Pareto-efficient symmetric equilibria with late PA, i.e., when Assumption PAF is violated.

Proposition 1.1 (SYM with PA violating Assumption PAF)**.** *Let costs c be arbitrarily small. Suppose there is a PA whose vote recommendation arrives* after *shareholders decided upon own research, i.e., Assumption PAF is violated.*

i. If $\ell_S \geq \ell_B + \ell_P$, then there exists a symmetric equilibrium in which shareholders invest in own research. The Pareto-efficient equilibrium is UNIS and leads to decision quality $\Pi(\sigma^{UNIS}) = \pi(N)$.

ii. Otherwise, there does not exist a symmetric equilibrium in which shareholders invest in own research. The Pareto-efficient equilibrium is Rubber-stamping and leads to decision quality is $\Pi(\sigma^{rubber}) = q_B$.

Proof. To show part i. of SOM Proposition 1.1, we use SOM Lemma 1.3 below, which shows that UNIS is an equilibrium (part iii.) and that there are two further equilibria: Rubber-stamping and Protest. It remains to show that UNIS Pareto-dominates rubber-stamping and the other trivial equilibrium (Protest) for $\ell_S \geq \ell_B + \ell_P$. We have

$$\Pi(\sigma^{UNIS}) = \pi(N) > q_S > q_B = \Pi(\sigma^{Rubber}) > 1 - q_B = \Pi(\sigma^{Protest}),$$

where $\sigma^{Protest}$ stands for the equilibrium in which shareholders acquire no information (NotSubscribe-NotInvest) and vote *no* unconditionally. Now, for costs c low enough (as it is assumed in the proposition), UNIS Pareto-dominates because of its higher decision quality.

To show part ii. of SOM Proposition 1.1, we use again SOM Lemma 1.3. For $\ell_S < \ell_B + \ell_P$, UNIS is not an equilibrium (by SOM Lemma 1.3) and only two equilibria remain, Rubber-stamping and Protest. Rubber-stamping Pareto-dominates because it leads to higher decision quality $\Pi(\sigma^{Rubber}) = q_B > 0.5 > 1 - q_B = \Pi(\sigma^{Protest})$. □

It is important to observe that Assumption BIB, $q_S \leq q_B$, implies $\ell_S < \ell_B + \ell_P$, which precludes UNIS here. Hence, we can summarize.

Remark. *Under Assumption BIB, unconditional information acquisition neither occurs with late PA (SOM Proposition 1.1), nor with an early PA (SOM Proposition 1.2). Conditional information acquisition occurs with an early PA (Proposition 2), but not with a late PA (SOM Proposition 1.1).*

Considering the benchmark setting without a PA, SOM Proposition 1.1 is very similar to Proposition 1, but differs in the condition for the two equilibria. In fact, the condition for an equilibrium with information acquisition becomes more demanding when a PA is admitted: $\ell_S \geq \ell_B + \ell_P$ means that a single shareholder has to be better informed, not only than the board, but than both the board and the PA together. The reason is that there is an additional deviation possibility, compared to the setting without a PA: A shareholder could invest in an own signal and buy the vote recommendation and then vote *no* only if both the vote recommendation and the own signal are against the board.[2] This deviation only changes the voting outcome, compared to the UNIS strategy profile, if i is pivotal and the PA's recommendation is for and i's signal is a: in UNIS i votes *no*, in this deviation she would vote *yes*. Conditional on that case, the deviation improves decision quality iff

$$\begin{aligned}
q_B q_P (1-q_S)\binom{N-1}{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} (1-q_S)^{\frac{N-1}{2}} &> (1-q_B)(1-q_P)q_S \binom{N-1}{\frac{N-1}{2}} (1-q_S)^{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} \\
q_B q_P (1-q_S) &> (1-q_B)(1-q_P)q_S \\
\frac{q_B}{1-q_B} \cdot \frac{q_P}{1-q_P} &> \frac{q_S}{1-q_S} \\
\ell_B + \ell_P &> \ell_S.
\end{aligned}$$

When this condition is satisfied, then for a sufficiently small fee f, the deviation is an improvement for i since it increases decision quality. Hence, $\ell_S \geq \ell_B + \ell_P$ is a necessary condition for UNIS to be an equilibrium when costs c and fee f are arbitrarily small.

The foundation for SOM Proposition 1.1 is SOM Lemma 1.3 that we establish next.

Lemma 1.3 (SYM with PA violating Assumption PAF: All Equilibria)**.** *Let costs c be arbitrarily small and let fee f be sufficiently smaller. Suppose there is a PA whose vote recommendation*

[2]This strategy is illustrated in Table A.2.

arrives after *shareholders decided upon own research, i.e., Assumption PAF is violated. Then in contrast to Lemma A.2 and SOM Lemma 1.2, we have:*

i. Protest (i.e., no shareholder invests in own research and all shareholders vote no) is a symmetric equilibrium for any $\ell_B, \ell_S \in (0, \infty)$. Its decision quality is $1 - q_B$.

ii. Rubber-stamping (i.e., no shareholder invests in research and all shareholders vote yes) is a symmetric equilibrium for any $\ell_B, \ell_S \in (0, \infty)$. Its decision quality is q_B.

iii. UNIS is a symmetric equilibrium if and only if $\ell_S \geq \ell_B + \ell_P$. Its decision quality is $\Pi(\sigma^{UNIS}) = \pi(N)$.

iv. There are no other symmetric equilibria. In particular, there is no equilibrium in which all shareholders subscribe to proxy advice and invest in own signal (Subscribe-Invest).

Proof. We address each part of SOM Lemma 1.3 separately.

i. The proof is identical to the proof of Lemma A.1, part i.

ii. The proof is identical to the proof of Lemma A.1, part ii.

iii. Suppose first that $\ell_S < \ell_B + \ell_P$. We show that there is a beneficial deviation for f low enough.

Consider shareholder i deviates to the following strategy σ_i': subscribe and invest (Subscribe-Invest) and vote *yes*, except if the PA's recommendation is *against* and the own signal is a. This strategy is illustrated Table A.2.

Pivotality in UNIS implies that among the $N-1$ other shareholders the signals are split in $\frac{N-1}{2}$ a-signals and $\frac{N-1}{2}$ b-signals. This deviation σ_i' does not alter the outcome after recommendation *against* because voting according to signal is like in UNIS. The difference to UNIS only occurs if the recommendation is for and i's signal is a: in UNIS i would vote *no*, in this deviation she would vote *yes*. Conditional on that case and conditional on pivotality, the deviation improves decision quality if and only if $\ell_B + \ell_P > \ell_S$, which holds by the assumption made at the beginning of this argument. Note that the deviation has the same research costs c but additional costs f. Hence, for f small enough (as it is assumed in this lemma) it strictly improves utility of i.

Suppose now that $\ell_S \geq \ell_B + \ell_P$. In order to show that UNIS is an equilibrium, we have to show that there is no individual deviation that improves utility. In the proof of SOM Lemma 1.2, part i., we consider all possible deviations from UNIS and show that there is no individual deviation that improves utility for $\ell_S > \ell_B + \ell_P$. Here, there are less deviation possibilities to consider, as Subscribe-InvestIFFfor and Subscribe-InvestIFF*against* cannot be played. For the other four information-acquisition strategies, all arguments from the proof of SOM Lemma 1.2 still apply under the assumption $\ell_S \geq \ell_B + \ell_P$ when simply using that $\ell_S \geq \ell_B + \ell_P$ implies $q_S > q_B$ and $q_S > q_P$.

iv. There are four information-acquisition strategies to consider.

(1) For NotSubscribe-NotInvest, i.e., shareholders neither subscribe to the PA nor invest in an own signal, both strategy profiles are symmetric equilibria, as already addressed in parts i. and ii. of this Lemma, SOM Lemma 1.3.

(2) For NotSubscribe-Invest, i.e., shareholders do not subscribe to the PA but invest in an own signal, UNIS is an equilibrium as established in SOM Lemma 1.3, part i. The other strategy with NotSubscribe-Invest, voting the opposite of the signal, is not an equilibrium. The proof is the same as in SOM Lemma 1.1.

(3) Consider Subscribe-NotInvest, i.e., shareholders subscribe to the PA but do not invest in an own signal. Since shareholders pay f, they must condition their voting strategy on the recommendation. (Otherwise, they could improve by using the same voting strategy, but saving fee f.) To condition the voting strategy on the recommendation means either to vote *yes* after for and *no* after *against*, or the opposite voting strategy. In either case, no shareholder is pivotal since all vote in the same way after a given vote recommendation.

    A shareholder can improve by not paying f and voting, e.g. *yes*. Hence, there is no symmetric equilibrium with this information-acquisition strategy.

(4) Consider now Subscribe-Invest, i.e., shareholders subscribe to the PA and invest in an own signal. Since shareholders pay both f and c, they must condition their voting strategy on both the vote recommendation and their own signal. Otherwise, they could improve exhibiting the same voting behavior, but saving costs. This means that in fact only two voting strategies remain. Case 1: The strategy to vote *yes* except if the vote recommendation is *against* and the own signal is a, when i votes according to the own signal, as illustrated in Table A.2. In this case 1, no shareholder is pivotal if the PA recommends for (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if the recommendation is *against*. If so, i votes according to the own signal. Hence, deviating to strategy UNIS would not change the outcome because either i is not pivotal or i would also vote according to the own signal. UNIS, however, saves fee f. Thus, the strategy profile of case 1 (Table A.2) cannot be a symmetric equilibrium.

    Case 2: The strategy to vote *no* except if both the vote recommendation is for and the own signal is b (for board), when i votes according to the own signal, as in Table A.3. The analogous argument as above for case 1 applies, as follows: In this case 2, no shareholder is pivotal if the PA recommends *against* (as the recommendation is common for all shareholders). Hence, shareholder i can only be pivotal if the vote recommendation is for. If so, i votes according to the own signal. Hence, deviating to strategy UNIS would not change the outcome because either i is not pivotal or i would also vote according to the own signal. UNIS, however, saves fee f. Thus, the strategy profile of case 2 (Table A.3) cannot be a symmetric equilibrium.

    Therefore, there cannot be a symmetric equilibrium with information-acquisition strategy Subscribe-Invest, in which shareholders unconditionally buy both the PA's recommendation and an own signal.

□

1.4 Derivation of Small Cost Condition

We derive the *small cost condition* discussed in Section 5.3, which is sufficient for the symmetric CAIS to be an equilibrium. If the costs c are not small, a shareholder has a profitable deviation from CAIS to Rubber-stamping, which does not involve any information acquisition costs. In contrast, CAIS has expected costs $C(\hat{\sigma}) = f + p^{dis}c$ for any shareholder. To compute the expected loss of benefits, focus on a potentially deviating shareholder i. Denote by $p^{split}(N-1, q_S)$ the probability that the other $N-1$ voters have their signals exactly split into $\frac{N-1}{2}$ a-signals and the same number of b signals:

$$p^{split}(N-1, q_S) := \binom{N-1}{\frac{N-1}{2}} (1-q_S)^{\frac{N-1}{2}} q_S^{\frac{N-1}{2}}.$$

Denote by $\Delta(\hat{\sigma}, \sigma')$ the difference in expected decision quality between the symmetric strategy profile CAIS, and the strategy profile σ' in which one voter has deviated from CAIS to Rubber-stamping, conditioning on the case that all other shareholders' signals are split:

$$\Delta(\hat{\sigma}, \sigma') := (1-q_B)\, q_P q_S \; - \; q_B(1-q_P)(1-q_S).$$

As any voter owns only the share $\frac{1}{N}$ of the firm, the symmetric CAIS equilibrium requires: $f + p^{dis}c < \frac{1}{N} p^{split}(N-1, q_S) \cdot \Delta(\hat{\sigma}, \sigma')$. With $p^{dis} \in (0, \frac{1}{2})$ and $f \leq \frac{c}{2}$, we have $f + p^{dis}c < c$. Hence,

$$c < \frac{1}{N} p^{split}(N-1, q_S) \cdot \Delta(\hat{\sigma}, \sigma') = \frac{1}{N} \binom{N-1}{\frac{N-1}{2}} (1-q_S)^{\frac{N-1}{2}} q_S^{\frac{N-1}{2}} \left[(1-q_B)\, q_P q_S \; - \; q_B(1-q_P)(1-q_S)\right]$$

is a sufficient condition for CAIS to be an equilibrium. Translated into costs per-share stake, $c^{rel} := \frac{c}{1/N}$, this *small cost condition* becomes

$$c^{rel} < p^{split}(N-1, q_S) \cdot \Delta(\hat{\sigma}, \sigma').$$

How restrictive this assumption is depends on the signal qualities and on the number of voters N. In Example 1 of the main text, we have $\Delta(\hat{\sigma}, \sigma') = 0.015$; and $p^{split} \approx .1171$ (.3456) when setting $N = 21$ ($N = 5$). Hence, the private costs of one shareholder had to be smaller than 0.18% (0.52%) of the per-share stake in this example. These values are 1.07% (3.15%) in another example that also satisfies all conditions of Proposition 2, where we set $q_B = q_S = .6$ and $q_P = .69$. These values are 0.39% (3.33%) in a third example, where we set $q_B = q_S = .7$ and $q_P = .8$.

2 Asymmetric Equilibria

The main text provides one key result for asymmetric equilibria (Proposition 3), while it summarizes the others. This section of the Supplementary Online Material (SOM) provides the detailed analysis for what happens when we drop the symmetry assumption. We first show that without a PA, the number of shareholders who invest in own research is bounded from above (SOM Section 2.1). We then show how admitting a PA alters this result. More specifically, we analyze asymmetric equilibria with a PA first by establishing three lemmata (SOM Section 2.2), then by comparing the shareholders' research incentives with and without a PA (SOM Section 2.3), and finally by comparing the resulting decision quality (SOM Section 2.3). As we show, the number of shareholders who invest or conditionally invest, as well as the decision quality, weakly increase due to the presence of a PA, confirming the conclusions from the analysis of symmetric equilibria.

2.1 Asymmetric Equilibria in Benchmark Setting without a Proxy Advisor

Consider again the benchmark setting in which no PA is admitted. While Proposition 1 stated that under Assumption BIB, there is no symmetric equilibrium in which *every* shareholder invests in own research, the next result extends this to asymmetric equilibria in some parameter range. It also states that generally, in equilibrium without a PA there are always some shareholders not investing in research.

Proposition 2.1 (ASYM without PA)**.** *Suppose no PA is admitted.*

i. If $\frac{\ell_B}{\ell_S} \geq \frac{N+1}{2}$, there does not exist an equilibrium in which any shareholder invests in own research. In any Pareto-efficient equilibrium $N' \in \{\frac{N+1}{2}, .., N\}$ shareholders (i.e., a majority) play Rubber-stamping and $N - N'$ play Protest, which leads to decision quality $\Pi(\sigma^{Rubber}) = q_B$.

ii. Otherwise, i.e., if $\frac{\ell_B}{\ell_S} < \frac{N+1}{2}$, in equilibrium the number of shareholders who invest in own research is at most $z_1 := N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$.[3] In the Pareto-efficient equilibrium under sufficiently small costs c, $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor = z_1$ shareholders play UNIS and $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ shareholders play Rubber-stamping.

Proof. The proof is organized in four paragraphs.

All Strategies. Consider all pure strategies. First, those who do not buy the signal have the following pure strategies: voting *yes* (Rubber-stamping) and voting *no* (Protest).

Second, a shareholder that invests into the own signal must condition his voting behavior on the signal and be pivotal in at least one draw of nature. Otherwise, i.e., when voting unconditionally or never being pivotal, there is an improvement by keeping the voting strategy and not investing into the signal, saving costs c. A shareholder that invests and conditions on the signal can either vote in line with his signal (i.e., vote *yes* if b (for board) and *no* if a (against board)), which we call UNIS, or do the opposite (*yes* iff a). The opposite (*yes* iff a) cannot be part of an equilibrium strategy. Indeed, if voting *no* after receiving signal b (for board) is a best response, then, conditional on signal b, state A, which does not match the board's proposal, must be more likely than B, the state that matches the board's proposal. Since the information technology of signals is monotonic, receiving signal a (against board) makes state A even more likely such that this voter also prefers to vote *no*

[3] The mathematical expression $\lfloor z \rfloor$ is defined as the largest integer that is lower or equal to z.

if the signal is a (against). Hence, the opposite voting strategy can be ruled out and only UNIS remains for those who buy the signal.

Conditions for Investing in Own Signal. For a given strategy profile σ, a given realization of signals, and a shareholder i, let us define two numbers d and δ_{-i}. For the voters who have not invested, let $\boxed{d}$ be the number of unconditional *yes*-votes minus the unconditional *no* votes; for the voters who have invested, let $\boxed{\delta_{-i}}$ be the number of a (against) signals received minus the number of b (for board) signals received when excluding the focal shareholder i. (Recall that in equilibrium those who received signal b will vote *yes* and those who received signal a will vote *no*).

Suppose shareholder i invested in research and is pivotal. Pivotality implies that the number of total votes of others, $N-1$, is fifty-fifty split in *yes* and *no* votes. This implies that $d = \delta_{-i}$.

The two necessary and jointly sufficient conditions for optimality of voting according to the own signal are: $\ell_B + 1\ell_S > \delta_{-i}\ell_S$, which is required for (and implies) optimality of voting *yes* when the own signal is b, and $\delta_{-i}\ell_S + 1\ell_S < \ell_B$, which is required for (and implies) optimality of voting *no* after the own signal a. This yields $d = \delta_{-i} \in (\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1)$. In fact, the interval is open. Suppose to the contrary, that $d = \frac{\ell_B}{\ell_S} - 1$. Then $\delta_{-i} = d$ implies $\delta_{-i}\ell_S + 1\ell_S = \ell_B$, i.e., an informed shareholder i is indifferent between voting *yes* and voting *no* after receiving signal a. After receiving signal b, this shareholder prefers to vote *yes*. Hence, if this shareholder would unconditionally vote *yes*, she would induce the same decision quality. Therefore, this shareholder could unilaterally improve (upon her strategy UNIS) by Rubber-stamping, i.e., not investing into an own signal and voting *yes* unconditionally, which saves costs $c > 0$. Analogously, it would be beneficial to switch to unconditionally voting *no* in case of $d = \frac{\ell_B}{\ell_S} + 1$.

Part i. For part i. of the proposition, we use in particular that we have $d = \delta_{-i} > \frac{\ell_B}{\ell_S} - 1$ when a shareholder invests, while $\frac{\ell_B}{\ell_S} \geq \frac{N+1}{2}$ by assumption. Thus, $d > \frac{N+1}{2} - 1$ and hence $d \geq \frac{N+1}{2}$ (as d is a natural number). Hence, the assumption that a shareholder invests in own research leads to the implication that at least $d \geq \frac{N+1}{2}$ more shareholders unconditionally vote *yes* than unconditionally vote *no*. The latter, however, implies that no voter is ever pivotal (since there is always a majority voting *yes*). This, in turn, contradicts the assumption that a shareholder invests in research. Thus, there cannot be an informed voter in equilibrium.

Without informed voter, each shareholder votes either unconditionally *yes* (Rubber-stamping) or *no* (Protest). Information quality is q_B if a majority votes unconditionally *yes* and $1 - q_B$ otherwise. Hence, in any Pareto-efficient equilibrium $N' \in \{\frac{N+1}{2}, .., N\}$ play the strategy of Rubber-stamping and $N - N'$ play the strategy of Protest such that decision quality is $\Pi^* = q_B$.

Part ii. In order to prove the upper bound for the number of informed shareholders, we use again that we have $d > \frac{\ell_B}{\ell_S} - 1$ when a shareholder invests. This implies $d \geq \lfloor \frac{\ell_B}{\ell_S} \rfloor$, as d is a natural number. To create a vote difference of d among those who did not invest, it takes at least d voters, e.g., when d vote *yes* while zero vote *no*. Hence, the number of informed voters is at most $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor = z_1$.

We now prove that $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ players voting always *yes* (strategy Rubber-stamping) and $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ players voting according to their private signals (strategy UNIS) is indeed an equilibrium. To this end, we show that no shareholder in these two groups has an incentive to deviate from her strategy.

- Consider a shareholder who invests in an own signal and votes according to it (strategy UNIS). She is pivotal if the others' votes are split, which happens if and only if there are $d = \lfloor \frac{\ell_B}{\ell_S} \rfloor = \delta_{-i}$ more a-signals than b-signals among the other informed shareholders. In that occasion, deviating to voting *no* after signal b is suboptimal because it would reject the board's proposal although $\ell_B + 1\ell_S \not< \delta_{-i}\ell_S$; and deviating to voting *yes* after signal a is suboptimal because it would accept the board's proposal although $\ell_B \not> \delta_{-i}\ell_S + 1\ell_S$. As pivotality occurs with positive probability, deviations certainly reduce decision quality, while they can only save costs c that are by assumption arbitrarily small.

- Consider now a shareholder who always votes *yes*. Even if she knew her signal in case of being pivotal, she would never want to deviate to voting *no*, as she is pivotal if and only if there are $\frac{N-1}{2} - (\lfloor\frac{\ell_B}{\ell_S}\rfloor - 1)$ players who obtained positive signals and $\frac{N-1}{2}$ players with negative ones: A negative signal would make her prefer to vote *no* if and only if $\ell_B + (\frac{N-1}{2} - \lfloor\frac{\ell_B}{\ell_S}\rfloor + 1) \cdot \ell_S < \frac{N-1}{2} \cdot \ell_S + \ell_S$, which is equivalent to $\ell_B - \lfloor\frac{\ell_B}{\ell_S}\rfloor \cdot \ell_S < 0$ and thus to $\frac{\ell_B}{\ell_S} < \lfloor\frac{\ell_B}{\ell_S}\rfloor$, which is a contradiction.

We will now prove that the equilibrium is Pareto-efficient. To this end, we first prove that for any equilibrium in which a positive number of players invests in own research, the difference d between players always voting *yes* and those always voting *no* must equal $\lfloor\frac{\ell_B}{\ell_S}\rfloor$. Above, we have already derived that $d \in (\frac{\ell_B}{\ell_S} - 1, \frac{\ell_B}{\ell_S} + 1)$, which can only happen for $d = \lfloor\frac{\ell_B}{\ell_S}\rfloor$ or $d = \lfloor\frac{\ell_B}{\ell_S}\rfloor + 1$. For any fixed strategy profile and signal distribution, let $\boxed{\delta}$ denote the difference between the number of informed shareholders voting *no* and informed shareholders voting *yes*. A shareholder always voting *yes* will then be pivotal if and only if $d - 1 = \delta$ or, equivalently, $d = \delta + 1$. Now, let us assume that this shareholder still had the opportunity for own research and obtained a signal which happens to contradict the board's proposal. This shareholder would like to vote *no* if $\ell_B < \delta\ell_S + \ell_S$, which is equivalent to $\ell_B < d\ell_S$ or $\frac{\ell_B}{\ell_S} < d$. Thus, whenever $d > \frac{\ell_B}{\ell_S}$ and costs are sufficiently low, we cannot have a rubber-stamping player in equilibrium because such a player would benefit from deviating to UNIS. As indeed $\lfloor\frac{\ell_B}{\ell_S}\rfloor + 1 > \frac{\ell_B}{\ell_S}$, $d = \lfloor\frac{\ell_B}{\ell_S}\rfloor + 1$ cannot result in an equilibrium. Thus, we overall must have $d = \lfloor\frac{\ell_B}{\ell_S}\rfloor$ in any equilibrium in which some shareholders invest in own research.

Overall, we now have established that apart from trivial equilibria in which no shareholder invests in private research, there can only be equilibria in which the difference between those always voting *yes* and those always voting *no* is exactly equal to $\lfloor\frac{\ell_B}{\ell_S}\rfloor$. Possible non-trivial equilibria are thus characterized by $\lfloor\frac{\ell_B}{\ell_S}\rfloor + \alpha$ shareholders always voting *yes* and α shareholders always voting *no*, with α being a non-negative integer. Define $\pi(l,k) := \sum_{i=k}^{l} \binom{l}{i} q_S^i (1-q_S)^{l-i}$, i.e., the probability that among l realizations of signals with precision q_S at least k are correct. Decision quality is then

$$\Pi(\sigma) = q_B \cdot \pi\left(N - (d+\alpha), \frac{N+1}{2} - d\right) + (1-q_B) \cdot \pi\left(N - (d+\alpha), \frac{N+1}{2}\right)$$

for $d = \lfloor\frac{\ell_B}{\ell_S}\rfloor$, as $N - (d+\alpha)$ shareholders play strategy UNIS, $d + \alpha$ play strategy Rubber-stamping, and α play Protest. Since the function π is increasing in its first argument, decision quality is maximized for $\alpha = 0$. Hence, in the equilibrium of this type that provides the highest decision quality, $N - \lfloor\frac{\ell_B}{\ell_S}\rfloor$ play strategy UNIS, while $\lfloor\frac{\ell_B}{\ell_S}\rfloor$ play strategy Rubber-stamping. The corresponding decision quality is larger than the highest decision quality of all equilibria in which no shareholder invests in own research, which is q_B. The reason is the following: whenever the proposal of the board is accepted, the decisions coincide, but when a proposal is rejected, we have at least $d+1$ more signals against the proposal than for the proposal, which makes it more likely that the proposal is wrong: $\ell_B \leq (d+1)\ell_S$ for $d = \lfloor\frac{\ell_B}{\ell_S}\rfloor$. □

To understand SOM Proposition 2.1, consider a shareholder who invested in own research. This investment can only be part of an equilibrium if this shareholder conditions on her own signal in some instance in which she is pivotal. In particular, this shareholder must vote *no* if the signal is a (against). This is a best response if pivotality implies that a sufficient number of other informed shareholders also have received information against the board's proposal. This, in turn, is possible in strategy profiles in which several uninformed shareholders Rubber-stamp the board's proposal. When the number of shareholders who Rubber-stamp is by roughly $\frac{\ell_B}{\ell_S}$ larger than the number of shareholders who vote unconditionally *no* (i.e., play Protest), then there might indeed be incentives to invest in own research and vote according to one's signal. If this difference, however, exceeds half of all shareholders, as considered in part i., then it is impossible to be pivotal in the first place. Otherwise, i.e., in the case addressed in part ii., it is possible to have informed shareholders, but their number is bounded from above by $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$. It turns out that the strategy profile with the highest decision quality is then $\sigma^{\mu,\nu}$, with $\mu = N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ shareholders investing in own research and voting according to signal (strategy UNIS), and $\nu = \lfloor \frac{\ell_B}{\ell_S} \rfloor$ shareholders playing strategy Rubber-stamping. This strategy profile is Pareto-efficient and yields the upper bound for the decision quality. For the description of decision quality it is helpful to define $\pi(l, k)$ as the probability that among l realizations of signals with precision q_S at least k are correct, i.e., $\pi(l, k) := \sum_{i=k}^{l} \binom{l}{i} q_S^i (1 - q_S)^{l-i}$. Then the decision quality in case of part ii. of SOM Proposition 2.1 is $\Pi(\sigma) = q_B \cdot \pi(z_1, z_1 - \frac{N-1}{2}) + (1 - q_B) \cdot \pi(z_1, \frac{N+1}{2})$, where still $z_1 := N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$.

Comparative statics imply that the maximal number of shareholders who invest is decreasing in the board's relative information quality $\frac{\ell_B}{\ell_S}$, starting with $N - 1$ for $\lfloor \frac{\ell_B}{\ell_S} \rfloor = 1$, decreasing down to $\frac{N+1}{2}$ for $\lfloor \frac{\ell_B}{\ell_S} \rfloor = \frac{N-1}{2}$, and then discontinuously jumping to 0.[4] This validates the insight we gained from the symmetric equilibria: Without a PA, well informed boards reduce shareholders' research incentives.[5]

2.2 Lemmata for Asymmetric Equilibria with a Proxy Advisor

Before analyzing the effects of introducing a PA on shareholders' research incentives (SOM Section 2.3) and on decision quality (SOM Section 2.4), we establish three lemmata about asymmetric equilibria when a PA is admitted. SOM Lemma 2.1 shows two properties of equilibrium behavior, SOM Lemma 2.2 provides bounds on the number of informed shareholders, and SOM Lemma 2.3 characterizes the Pareto-efficient strategy profiles.

Lemma 2.1 (ASYM with PA: Equilibrium Behavior)**.** *Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee $f > 0$ be sufficiently smaller. In equilibrium the following statements hold:*

i. There is no shareholder who buys the vote recommendation and unconditionally invests in own research, i.e., uses Subscribe-Invest.

ii. Every shareholder who acquires an own signal votes according to this signal, i.e., votes yes if the signal is b (board) and vote no if the signal is a (against).

Proof. We first show part i. and then use part i. to show part ii.

i. Suppose shareholder i uses Subscribe-Invest. Since i pays both f and c she must condition her voting strategy on both the PA's vote recommendation and the own signal. This excludes

[4] Only if $\frac{\ell_B}{\ell_S} < 1$, which is the negation of Assumption BIB, all N shareholders could be informed.

[5] Their overall effect on decision quality might however still be positive -- we will discuss comparative-static effects on decision quality further below in SOM Section 2.4.

unconditional voting (such as always *yes*) and voting conditional only on one type of information (such as only voting according to the vote recommendation, or only voting according to the signal). Indeed, compared to these strategies, the shareholder could improve by exhibiting the same voting behavior, but saving costs. This means that only one type of voting strategy remains, namely conditioning on both the vote recommendation and the own signal. Consider one such strategy, namely voting *yes* except if the vote recommendation is *against* and the signal is a, then the shareholder votes *no* (as in Table A.2). In this case, the shareholder votes *yes* after the for recommendation, independently of the own signal. She could improve by keeping the same voting behavior, but changing the information-acquisition strategy, switching to Subscribe-InvestIFF*against*, which saves costs c with positive probability. Likewise, there is an improvement for all strategies of this type. In particular, consider voting *no* except if the vote recommendation is for and the own signal is b (for board), as in Table A.3. Here, there is an improvement to Subscribe-InvestIFFfor.

ii. There are four information-acquisition strategies that involve investing in an own signal: NotSubscribe-Invest, Subscribe-Invest, Subscribe-InvestIFFfor, and Subscribe-InvestIFF*against*.

Suppose there is a shareholder i who plays NotSubscribe-Invest and violates the assertion. This shareholder either votes independently of the own signal or votes for the opposite of what the signal indicates. In the former case, i could improve by using the same voting strategy, but not acquiring a signal. That would lead to the same decision quality but reduce her costs. In the latter case, the shareholder is either never pivotal and could again improve by not acquiring the signal, or she is pivotal with positive probability. If pivotal with positive probability, i would only find it optimal to vote for the opposite of what the signal indicates if, conditional on signal b and pivotality, state A that does not match the board's proposal is at least as likely as state B that does (and conditional on signal a and pivotality, state B is at least as likely as A). However, as the signal is drawn independently of the occurrence of pivotality and is informative as $q_S > 0.5$, this is impossible (conditional on signal b, state B is strictly more likely than it would be when conditioning on signal a). Therefore, a shareholder who uses NotSubscribe-Invest in equilibrium plays strategy UNIS.

There is no shareholder who plays Subscribe-Invest, as shown in part i. of this proof.

Suppose there is a shareholder i who plays Subscribe-InvestIFFfor and violates the assertion. After vote recommendation for, this shareholder either votes independently of the signal or for the opposite of what the signal indicates. There are improvements by switching to not acquiring the signal or to voting according to the signal, in complete analogy to the information-acquisition strategy NotSubscribe-Invest above. Likewise, this holds for Subscribe-InvestIFF*against*.

□

Lemma 2.2 (ASYM with PA: Bounds)**.** *Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee $f > 0$ be sufficiently smaller. In equilibrium the following holds.*

i. If $\frac{\ell_B+\ell_P}{\ell_S} < \frac{N+1}{2}$, then the number of shareholders who invest in own research is at most $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ after vote recommendation for and at most $N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ after vote recommendation $against$.

ii. If $\frac{|\ell_B-\ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B+\ell_P}{\ell_S}$, then no shareholder invests in own research after vote recommendation for and at most $N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ shareholders invest after vote recommendation $against$.

iii. Otherwise, i.e., if $\frac{N+1}{2} \leq \frac{|\ell_B - \ell_P|}{\ell_S}$*, no shareholder invests in own research.*

Proof. We first establish two statements and then proceed with the three parts of SOM Lemma 2.2.

(B1) In equilibrium, the number of shareholders who invest in own research is at most $\max\{N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor, 0\}$ after vote recommendation for.

(B2) In equilibrium, the number of shareholders who invest in own research is at most $\max\{N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor, 0\}$ after vote recommendation *against*. This holds for both (i) $\ell_B \geq \ell_P$ and (ii) $\ell_B < \ell_P$.

Ad B1 Suppose statement (B1) is violated. Then there is an equilibrium where the number of informed shareholders after vote recommendation for, x_1, satisfies $x_1 > N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and $x_1 > 0$.

Since the information-acquisition strategy Subscribe-Invest is never part of an equilibrium (by part i. of SOM Lemma 2.1), $x_1 > 0$ implies that every informed player after vote recommendation for either plays NotSubscribe-Invest or Subscribe-InvestIFFfor.

Suppose first that there is a shareholder i who plays Subscribe-InvestIFFfor in equilibrium. This player must be pivotal after vote recommendation for with positive probability, otherwise she could improve by saving costs c without affecting decision quality. Pivotality implies that $\frac{N-1}{2}$ of the $N-1$ other shareholders vote *yes* and $\frac{N-1}{2}$ vote *no*. Let $\boxed{d}$ be the difference between the *yes*-votes and *no*-votes of uninformed players (i.e., those who have not acquired a signal) after vote recommendation for. By part ii. of SOM Lemma 2.1, all shareholders who have acquired a signal after vote recommendation for must vote according to it. Therefore, conditional on having received recommendation for and on pivotality, shareholder i knows that among the other informed shareholders, exactly $\delta_{-i} = d$ more have received signal a than b. (Again $\boxed{\delta_{-i}}$ designates the number of a (against) signals received minus the number of b (for board) signals received when excluding the focal shareholder i.) However, as the number of uninformed players $y_1 := N - x_1$ satisfies $y_1 < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$, the signal difference that makes player i pivotal satisfies $\delta_{-i} = d \leq y_1 < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$. As δ_{-i} is a natural number, $\delta_{-i} < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ implies $\delta_{-i} \leq \frac{\ell_B+\ell_P}{\ell_S} - 1$, which is equivalent to $\ell_B + \ell_P \geq \delta_{-i}\ell_S + 1\ell_S$. The final inequality means that state B, which matches the board's proposal, is more likely than state A, which does not match the proposal, when i is pivotal after vote recommendation for, even if i has received signal a. Hence, shareholder i could improve by voting *yes* after vote recommendation for and not investing in the own signal, which would weakly increase decision quality and strictly decrease costs, contradicting the assumption that the strategy profile with i playing Subscribe-InvestIFFfor was an equilibrium.

Suppose now that there is no shareholder who plays Subscribe-InvestIFFfor in equilibrium. Then all informed shareholders play NotSubscribe-Invest. By part ii. of SOM Lemma 2.1, all of them play UNIS. Consider the unilateral deviation of a shareholder i from UNIS to strategy CAIS. By the assumption on costs that f is sufficiently lower than c, this saves costs. Moreover, this would weakly improve decision quality: after the *against* recommendation the decision is the same, but after the for recommendation the decision quality weakly improves. The argument for the latter claim is exactly as before when considering a player using Subscribe-InvestIFFfor. When shareholder i is pivotal after vote recommendation for, we have $\delta_{-i} = d \leq y_1 = N - x_1 < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$, which implies $\ell_B + \ell_P \geq \delta_{-i}\ell_S + 1\ell_S$, meaning that state B is more likely than state A even if i has received signal a. Hence, the deviation to strategy CAIS is an improvement for shareholder i, contradicting the assumption that the strategy profile with no shareholder playing Subscribe-InvestIFFfor was an equilibrium.

Hence, if (B1) is violated, we have that no shareholder plays Subscribe-InvestIFFfor in equilibrium and that a strategy profile without a shareholder playing Subscribe-InvestIFFfor cannot be an equilibrium -- a contradiction.

Ad B2(i) Suppose statement (B2) is violated and $\ell_B \geq \ell_P$ holds. Then there is an equilibrium where the number of informed shareholders after the *against* recommendation, x_2, satisfies $x_2 > N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ and $x_2 > 0$.

Since the information-acquisition strategy Subscribe-Invest is never part of an equilibrium (by part i. of SOM Lemma 2.1), $x_2 > 0$ implies that every informed player after the *against* recommendation either plays NotSubscribe-Invest or Subscribe-InvestIFF$against$.

Suppose first that there is a shareholder i who plays Subscribe-InvestIFF$against$ (e.g., strategy CAIS) in equilibrium. This shareholder must be pivotal with positive probability, otherwise she could improve by saving costs without affecting the decision quality. By part ii. of SOM Lemma 2.1, all shareholders who are informed after vote recommendation *against* vote according to their signal. In particular, shareholder i must vote *no* after signal a, which requires: $\ell_B < \ell_P + \delta_{-i}\ell_S + \ell_S$, where δ_{-i} is agian the vote difference of the other informed shareholders. (A weak inequality cannot be part of an equilibrium strategy as the deviation to voting always *yes* after vote recommendation *against* and saving costs c would be an improvement.) That is, $\delta_{-i} > \frac{|\ell_B - \ell_P|}{\ell_S} - 1$. Since all informed shareholders vote according to their signal, the condition above and pivotality imply that at least $\frac{|\ell_B - \ell_P|}{\ell_S}$ shareholders are uninformed, which is in contradiction to the assumption $x_2 > N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ as this inequality implies that strictly less than $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ are uninformed.[6] Hence, in equilibrium no shareholder can play Subscribe-InvestIFF$against$.

Suppose now that there is no player who plays Subscribe-InvestIFF$against$ in equilibrium. Then all informed shareholders after the *against* recommendation play strategy UNIS (by part ii. of SOM Lemma 2.1). Consider the unilateral deviation of a shareholder i from UNIS to Subscribe-InvestIFFfor, voting unconditionally *yes* after vote recommendation *against*. By the assumption on costs that f is sufficiently lower than c, this saves costs. Moreover, this would weakly improve decision quality because after the for recommendation, the decision is the same, but after the *against* recommendation, the decision weakly improves, as we show now. For the given strategy profile in the assumed equilibrium, we define the following numbers for the case that the vote recommendation is *against*: Let $y_2 = N - x_2$ be the number of uninformed shareholders, let d be the difference of *yes*-votes and *no*-votes among the y_2 uninformed shareholders, and let δ_{-i} be the difference of a-signals and b-signals of the x_2 informed shareholders. When shareholder i is pivotal after vote recommendation *against*, we have $\delta_{-i} = d \leq y_2 = N - x_2 < \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$. For $\ell_B \geq \ell_P$, and since δ_{-i} is a natural number, this implies $\delta_{-i} \leq \frac{\ell_B - \ell_P}{\ell_S} - 1$, which is equivalent to $\ell_B \geq \ell_P + \delta_{-i}\ell_S + 1\ell_S$. The final inequality means that state B, which matches the board's proposal, is more likely than state A, which does not match the proposal, when shareholder i is pivotal after vote recommendation *against*, even if i has received signal a. Hence, the deviation weakly

[6] For an integer x, $x \geq \lfloor \tau \rfloor$ is equivalent to $x > \tau - 1$.

improves decision quality, while it strictly saves costs, contradicting the assumption that the strategy profile with no shareholder playing Subscribe-InvestIFF*against* was an equilibrium.

Hence, if B2(i) is violated, we have that no shareholder plays Subscribe-InvestIFF*against* in equilibrium and that a strategy profile without a shareholder playing Subscribe-InvestIFF*against* cannot be an equilibrium -- a contradiction.

Ad B2(ii) Suppose statement (B2) is violated and $\ell_B < \ell_P$ holds. Then there is an equilibrium where the number of informed shareholders after the *against* recommendation, x_2, satisfies $x_2 > N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ and $x_2 > 0$.

Since the information-acquisition strategy Subscribe-Invest is never part of an equilibrium (by part i. of SOM Lemma 2.1), $x_2 > 0$ implies that every informed player after the *against* recommendation either plays NotSubscribe-Invest or Subscribe-InvestIFF*against*.

Suppose first that there is a shareholder i who plays Subscribe-InvestIFF*against* (e.g., strategy CAIS) in equilibrium. For the given strategy profile in the assumed equilibrium, we define the following numbers for the case that the vote recommendation is *against*: Let $y_2 = N - x_2$ be the number of uninformed shareholders, let $\boxed{\tilde{d}} = -d$ be the difference of *no*-votes and *yes*-votes among the y_2 uninformed shareholders, and let $\boxed{\tilde{\delta}_{-i}} = -\delta_{-i}$ be the difference of b-signals and a-signals (in favor of the proposal) of the x_2 informed shareholders who are not i. When shareholder i is pivotal after vote recommendation *against*, we have $\tilde{\delta}_{-i} = \tilde{d} \leq y_2 = N - x_2 < \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$. For $\ell_B < \ell_P$ and since $\tilde{\delta}_{-i}$ is a natural number, this implies $\tilde{\delta}_{-i} \leq \frac{\ell_P - \ell_B}{\ell_S} - 1$, which is equivalent to $\ell_B + \tilde{\delta}_{-i}\ell_S + 1\ell_S \leq \ell_P$. The final inequality means that state B, which matches the board's proposal, is less likely than state A, which does not match the proposal, when shareholder i is pivotal after vote recommendation *against*, even if i has received signal b. Hence, i could improve by deviating to unconditionally voting *no* after vote recommendation *against* and not investing in an own signal. This deviation weakly improves decision quality, while it strictly saves costs, contradicting the assumption that the strategy profile with i playing Subscribe-InvestIFF*against* was an equilibrium.

Suppose now that there is no player who plays Subscribe-InvestIFF*against*. Then all informed shareholders after the *against* recommendation play UNIS. Consider the unilateral deviation of a shareholder i from UNIS to Subscribe-InvestIFF*for*, unconditionally voting *no* after vote recommendation *against*. By the assumption on costs that f is sufficiently lower than c, this saves costs. Moreover, this would weakly improve decision quality by the same argument used immediately above: When shareholder i is pivotal after vote recommendation *against*, we have $\tilde{\delta}_{-i} = \tilde{d} \leq y_2 = N - x_2 < \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$, which implies $\ell_B + \tilde{\delta}_{-i}\ell_S + 1\ell_S \leq \ell_P$. The final inequality means that state B is less likely than state A when shareholder i is pivotal after vote recommendation *against* even if i has received signal b.

Hence, if B2(ii) is violated, we have that no shareholder plays Subscribe-InvestIFF*against* in equilibrium and that a strategy profile without a shareholder playing Subscribe-InvestIFF*against* cannot be an equilibrium -- a contradiction.

Now, we can use (B1) and (B2) to address the three parts of SOM Lemma 2.2.

i. By the assumption $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$, we have $\max\{N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor, 0\} = N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ and the result after vote recommendation *for* follows directly from statement (B1). Likewise, we have $\frac{|\ell_B - \ell_P|}{\ell_S} \leq \frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$, implying $\max\{N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor, 0\} = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ and the result after vote recommendation *against* follows directly from statement (B2).

ii. By the assumption $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2}$, we have $\max\{N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor, 0\} = N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ and the result after vote recommendation *against* follows directly from statement (B2).

It remains to show that after vote recommendation for, no shareholder invests in own research. Suppose, in contrast, that there is an equilibrium where the number x_1 of informed shareholders after vote recommendation for satisfies $x_1 > 0$.

Statement (B1) implies $x_1 \leq N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$. Hence, the number of uninformed shareholders after vote recommendation for satisfies $y_1 = N - x_1 \geq \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$, which implies $y_1 > \frac{\ell_B+\ell_P}{\ell_S} - 1$. The assumption $\frac{N+1}{2} \leq \frac{\ell_B+\ell_P}{\ell_S}$ now implies $\frac{N+1}{2} \leq \frac{\ell_B+\ell_P}{\ell_S} < y_1 + 1$. Hence, there are at least $\frac{N+1}{2}$ uninformed shareholders after vote recommendation for. Therefore, no shareholder can be pivotal in that case. Any of the $x_1 > 0$ shareholders who invest in an own signal can beneficially deviate to (still) buying the vote recommendation, but not investing in an own signal after vote recommendation for, as this saves costs and does not affect decision quality.

iii. Suppose, in contrast, that there is an equilibrium where either (1) the number of informed shareholders after the for recommendation, x_1, satisfies $x_1 > 0$, or (2) the number of informed shareholders after the *against* recommendation, x_2, satisfies $x_2 > 0$, or both. We show that there is a contradiction, first for statement (1) and then for statement (2).

The first case (1) is excluded by statement B1, exactly as shown immediately above.

In the second case (2), statement B2 implies $x_2 \leq N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$. Hence, the number of uninformed shareholders after vote recommendation *against* satisfies $y_2 = N - x_2 \geq \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$, which implies $y_2 > \frac{|\ell_B-\ell_P|}{\ell_S} - 1$. The assumption $\frac{N+1}{2} \leq \frac{|\ell_B-\ell_P|}{\ell_S}$ now implies $\frac{N+1}{2} \leq \frac{|\ell_B-\ell_P|}{\ell_S} < y_2 + 1$. Hence, there are at least $\frac{N+1}{2}$ uninformed shareholders after vote recommendation *against*. Therefore, no shareholder can be pivotal in that case. Any of the $x_2 > 0$ shareholders who invest in an own signal can beneficially deviate to (still) buying the vote recommendation, but not investing in an own signal after vote recommendation *against*, as this saves costs and does not affect decision quality.

$\square$

Lemma 2.3 (ASYM with PA: Pareto-efficient Equilibria)**.** *Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee $f > 0$ be sufficiently smaller. Then the following strategy profiles are Pareto-efficient equilibria.*[7]

i. If $\frac{\ell_B+\ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B \geq \ell_P$, then $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ shareholders play strategy Rubber-stamping, $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ play strategy CAIS, and $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ play strategy UNIS. Hence, decision quality is $\Pi^* = q_B q_P \pi(N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor) + (1-q_B)(1-q_P)\pi(N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor, \frac{N+1}{2}) + q_B(1-q_P)\pi(N - \lfloor \frac{\ell_B-\ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B-\ell_P}{\ell_S} \rfloor) + (1-q_B)q_P\pi(N - \lfloor \frac{\ell_B-\ell_P}{\ell_S} \rfloor, \frac{N+1}{2})$.

ii. If $\frac{\ell_B+\ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B < \ell_P$, then $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ shareholders play Follow (buy recommendation and follow it), $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ play strategy CAIS, and $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ play strategy UNIS. Hence, decision quality is $\Pi^* = q_B q_P \pi(N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor) + (1-q_B)(1-q_P)\pi(N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor, \frac{N+1}{2}) + q_B(1-q_P)\pi(N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor, \frac{N+1}{2}) + (1-q_B)q_P\pi(N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor)$.

[7]We are particularly thankful to Stefan Kloessner who foresaw this result during a walk in the forest.

iii. *If* $\frac{|\ell_B-\ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B+\ell_P}{\ell_S}$ *and* $\ell_B \geq \ell_P$*, then* $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ *shareholders play strategy Rubber-stamping and* $N-\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ *play strategy CAIS. Hence, decision quality is* $\Pi^* = q_Bq_P + q_B(1-q_P)\pi(N-\lfloor\frac{\ell_B-\ell_P}{\ell_S}\rfloor, \frac{N+1}{2}-\lfloor\frac{\ell_B-\ell_P}{\ell_S}\rfloor) + (1-q_B)q_P\pi(N-\lfloor\frac{\ell_B-\ell_P}{\ell_S}\rfloor, \frac{N+1}{2})$.

iv. *If* $\frac{|\ell_B-\ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B+\ell_P}{\ell_S}$ *and* $\ell_B < \ell_P$*, then* $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ *shareholders play strategy Protest and* $N-\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ *play strategy CAIS. Hence, decision quality is* $\Pi^* = q_Bq_P + q_B(1-q_P)\pi(N-\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor, \frac{N+1}{2}) + (1-q_B)q_P\pi(N-\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor, \frac{N+1}{2}-\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor)$.

v. *If* $\frac{|\ell_B-\ell_P|}{\ell_S} \geq \frac{N+1}{2}$ *and* $\ell_B \geq \ell_P$*, then* $N' \in \{\frac{N+1}{2}, .., N\}$ *shareholders play strategy Rubber-stamping and* $N-N'$ *play strategy Protest. Hence, decision quality is* $\Pi^* = q_B$.

vi. *If* $\frac{|\ell_B-\ell_P|}{\ell_S} \geq \frac{N+1}{2}$ *and* $\ell_B < \ell_P$*, then* $\frac{N-1}{2}$ *shareholder play strategy Rubber-stamping,* $\frac{N+1}{2}$ *play Protest, and* 1 *plays Follow. Hence, decision quality is* $\Pi^* = q_P$.

SOM Lemma 2.3 is illustrated in the lower panel of Figure 3 of the main text, where any region of the figure corresponds to one case of the lemma. In each region a Pareto-efficient equilibrium is indicated. The comparison with the Pareto-efficient equilibria in the benchmark setting without proxy advisor, illustrated in the upper panel of Figure 3, is discussed in the next two subsections. The remainder of this subsection is dedicated to prove SOM Lemma 2.3.

Proof. We first determine the theoretically maximal decision quality. Then we show that in every area of the parameter space the claimed strategy profile reaches this maximum, is an equilibrium, and is Pareto-efficient.

Generally, decision quality is maximal for a given number of signals if for any realization of these signals, the alternative that is more likely to match the true state is implemented. Moreover, additional signals increase the maximal decision quality. In our model, we always have the signal of the board of quality q_B and the signal of the PA of quality q_P. Additionally, we can have a number of signals of quality q_S. This number is restricted by SOM Lemma 2.2 and depends on whether the PA's signal agrees with the board (i.e., the vote recommendation is for) or not (i.e., the vote recommendation is *against*). Let $\boxed{\bar{n}_1}$ denote the maximal number of signals of quality q_S when the PA's signal agrees with the board's (vote recommendation for) and let $\boxed{\bar{n}_2}$ denote the maximal number of signals of quality q_S when it disagrees (vote recommendation *against*). For any realization of signals of quality q_S let $\boxed{\delta}$ be the number of a-signals minus the number of b-signals, i.e., the difference against the board, and define $\delta = 0$ if there is no such signal.

Then maximal decision quality under the constraints given by the upper bounds $\bar{n}_1$ and $\bar{n}_2$ is reached if and only if the following two conditions are satisfied:

(C1) When the PA's signal is aligned with the board's, then $\bar{n}_1$ signals of quality q_S are generated and the proposal is accepted if $\delta < \frac{\ell_B+\ell_P}{\ell_S}$ and rejected if $\delta > \frac{\ell_B+\ell_P}{\ell_S}$ (and either accepted or rejected if $\delta = \frac{\ell_B+\ell_P}{\ell_S}$).

(C2) When the PA's signal is *against* board, then $\bar{n}_2$ additional signals are generated and the proposal is accepted if $\delta < \frac{\ell_B-\ell_P}{\ell_S}$ and rejected if $\delta > \frac{\ell_B-\ell_P}{\ell_S}$ (and either accepted or rejected if $\delta = \frac{\ell_B+\ell_P}{\ell_S}$).

Indeed, when the PA's signal agrees with the board's and $\bar{n}_1 = 0$, the proposal of the board must be accepted. When the PA's signal disagrees with the board's and $\bar{n}_2 = 0$, the proposal must be accepted if $q_B > q_P$ and rejected if $q_B < q_P$. When the PA's signal agrees with the board's and $\bar{n}_1 > 0$, then the condition for acceptance in C1 is equivalent to the following.

$$\begin{aligned}
\delta &< \frac{\ell_B + \ell_P}{\ell_S} \\
\delta \ell_S &< \ell_B + \ell_P \\
\delta \log(\frac{q_S}{1-q_S}) &< \log(\frac{q_B}{1-q_B}) + \log(\frac{q_P}{1-q_P}) \\
\left(\frac{q_S}{1-q_S}\right)^{\delta} &< \frac{q_B}{1-q_B} \cdot \frac{q_P}{1-q_P},
\end{aligned}$$

which is indeed the condition for state B being more likely than state A, conditional on the signal realizations. The analogous arguments hold for $\delta > \frac{\ell_B+\ell_P}{\ell_S}$ and Condition C2. Observe that in C2, $\delta < \frac{\ell_B-\ell_P}{\ell_S}$ is negative when $q_B < q_P$, requiring more b-signals than a-signals to accept the proposal, as it should be. In the knife-edge case where $\delta = \frac{\ell_B+\ell_P}{\ell_S}$, respectively $\delta = \frac{\ell_B-\ell_P}{\ell_S}$, both states are equally likely, conditional on the signal realizations, and hence it does not matter for decision quality which decision is made. It is an important part of the Conditions C1 and C2 that the maximal possible number of signals of quality q_S is generated, as any lower number of signals would lead to a strictly lower decision quality even if the decisions, given the realizations of signals, were according to the inequalities in C1 and C2.

We now set out to show for each case of SOM Lemma 2.3 that the claimed strategy satisfies the two conditions C1 and C2, with upper bounds $\bar{n}_1$ and $\bar{n}_2$ derived from SOM Lemma 2.2, and hence the maximum decision quality is attained. Moreover, we show that no shareholder has an incentive to deviate.

i. For $\frac{\ell_B+\ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B \geq \ell_P$, part i. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and $\bar{n}_2 = N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$. With the proposed strategy profile, we have $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ shareholders who play strategy UNIS, which makes $\bar{n}_1$ informed shareholders after the for recommendation. After the *against* recommendation, we have additional $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ informed shareholders who play strategy CAIS. This makes $N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor = \bar{n}_2$ informed shareholders after the *against* recommendation.

   After the *against* recommendation, the $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ Rubber-stamping shareholders vote *yes* and all other shareholders (those who play strategy CAIS and those who play strategy UNIS) invest in an own signal and vote according to it. Thus, the proposal is accepted if and only if $\delta < \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$. If $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ is even, then $\bar{n}_2$ is odd and thus δ is odd. If $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ is odd, then $\bar{n}_2$ is even and thus δ is even. Thus, for any realization of signals we have $|\delta - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor| \geq 1$ because exactly one of these integers is even and one of them is odd.[8] Taken together, if the vote recommendation is *against* and $\delta < \frac{\ell_B-\ell_P}{\ell_S}$, then $\delta < \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ and the proposal is accepted, as required by C2. If the vote recommendation is *against* and $\delta > \frac{\ell_B-\ell_P}{\ell_S}$, then $\delta > \frac{\ell_B-\ell_P}{\ell_S} \geq \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ and the proposal is rejected, as required by C2. Hence, Condition C2 is satisfied.

   Suppose the recommendation is for. Then, $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ shareholders vote *yes* (those playing strategy Rubber-stamping plus those playing strategy CAIS), while $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor = \bar{n}_1$ shareholders invest in a signal of quality q_S and vote according to it. Thus, the proposal is accepted if and only

[8] Hence, $\delta = \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ is not possible.

if $\delta < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$. Of the integers $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and δ exactly one is even and one is odd.[9] (δ is even iff $\bar{n}_1$ is even, while $\bar{n}_1 = N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$, whereas N is always odd.) If $\delta < \frac{\ell_B+\ell_P}{\ell_S}$, then $\delta < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and the proposal is accepted, as required by C1. If $\delta > \frac{\ell_B+\ell_P}{\ell_S}$, then $\delta > \frac{\ell_B+\ell_P}{\ell_S} \geq \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and the proposal is rejected, as required by C1. Hence, Condition C1 is satisfied.

Hence, the proposed strategy profile satisfies Conditions C1 and C2. Thus, decision quality is maximized under the constraints that the maximal numbers of informed shareholders after recommendations for and *against* must not exceed upper bounds $\bar{n}_1$ and $\bar{n}_2$, respectively. Deviating to a strategy that lowers decision quality is not an improvement as costs are assumed to be sufficiently small and hence cannot compensate any loss in decision quality. Potentially beneficial deviations must therefore either reduce costs without affecting decision quality, or increase decision quality (which requires relaxing at least one of the constraints that impose upper bounds $\bar{n}_1$ and $\bar{n}_2$).[10]

The shareholders who Rubber-stamp cannot reduce costs, as their costs are zero. Suppose shareholder i deviates from strategy Rubber-stamping and improves decision quality. She is pivotal after the for recommendation if the others' votes are split (there are $\frac{N+1}{2}$ *yes*-votes and $\frac{N+1}{2}$ *no*-votes). As $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor - 1$ others play strategy Rubber-stamping and $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ play strategy CAIS, there are $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - 1$ unconditional *yes*-votes after recommendation for. The remaining $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ shareholders play UNIS and are thus informed. Hence, i is pivotal after for if and only if $\delta = \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - 1$ (where δ is again the signal difference against the board among the informed). With Rubber-stamping, i would vote *yes*. To change the decision with a deviation, she must deviate to voting *no*. This deviation would be most likely to increase decision quality if she had received signal a. Decision quality would improve in that case iff $\ell_B + \ell_P < (\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - 1)\ell_S + 1\ell_S$, which is equivalent to $\frac{\ell_B+\ell_P}{\ell_S} < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$, but clearly not true. Hence, the Rubber-stamping shareholder i cannot improve decision quality after vote recommendation for.

The Rubber-stamping shareholder i is pivotal after the *against* recommendation if the others' votes are split. As $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor - 1$ others play strategy Rubber-stamping, there are $\lfloor \frac{\ell_B-\ell_P}{\ell_S} \rfloor - 1$ unconditional *yes*-votes after recommendation *against*. The remaining $N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ shareholders play strategy UNIS or strategy CAIS and are thus informed. Hence, i is pivotal after recommendation *against* if and only if $\delta = \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor - 1$. With Rubber-stamping, i would vote *yes*. To change the decision with a deviation she must deviate to voting *no*. This deviation would be most likely to increase decision quality if she had received signal a. Decision quality would improve in that case iff $\ell_B < \ell_P + (\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor - 1)\ell_S + 1\ell_S$, which is equivalent to $\frac{\ell_B-\ell_P}{\ell_S} < \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ and clearly not true, as $|\ell_B - \ell_P| = \ell_B - \ell_P$ for $q_B \geq q_P$ (which holds by assumption in Case i. of SOM Lemma 2.3). Therefore, a shareholder who Rubber-stamps cannot beneficially deviate.

The shareholders who play strategy UNIS bear the costs c. The following information-acquisition strategies have lower costs (than NotSubscribe-Invest): NotSubscribe-NotInvest, Subscribe-NotInvest, Subscribe-InvestIFFfor, and Subscribe-InvestIFF$against$, by the assumption that

[9]Hence, $\delta = \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ is not possible.

[10]We have to check for deviations that might potentially increase decision quality, although we have already established that decision quality in equilibrium is maximal under the constraints given by SOM Lemma 2.2. While the bounds of SOM Lemma 2.2 hold in any equilibrium, decision quality after deviations from a strategy profile is not *perse* restricted by the respective upper bound in SOM Lemma 2.2.

f is sufficiently lower than c. Suppose one shareholder deviates to a strategy that involves one of these information-acquisition strategies. If this shareholder uses NotSubscribe-NotInvest, Subscribe-NotInvest, or Subscribe-InvestIFF*against*, then the number of informed shareholders after for is smaller than $\bar{n}_1$, namely $\bar{n}_1 - 1$, and hence C1 is violated. If this shareholders uses Subscribe-InvestIFFfor, then the number of informed shareholders after recommendation *against* is smaller than $\bar{n}_2$, namely $\bar{n}_2 - 1$, and hence C2 is violated. Thus, any cost-saving deviation of a shareholder who plays strategy UNIS decreases decision quality. To improve decision quality, while at most $\bar{n}_1$ or $\bar{n}_2$ shareholders are informed after recommendation for or *against*, respectively, is not possible because decision quality is already maximal (by satisfying C1 and C2). To improve decision quality by deviating from strategy UNIS and increasing the number of informed shareholders after for or *against* is also not possible, as a shareholder who plays strategy UNIS is already informed in both cases. Thus, there is no beneficial deviation for a shareholder who plays strategy UNIS.

The shareholders who play strategy CAIS and hence use Subscribe-InvestIFF*against* could save costs by deviating and using the information-acquisition strategies NotSubscribe-NotInvest or Subscribe-NotInvest. Indeed, both would save costs by the assumption that f is sufficiently smaller than c. Deviating to the other information-acquisition strategies would increase costs by the same assumption and by the fact that the recommendation for is more likely than *against*: $q_B q_P > q_B(1 - q_P) + (1 - q_B)q_P$. If a shareholder deviates to a strategy involving NotSubscribe-NotInvest or Subscribe-NotInvest, this reduces decision quality because the number of signals after recommendation *against* is smaller than $\bar{n}_2$ and thus, Condition C2 is violated.

Hence, it remains to check whether a shareholder i who plays strategy CAIS can improve decision quality with some deviation. After recommendation for, she votes *yes* according to CAIS and hence has to vote *no* and be pivotal to affect the decision with her deviation. After recommendation for, $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - 1$ shareholders vote *yes* without being informed when excluding i (summing up those who play strategy Rubber-stamping and those who play strategy CAIS, minus one player). Pivotality after recommendation for requires that the votes of the $N - 1$ others are split, which occurs if and only if the signal difference of the $\bar{n}_1$ informed shareholders, δ_{-i}, satisfies $\delta_{-i} = \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - 1$ because this is the number of shareholders who vote *yes* without being informed. Conditioning on pivotality, voting *no* would be most likely to increase decision quality if the own signal was a. Decision quality would improve in that case iff $\ell_B + \ell_P < (\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - 1)\ell_S + 1\ell_S$, which is equivalent to $\frac{\ell_B+\ell_P}{\ell_S} < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and clearly not true. After the *against* recommendation, a shareholder i who plays strategy CAIS votes according to the own signal, as do all others who play strategy CAIS or strategy UNIS. The $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ Rubber-stamping shareholders vote *yes*. In order to affect the decision with her deviation, shareholder i has to vote *yes* after signal a or *no* after signal b and be pivotal. Pivotality after recommendation *against* requires that the votes of the $N - 1$ others are split, which occurs if and only if $\delta_{-i} = \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$, where δ_{-i} is again the signal difference against the proposal among the informed shareholders, excluding i. Conditional on pivotality after recommendation *against*, deviating to voting *yes* after signal a would improve decision quality iff $\ell_B > \ell_P + \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor \ell_S + 1\ell_S$, which is equivalent to $\frac{\ell_B-\ell_P}{\ell_S} > \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor + 1$, which is clearly not true. Deviating to *no* after signal b would improve decision quality in that case iff $\ell_B + 1\ell_S < \ell_P + \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor \ell_S$, which is equivalent to $\frac{\ell_B-\ell_P}{\ell_S} + 1 < \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$, which is clearly not true either. Thus, there is no beneficial deviation for a shareholder who plays strategy CAIS.

Taken together, above arguments imply that the proposed strategy profile is an equilibrium as no shareholder can profitably deviate. This equilibrium maximizes decision quality under the constraints of SOM Lemma 2.2 as it satisfies the Conditions C1 and C2 for these constraints. By SOM Lemma 2.2 there cannot be an equilibrium with looser constraints. Hence, among all equilibria it attains maximal decision quality. A Pareto-superior equilibrium would require lower costs for some shareholders without increasing the costs of any other shareholder. The total costs after vote recommendation for are $\bar{n}_1 c + \left(\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor \right) f$; and after vote recommendation $against$, total costs are $\bar{n}_2 c + \left(\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor \right) f$. Reducing the number of informed shareholders in either case to save costs c would lead to a violation of the Conditions C1 or C2. Reducing the number of shareholders who subscribe to the PA (by an integer $t \in \{0, 1, ..., \left(\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor \right) \}$) to decrease costs related to fee f would either lead to a decrease in the number of informed shareholders or would require a "compensation" by having t more shareholders who unconditionally invest in an own signal (Subscribe-NotInvest). The former violates the Conditions C1 or C2, the latter is even more costly as $tc > tf$ holds by assumption. Thus, the proposed equilibrium is Pareto-efficient.

Finally, decision quality for this equilibrium can be constructed by using the probability that under the equilibrium strategy profile the decision is correct, considering the four combinations of whether the board's signal is correct or wrong and whether the PA's signal is correct or wrong, together with the probability $\pi(l, k)$ that among l realizations with precision q_S at least k are correct: $\Pi^* = q_B q_P \pi(N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor) + (1-q_B)(1-q_P)\pi(N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor, \frac{N+1}{2}) + q_B(1-q_P)\pi(N - \lfloor \frac{\ell_B-\ell_P}{\ell_S} \rfloor, \frac{N+1}{2} - \lfloor \frac{\ell_B-\ell_P}{\ell_S} \rfloor) + (1-q_B) q_P \pi(N - \lfloor \frac{\ell_B-\ell_P}{\ell_S} \rfloor, \frac{N+1}{2})$.

ii. For $\frac{\ell_B+\ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B < \ell_P$, part i. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and $\bar{n}_2 = N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$. With the proposed strategy profile, we have $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ shareholders who play strategy UNIS, which makes $\bar{n}_1$ informed shareholders after the for recommendation. After the $against$ recommendation, we have additional $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ informed shareholders who play strategy CAIS. This makes $N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor = \bar{n}_2$ informed shareholders after the $against$ recommendation. The remaining $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ shareholders play Follow, i.e., they are uninformed and always vote according to the vote recommendation.

Suppose the vote recommendation is for. Then, $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ shareholders vote *yes* (those playing Follow plus those playing strategy CAIS), while $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor = \bar{n}_1$ shareholders invest into a signal of quality q_S and vote according to it. Thus, the proposal is accepted if and only if $\delta < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$, where δ is again the signal difference against the board. Of the integers $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and δ, exactly one is even and one is odd (see above). If $\delta < \frac{\ell_B+\ell_P}{\ell_S}$, then $\delta < \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and the proposal is accepted, as required by C1. If $\delta > \frac{\ell_B+\ell_P}{\ell_S}$, then $\delta > \frac{\ell_B+\ell_P}{\ell_S} \geq \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ and the proposal is rejected, as required by C1. Hence, Condition C1 is satisfied.

After the $against$ recommendation, the $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ shareholders who play Follow vote *no* and all other shareholders (who play strategy CAIS or strategy UNIS) invest into an own signal and vote according to it. Thus, the proposal is accepted if and only if $\tilde{\delta} > \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$, where $\boxed{\tilde{\delta}} = -\delta$ is the number of b-signals minus the number of a-signals (that is the vote difference in favor of the board). Of the integers $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ and $\tilde{\delta}$ exactly one is even and one is odd. ($\tilde{\delta}$ is even iff $\bar{n}_2$ is even, while $\bar{n}_2 = N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$, whereas N is always odd.) Thus, for any realization of signals we have $|\tilde{\delta} - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor| \geq 1$. The condition $\tilde{\delta} > \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ is hence equivalent to $\tilde{\delta} > \frac{|\ell_B-\ell_P|}{\ell_S}$ and (since $\ell_B < \ell_P$) to $-\delta > \frac{\ell_P-\ell_B}{\ell_S}$ and finally to $\delta < \frac{\ell_B-\ell_P}{\ell_S}$, as required by C2.

Hence, the proposed strategy profile satisfies Conditions C1 and C2. Potentially beneficial deviations must either reduce costs without affecting decision quality; or increase decision quality.

Consider a shareholder i who plays Follow. The only possibility to save costs is to deviate to information-acquisition strategy NotSubscribe-NotInvest. Hence, only the strategies Rubber-stamping and Protest save costs. Suppose i deviates to Rubber-stamping. Consider the realization of signals such that the vote recommendation is *against* and $\tilde{\delta} = \frac{|\ell_B - \ell_P|}{\ell_S} - 1$. By the assumption $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$, we have $\frac{|\ell_B - \ell_P|}{\ell_S} \leq \frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$ and $\bar{n}_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor \geq \frac{N+1}{2} > \frac{|\ell_B - \ell_P|}{\ell_S}$ such that this realization occurs with positive probability. (Indeed, after recommendation *against* $\tilde{\delta}$ attains all of the values in $\{-(N - \frac{|\ell_B - \ell_P|}{\ell_S}), -(N - \frac{|\ell_B - \ell_P|}{\ell_S}) + 2, ..., (N - \frac{|\ell_B - \ell_P|}{\ell_S}) - 2, N - \frac{|\ell_B - \ell_P|}{\ell_S}\}$ with positive probability.) Then excluding shareholder i we have $\frac{N-1}{2}$ *yes*-votes and the same number of *no*-votes. Shareholder i is pivotal and makes the proposal accepted when Rubber-stamping. This violates Condition C2 because the proposal is accepted although $\delta > \frac{\ell_B - \ell_P}{\ell_S}$. Indeed, $\tilde{\delta} = \frac{|\ell_B - \ell_P|}{\ell_S} - 1$ is equivalent to $-\delta = \frac{\ell_P - \ell_B}{\ell_S} - 1$ (because $\tilde{\delta} = -\delta$ and $\ell_B < \ell_P$) and further equivalent to $\delta = \frac{\ell_B - \ell_P}{\ell_S} + 1$.

Now, suppose that i deviates to strategy Protest. Consider the realization of signals such that the vote recommendation is for and $\delta = \frac{\ell_B + \ell_P}{\ell_S} - 1$. As $\bar{n}_1 \geq \frac{N+1}{2} \geq \frac{\ell_B + \ell_P}{\ell_S} - 1$, this realization occurs with positive probability. (Indeed, after recommendation for, δ attains all of the values in $\{-(N - \frac{\ell_B + \ell_P}{\ell_S}), -(N - \frac{\ell_B + \ell_P}{\ell_S}) + 2, ..., (N - \frac{\ell_B + \ell_P}{\ell_S}) - 2, N - \frac{\ell_B + \ell_P}{\ell_S}\}$ with positive probability.) Then, excluding shareholder i we have $\frac{N-1}{2}$ *yes*-votes and the same number of *no*-votes. Thus, shareholder i is pivotal and makes the proposal rejected when playing Protest. This violates Condition C1 because the proposal is rejected although $\delta = \frac{\ell_B + \ell_P}{\ell_S} - 1 < \frac{\ell_B + \ell_P}{\ell_S}$.

Now, suppose that i deviates from Follow in order to increase decision quality. To affect the decision, i must either vote *no* after recommendation for or vote *yes* after recommendation *against*, or both. After for, the $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor - 1$ other shareholders who Follow vote *yes*, as do the CAIS players. In sum, $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - 1$ shareholders vote *yes* when excluding i, while the $\bar{n}_1$ other shareholders play strategy UNIS and vote according to their signal. Hence, i is pivotal after recommendation for iff $\delta = \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - 1$. Voting *no* after recommendation for would be most likely to increase decision quality if i had received signal a. Decision quality would improve in that case iff $\ell_B + \ell_P < (\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - 1)\ell_S + 1\ell_S$, which is equivalent to $\frac{\ell_B + \ell_P}{\ell_S} < \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ and clearly not true.

After recommendation *against*, the $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor - 1$ shareholders who Follow vote *no*, while all $\bar{n}_2$ other shareholders (who play strategy CAIS or strategy UNIS) vote according to their signal. Hence, i is pivotal after recommendation *against* iff $\tilde{\delta} = \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor - 1$. Voting *yes* after recommendation *against* would be most likely to increase decision quality if i had received signal b. Decision quality would improve in that case iff $\ell_B + (\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor - 1)\ell_S + 1\ell_S > \ell_P$, which is equivalent to $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor \ell_S > \ell_P - \ell_B$ and with $\ell_B < \ell_P$ also equivalent to $\lfloor \frac{\ell_P - \ell_B}{\ell_S} \rfloor > \frac{\ell_P - \ell_B}{\ell_S}$, which is clearly not true.[11] Therefore, a shareholder who plays Follow cannot beneficially deviate.

The shareholders who play strategy UNIS bear the costs c. Any cost-saving deviation from strategy UNIS affects the number of informed shareholders either after recommendation for or after recommendation *against* or both. Hence, either Condition C1 or Condition C2 or both

[11] Notice that we don't apply the $\lfloor ... \rfloor$-operator to negative numbers.

are violated in case of such a deviation. Therefore, any cost-saving deviation of a shareholder from strategy UNIS decreases decision quality. To improve decision quality, while at most $\bar{n}_1$ or $\bar{n}_2$ shareholders are informed after recommendation for or *against*, respectively, is not possible because decision quality is already maximal (by satisfying C1 and C2). To improve decision quality by increasing the number of informed shareholders after recommendation for or *against* is also not possible for a shareholder who plays strategy UNIS as she is already informed in both cases. Thus, there is no beneficial deviation for a shareholder who plays strategy UNIS.

The shareholders who play stragegy CAIS and hence use Subscribe-InvestIFF*against* could save costs by the information-acquisition strategies NotSubscribe-NotInvest or Subscribe-NotInvest. If a shareholder deviates to a strategy involving one of these information-acquisition strategies (NotSubscribe-NotInvest or Subscribe-NotInvest), this reduces decision quality because the number of signals after recommendation *against* is smaller than $\bar{n}_2$ and thus Condition C2 is violated. Hence, it remains to check whether a shareholder i who plays strategy CAIS can improve decision quality with some deviation. After recommendation for, she votes *yes* according to CAIS. Moreover, the other shareholders who play strategy CAIS and those who play Follow vote *yes*, which makes $\lfloor\frac{\ell_B+\ell_P}{\ell_S}\rfloor - 1$ unconditional *yes*-votes when excluding i. The other $\bar{n}_1$ shareholders play strategy UNIS and follow their signal. Shareholder i has to vote *no* and be pivotal to affect the decision after recommendation for with her deviation. Pivotality after recommendation for requires that the votes of the $N-1$ others are split, which occurs if and only if the signal difference against the proposal of the (other) informed shareholders, δ_-i, satisfies $\delta_-i = \lfloor\frac{\ell_B+\ell_P}{\ell_S}\rfloor - 1$ because this is the number of (other) shareholders who vote *yes* without being informed (they play Follow or strategy CAIS), excluding i. Voting *no* would be most likely to increase decision quality when the own signal was a. Decision quality would improve in that case iff $\ell_B+\ell_P < (\lfloor\frac{\ell_B+\ell_P}{\ell_S}\rfloor-1)\ell_S+1\ell_S$, which is equivalent to $\frac{\ell_B+\ell_P}{\ell_S} < \lfloor\frac{\ell_B+\ell_P}{\ell_S}\rfloor$ and clearly not true.

After the *against* recommendation, $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ shareholders who play Follow vote *no*. A shareholder i who plays strategy CAIS votes according to her signal as do all others who play strategy CAIS or UNIS. In order to affect the decision with her deviation, shareholder i has has to vote *yes* after signal a or *no* after signal b and be pivotal. Pivotality after recommendation *against* requires that the votes of the $N-1$ others are split, which occurs if and only if $\tilde{\delta}_{-i} = \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$, where $\tilde{\delta}_{-i}$ is again the number of b-signals minus the number of a-signals, i.e., the signal difference in favor of the proposal, among the informed shareholders, excluding i. Deviating to voting *yes* after signal a would improve decision quality in that case iff $\ell_B+\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor\ell_S > \ell_P+1\ell_S$, which is equivalent to $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor\ell_S > \ell_P-\ell_B+1\ell_S$ and by $\ell_B<\ell_P$ to $\lfloor\frac{\ell_P-\ell_B}{\ell_S}\rfloor > \frac{\ell_P-\ell_B}{\ell_S}+1$, which is clearly not true. Deviating to voting *no* after signal b would improve decision quality in case of pivotality iff $\ell_B+\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor\ell_S+1\ell_S < \ell_P$, which is equivalent to $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor\ell_S+1\ell_S < \ell_P-\ell_B$ and by $\ell_B<\ell_P$ to $\lfloor\frac{\ell_P-\ell_B}{\ell_S}\rfloor+1 < \frac{\ell_P-\ell_B}{\ell_S}$, which is clearly not true either.

Thus, there is no beneficial deviation for a shareholder who plays strategy CAIS either.

Taken together, the above arguments imply that the proposed strategy profile is an equilibrium, as there is no shareholder who can beneficially deviate. Moreover, it maximizes decision quality among all potential equilibria, as it satisfies the Conditions C1 and C2 for the constraints given by SOM Lemma 2.2. There cannot be a Pareto-superior equilibrium with higher decision quality. It remains to verify that there is no Pareto-superior equilibrium with the same decision quality, but lower costs. Any reduction of signal costs c would lead to a violation of either

Condition C1 or C2 or both. Any reduction of costs related to the fee f would either also induce such a violation or would have to be compensated by shareholders who unconditionally invest c into an own signal, which is more costly by the assumption that f is sufficiently smaller than c. Hence, the equilibrium is Pareto-efficient.

iii. For $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$ and $\ell_B \geq \ell_P$, part iii. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = 0$ and $\bar{n}_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$. With the proposed strategy profile, we have $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders who play strategy CAIS and $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ who Rubber-stamp, which makes $\bar{n}_1 = 0$ informed shareholders after the for recommendation. After the *against* recommendation, we have $\bar{n}_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ informed shareholders who play strategy CAIS.

As $\bar{n}_1 = 0$, we have $\delta = 0$ after recommendation for. Since $\delta = 0 < \frac{\ell_B + \ell_P}{\ell_S}$, Condition C1 requires that the proposal is accepted. With the proposed strategy profile all N shareholders vote *yes* after recommendation for such that the proposal is always accepted. Hence, Condition C1 is satisfied.

After the *against* recommendation, the $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders who play strategy Rubber-stamping vote *yes*, and all other shareholders (who play strategy CAIS) invest into an own signal and vote according to it. Thus, the proposal is accepted if and only if $\delta < \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$, where δ is again the number of a-signals minus the number of b-signals. Of the integers $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ and δ exactly one is even and one is odd.[12] Thus, for any realization of signals we have $|\delta - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor| \geq 1$. The condition $\delta < \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ is hence equivalent to $\delta < \frac{|\ell_B - \ell_P|}{\ell_S}$ and (since $\ell_B \geq \ell_P$) to $\delta < \frac{\ell_B - \ell_P}{\ell_S}$, as required by C2.

Hence, the proposed strategy profile satisfies Conditions C1 and C2 and hence maximizes decision quality under the constraints that impose upper bounds $\bar{n}_1$ and $\bar{n}_2$. Potentially beneficial deviations must either reduce costs without affecting decision quality, or increase decision quality.

We describe the proposed strategy profile by $\hat{\sigma}^{\mu,\nu}$, where $\mu = \bar{n}_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play strategy CAIS and the remaining $\nu = \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play strategy Rubber-stamping. We have to show that no player has a deviation incentive. We begin with a shareholder who plays strategy Rubber-stamping, then turn to a shareholder who plays strategy CAIS.

A shareholder j who plays strategy Rubber-stamping does not acquire any information and votes *yes*. In the strategy profile $\hat{\sigma}^{\mu,\nu}$, a Rubber-stamping player is pivotal iff the PA has recommended *against* and among the shareholders who play strategy CAIS, $\nu - 1$ more have received signal a than have received b.

In order to improve decision quality by a deviation of shareholder j, this deviation must change the voting outcome, i.e., j must vote *no* in some instance of pivotality. The most attractive starting point for voting *no* occurs when the PA recommends *against* and the own signal is a. Suppose that shareholder j would then vote *no*. This is a strict improvement of decision quality iff $(\nu - 1)\ell_S + \ell_P + \ell_S > \ell_B$, or equivalently iff $\nu > \frac{\ell_B - \ell_P}{\ell_S}$. However, setting $\nu = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$ precludes this. Hence, a Rubber-stamping shareholder cannot improve decision quality by deviating when $\mu = \bar{n}_2$. Considering that any deviation on the information-acquisition stages moreover means that more costs are incurred, there is no beneficial deviation for any Rubber-stamping shareholder.

[12] Hence, $\delta = \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ is not possible.

Let us now turn to shareholders who play strategy CAIS. First, we show that there is no deviation that improves decision quality. We then proceed by showing that all deviations with identical decision quality come at the same or higher costs. Together, these two assertions then prove that no individual deviation can improve utility.

Concerning decision quality, it is obvious that improvements are impossible when the PA's recommendation is for, as in this case all shareholders vote *yes* and no shareholder is pivotal, given that $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor < \frac{N-1}{2}$ (and hence, the number of *yes*-votes of CAIS players is $N - \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor > \frac{N+1}{2} + 1$). In the special case of $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor = \frac{N-1}{2}$, a CAIS player i is pivotal after recommendation for. A deviation to voting *no* would be most likely to increase decision quality if the own signal was a. It would improve decision quality in that case iff $\ell_B + \ell_P < 1\ell_S$, which is precluded by the assumption that $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor \geq \frac{N+1}{2} > 1$. Thus, a deviation may only improve decision quality by changing the outcome when the PA recommends *against* and the shareholder is pivotal. Pivotality implies that among the $\mu - 1$ other shareholders who play strategy CAIS, the difference between the numbers of a and b-signals must equal ν, the number of Rubber-stamping shareholders. Those signals are thus split into $\frac{N-1}{2}$ a-signals and $\frac{N-1}{2} - \nu$ b-signals. Shareholder i may improve decision quality by always voting *yes* in these instances if and only if $l_B + (\frac{N-1}{2} - \nu) \cdot \ell_S > l_P + \frac{N-1}{2}\ell_S + \ell_S$, which is equivalent to $\ell_B - \ell_P > (\nu + 1)\ell_S$ or $\frac{\ell_B - \ell_P}{\ell_S} > \nu + 1$, and thus to $\nu < \frac{\ell_B - \ell_P}{\ell_S} - 1$. This, however, is precluded by $\nu = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$. Similarly, shareholder i may improve decision quality by always voting *no* in these instances if and only if $l_B + (\frac{N-1}{2} - \nu) \cdot \ell_S + \ell_S < l_P + \frac{N-1}{2}\ell_S$, which turns out to be equivalent to $\nu > \frac{\ell_B - \ell_P}{\ell_S} + 1$, which is again at odds with $\nu = \lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor$. Thus, it is impossible to improve decision quality by a deviating strategy.

The only possibility remaining for a CAIS player in order to improve utility is thus to look for strategies that attain the same decision quality as $\hat{\sigma}^{\mu,\nu}$, but at lower costs. As the costs associated with $\hat{\sigma}^{\mu,\nu}$ are the fee f as well as costs c in case of the PA recommending *against*, there are two possibilities for reducing costs: the first one would be to get rid of conditional costs c, which, however, would result in reduced decision quality, as such a deviation violates Condition C2. The second alternative is to get rid of the fee f, by always voting according to the own signal. While this preserves the maximal decision quality and saves fee f, it comes at additional costs c when the PA recommends for. (Thus, in expectation, costs decrease by f, but increase by c times the probability of a for recommendation (which is $q_B q_P + (1 - q_B)(1 - q_P)$.) As we have assumed that f is sufficiently smaller than c, this is an overall increase of costs.

Taken together, we have shown that there is no utility improving deviation for the CAIS players, and neither for the Rubber-stamping players. Hence, the proposed strategy profile is an equilibrium. Moreover, it maximizes decision quality among all potential equilibria, as it satisfies the Conditions C1 and C2 for the constraints given by SOM Lemma 2.2. There cannot be a Pareto-superior equilibrium with higher decision quality. It remains to verify that there is no Pareto-superior equilibrium with the same decision quality, but lower costs. Any reduction of signal costs c would lead to a violation of Condition C2. Any reduction of costs related to the fee f would either also induce such a violation or had to be compensated by shareholders who unconditionally invest in a signal c, which is more costly by the assumption that f is sufficiently smaller than c. Hence, the strategy profile is Pareto-efficient.

Finally, decision quality follows again from the probability that under the equilibrium strategy profile the decision is correct, considering the four combinations of whether the board's signal is correct or wrong and whether the PA's signal is correct or wrong, together with the probability $\pi(l, k)$ that among l realizations with precision q_S at least k are correct: $\Pi(\sigma) =$

$q_Bq_P + q_B(1-q_P)\pi(z_2, z_2 - \frac{N-1}{2}) + (1-q_B)q_P\pi(z_2, \frac{N+1}{2}) > q_B.$

iv. For $\frac{|\ell_B-\ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B+\ell_P}{\ell_S}$ and $\ell_B < \ell_P$, part ii. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = 0$ and $\bar{n}_2 = N - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$. With the proposed strategy profile, we have $N - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ shareholders who play strategy CAIS and $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ who play strategy Protest, which makes $\bar{n}_1 = 0$ informed shareholders after the for recommendation. After the *against* recommendation, we have $\bar{n}_2 = N - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ informed shareholders who play strategy CAIS.

As $\bar{n}_1 = 0$, we have $\delta = 0$ after recommendation for. Since $\delta = 0 < \frac{\ell_B+\ell_P}{\ell_S}$, Condition C1 requires that the proposal is accepted. With the proposed strategy profile, there are $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ *no*-votes by the Protest players versus $N - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ *yes*-votes by the CAIS players after recommendation for. By the assumption $\frac{|\ell_B-\ell_P|}{\ell_S} < \frac{N+1}{2}$, we have $\bar{n}_2 = N - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor \geq \frac{N+1}{2} > \frac{|\ell_B-\ell_P|}{\ell_S}$ such that the proposal is always accepted after recommendation for, as required by Condition C1. After the *against* recommendation, the $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ shareholders who play strategy Protest vote unconditionally *no* and all other shareholders (who play strategy CAIS) invest into an own signal and vote according to it. Thus, the proposal is accepted if and only if $\tilde{\delta} > \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$, where $\tilde{\delta} = -\delta$ is again the number of b-signals minus the number of a-signals, i.e., the signal difference in favor of the proposal. Of the integers $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ and $\tilde{\delta}$, exactly one is even and one is odd. Thus, for any realization of signals we have $|\tilde{\delta} - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor| \geq 1$. The condition $\tilde{\delta} > \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ is hence equivalent to $\tilde{\delta} > \frac{|\ell_B-\ell_P|}{\ell_S}$ and (since $\ell_B < \ell_P$) to $-\delta > \frac{\ell_P-\ell_B}{\ell_S}$ and finally to $\delta < \frac{\ell_B-\ell_P}{\ell_S}$, as required by C2.

Hence, the proposed strategy profile satisfies Conditions C1 and C2. Potentially beneficial deviations must either reduce costs without affecting decision quality, or increase decision quality.

Consider a shareholder i who plays Protest. There is no possibility to save costs as all other information-acquisition strategies are more costly than NotSubscribe-NotInvest. Now, suppose i deviates from strategy Protest in order to increase decision quality.

After vote recommendation for, a majority of $\bar{n}_2 = N - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor \geq \frac{N+1}{2}$ shareholders (who play strategy CAIS) vote *yes*. Hence, shareholder i who plays Protest is not pivotal and hence cannot affect decision quality.

After recommendation *against*, the $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor - 1$ other shareholders who play strategy Protest vote *no*, while all $\bar{n}_2$ shareholders who play strategy CAIS vote according to their signal. Hence, i is pivotal after recommendation *against* iff $\tilde{\delta} = \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor - 1$, where again $\tilde{\delta} = -\delta$. Under playing Protest, i would vote *no*. To effectively deviate, i has to be pivotal and vote *yes*. Voting *yes* after recommendation *against* would be most likely to increase decision quality if i had received signal b. Decision quality would improve in that case iff $\ell_B + (\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor - 1)\ell_S + 1\ell_S > \ell_P$, which is equivalent to $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor\ell_S > \ell_P - \ell_B$ and with $\ell_B < \ell_P$ also equivalent to $\lfloor\frac{\ell_P-\ell_B}{\ell_S}\rfloor > \frac{\ell_P-\ell_B}{\ell_S}$, which is clearly not true. Therefore, a shareholder who plays strategy Protest cannot beneficially deviate.

It remains to show that shareholders who play strategy CAIS cannot beneficially deviate. First, we address possibilities to improve by reducing costs at the same decision quality, then we turn to deviations intended to increase decision quality.

As the costs associated with the proposed strategy profile are the fee f as well as costs c in case of the PA recommending *against*, there are two possibilities for reducing costs: the first one would be to get rid of conditional costs c, which, however, would result in reduced decision quality, as it violates Condition C2. The second alternative is to get rid of the fee f, by always voting according to the individual signal. While this preserves the optimal decision quality and saves fee f, it comes with additional costs c when the PA recommends for. As we have assumed that f is sufficiently smaller than c, this is an overall increase of costs.

Consider a shareholder i who plays strategy CAIS. After recommendation for, the other CAIS players vote *yes* while the Protest players vote *no* such that we have $N - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor - 1$ *yes*-votes when excluding i, and $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ no-votes. If $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor < \frac{N-1}{2}$, then there is a majority voting *yes* and i is not pivotal. In that case, i cannot affect decision quality after recommendation for. Otherwise, we have $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor = \frac{N-1}{2}$. Then i is pivotal after recommendation for. Under strategy CAIS, i votes *yes* and the proposal is accepted. To affect the outcome after recommendation for by deviating, i has to vote *no* and be pivotal under this deviation. Such a deviation would be most likely to increase decision quality if i had received signal a. This deviation would increase decision quality in that case iff $\ell_B + \ell_P < 1\ell_S$, which is equivalent to $\frac{\ell_B+\ell_P}{\ell_S} < 1$ and precluded by the assumption $\frac{\ell_B+\ell_P}{\ell_S} \geq \frac{N+1}{2} > 1$.

After recommendation *against*, shareholder i who plays strategy CAIS votes according to the own signal, as do all others who play strategy CAIS, while the $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$ Protest players vote *no*. In order to affect the decision with her deviation, shareholder i has to vote *yes* after signal a or *no* after signal b and be pivotal. Pivotality after recommendation *against* requires that the votes of the $N-1$ others are split, which occurs if and only if $\tilde{\delta}_{-i} = \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor$, where $\tilde{\delta}_{-i}$ is again the number of b-signals minus the number of a-signals, i.e., the signal difference in favor of the proposal, among the informed shareholders, excluding i. Deviating to voting *yes* after signal a would improve decision quality in that case iff $\ell_B + \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor\ell_S > \ell_P + 1\ell_S$, which is equivalent to $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor\ell_S > \ell_P - \ell_B + 1\ell_S$ and by $\ell_B < \ell_P$ to $\lfloor\frac{\ell_P-\ell_B}{\ell_S}\rfloor > \frac{\ell_P-\ell_B}{\ell_S} + 1$, which is clearly not true. Deviating to *no* after signal b would improve decision quality in case of pivotality iff $\ell_B + \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor\ell_S + 1\ell_S < \ell_P$, which is equivalent to $\lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor\ell_S + 1\ell_S < \ell_P - \ell_B$ and by $\ell_B < \ell_P$ to $\lfloor\frac{\ell_P-\ell_B}{\ell_S}\rfloor + 1 < \frac{\ell_P-\ell_B}{\ell_S}$, which is clearly not true either. Hence, a shareholder who plays strategy CAIS cannot beneficially deviate either.

Taken together, we thus have shown that there is no utility improving deviation strategy for the CAIS players, and neither for the Protest players. Hence, the proposed strategy profile is an equilibrium. Moreover, it maximizes decision quality among all potential equilibria, as it satisfies the Conditions C1 and C2 for the constraints given by SOM Lemma 2.2. There cannot be a Pareto-superior equilibrium with higher decision quality. It remains to verify that there is no Pareto-superior equilibrium with the same decision quality, but lower costs. Any reduction of signal costs c would lead to a violation of Condition C2. Any reduction of costs related to the fee f would either also induce such a violation or would have to be compensated by shareholders who unconditionally invest c into an own signal, which is more costly by the assumption that f is sufficiently smaller than c. Hence, the equilibrium is Pareto-efficient.

Finally, decision quality follows again from the probability that under the equilibrium strategy profile the decision is correct, considering the four combinations of whether the board's signal is correct or wrong and whether the PA's signal is correct or wrong, together with the probability $\pi(l,k)$ that that among l realizations with precision q_S at least k are correct: The decision quality in this profile is $\Pi^* = q_B q_P + q_B(1-q_P)\pi(N - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor, \frac{N+1}{2}) + (1-q_B)q_P\pi(N - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor, \frac{N+1}{2} - \lfloor\frac{|\ell_B-\ell_P|}{\ell_S}\rfloor)$.

v. For $\frac{|\ell_B - \ell_P|}{\ell_S} \geq \frac{N+1}{2}$ and $\ell_B \geq \ell_P$, part iii. of SOM Lemma 2.2 applies (with $\ell_B > \ell_P$, since zero cannot be larger than or equal to $\frac{N+1}{2}$). Hence, $\bar{n}_1 = \bar{n}_2 = 0$. By definition the signal difference is $\delta = 0$ when there are no informed shareholders. Hence, Condition C1 requires to accept the board's proposal after recommendation for iff $0 = \delta < \frac{\ell_B + \ell_P}{\ell_S}$, which is always satisfied. Condition C2 requires to accept the proposal after recommendation *against* iff $0 = \delta < \frac{|\ell_B - \ell_P|}{\ell_S}$, which is also always satisfied, as we have $\frac{|\ell_B - \ell_P|}{\ell_S} \geq \frac{N+1}{2} > 0$. Hence, to maximize decision quality, the proposal must be accepted.

In the proposed strategy profiles, $N' \in \{\frac{N+1}{2}, .., N\}$ shareholders play strategy Rubber-stamping and $N - N'$ play strategy Protest. The N' Rubber-stampers vote *yes*, while the $N - N'$ shareholders who play stragegy Protest vote *no*. As $N' \geq \frac{N+1}{2} > N - N'$, the proposal is accepted. Hence, any of these strategy profiles satisfies C1 and C2 and thus maximizes decision quality.

It remains to show that there are no profitable deviations. First, neither a Rubber-stamping shareholder nor a Protest player can save costs, as NotSubscribe-NotInvest has no costs. A Protest playing shareholder cannot improve decision quality as she is not pivotal. Indeed, there are at least $N' \geq \frac{N+1}{2}$ *yes*-votes. If $N' \geq \frac{N+1}{2} + 1$, then no Rubber-stamping shareholder i is pivotal either, as there are at least $\frac{N+1}{2}$ *yes*-votes without i. Otherwise, i.e., if $N' = \frac{N+1}{2}$, a Rubber-stamping shareholder i is pivotal, as there are $\frac{N-1}{2}$ *yes*-votes, excluding i, and the same number of *no*-votes. To change the outcome by a deviation, shareholder i has to vote *no*. This deviation would be most likely to increase decision quality after the recommendation *against* if i had received signal a. This deviation would improve decision quality in that case iff $\ell_B < \ell_P + 1\ell_S$, which is equivalent to $\frac{\ell_B - \ell_P}{\ell_S} < 1$, but precluded by $\frac{\ell_B - \ell_P}{\ell_S} = \frac{|\ell_B - \ell_P|}{\ell_S} \geq \frac{N+1}{2} > 1$ (for $\ell_B \geq \ell_P$).

Therefore, the proposed strategy profiles are equilibria. Moreover, they are not Pareto-dominated by any other equilibrium because they not only maximize decision quality under the constraints of SOM Lemma 2.2, but also lead to no costs.

Decision quality in such a strategy profile equals the probability that the board has received the correct signal, i.e., $\Pi^* = q_B$.

vi. For $\frac{|\ell_B - \ell_P|}{\ell_S} \geq \frac{N+1}{2}$ and $\ell_B < \ell_P$, part iii. of SOM Lemma 2.2 applies. Hence, $\bar{n}_1 = \bar{n}_2 = 0$. By definition the signal difference is $\delta = 0$ when there are no informed shareholders. Hence, Condition C1 requires to accept the board's proposal after recommendation for iff $0 = \delta < \frac{\ell_B + \ell_P}{\ell_S}$, which is always satisfied. Condition C2 requires to reject after recommendation *against* iff $\delta < \frac{\ell_B - \ell_P}{\ell_S}$ which is also always satisfied, as we have $\delta = 0$ and $\ell_B - \ell_P < 0$. Hence, to maximize decision quality, the proposal must be accepted if and only if the vote recommendation is for.

In the proposed strategy profiles, $\frac{N-1}{2}$ shareholders play strategy Rubber-stamping and $\frac{N-1}{2}$ play strategy Protest, while one shareholder plays Follow. The votes of the Rubber-stamping shareholders and the votes of the Protest players cancel each other out, while the shareholder who plays Follow determines the outcome according to the vote recommendation. Hence, this strategy profile satisfies both C1 and C2 and thus maximizes decision quality.

It remains to show that there are no profitable deviations. First, neither a Rubber-stamping shareholder nor a Protest player can save costs, as NotSubscribe-NotInvest does not involve any costs. A Rubber-stamping player i cannot improve decision quality after recommendation *against* because she is not pivotal after it. (Indeed, after recommendation *against*, there are $\frac{N-1}{2} + 1$ *no*-votes by the Protest-voters and the Follow-voter.) To change the outcome, shareholder i has to vote *no* after recommendation for. This deviation would be most likely to increase decision qualityif i had received signal a. This deviation would improve decision quality in that case iff $\ell_B + \ell_P < 1\ell_S$, which is equivalent to $\frac{\ell_B+\ell_P}{\ell_S} < 1$, but precluded by $\frac{\ell_B+\ell_P}{\ell_S} > \frac{|\ell_B-\ell_P|}{\ell_S} \geq \frac{N+1}{2} > 1$. Hence, a Rubber-stamping shareholder cannot beneficially deviate.

A Protest playing shareholder i cannot improve decision quality after recommendation for as she is not pivotal. (Indeed, after recommendation for, there are at least $\frac{N-1}{2} + 1$ *yes*-votes and $\frac{N-1}{2} - 1$ *no*-votes when excluding i.) To change the outcome, shareholder i has to vote *yes* after recommendation *against*. This deviation is most attractive if i had received signal b. This deviation would improve decision quality in that case iff $\ell_B + 1\ell_S > \ell_P$, which is equivalent to $1 > \frac{\ell_P-\ell_B}{\ell_S}$, but precluded by $\frac{\ell_P-\ell_B}{\ell_S} = \frac{|\ell_B-\ell_P|}{\ell_S} \geq \frac{N+1}{2} > 1$ (for $\ell_B < \ell_P$).

Now, consider the shareholder who plays Follow. She could save costs, which consist of the fee f, only by switching to information-acquisition strategy NotSubscribe-NotInvest. Without subscribing to the vote recommendation, i cannot condition the voting behavior on it. Therefore, either C1 or C2 or both are violated when saving costs. Hence, decision quality suffers, and this type of deviation is not beneficial. Now, suppose that i who plays Follow deviates in order to increase decision quality. To change the outcome, shareholder i has to vote *yes* after recommendation *against* and/or *no* after recommendation for. A deviation including the former would be most likely to increase decision quality if i had invested into an own signal and received signal b. This deviation would improve decision quality in that case iff $\ell_B + 1\ell_S > \ell_P$, which is equivalent to $1 > \frac{\ell_P-\ell_B}{\ell_S}$, but precluded as shown already above. A deviation including the latter behavior, i.e., voting *no* after recommendation for, would be most likely to increase decision quality if i had invested into an own signal and received signal a. This deviation would improve decision quality in that case iff $\ell_B + \ell_P < 1\ell_S$, which is equivalent to $\frac{\ell_P+\ell_B}{\ell_S} < 1$, but precluded as shown already above.

Therefore, the proposed strategy profile is an equilibrium. Moreover, by satisfying C1 and C2 for $\bar{n}_1 = \bar{n}_2 = 0$ it maximizes decision quality under the constraints given by SOM Lemma 2.2. Costs for all shareholders together are f. Any strategy profile with lower costs necessarily violates C1 or C2 and hence induces lower decision quality. Thus, the proposed strategy profile cannot be Pareto-dominated by any other equilibrium.

Decision quality in such a strategy profile equals the probability that the PA has received the correct signal, i.e., $\Pi^* = q_P$.

□

2.3 Research Incentives Increase with a Proxy Advisor

Analogously to the analysis of symmetric equilibria in Section 3, the negative result obtained without a PA can be mitigated when a PA is admitted. Again, the basic idea is that the PA's recommendation is used as a condition to invest in own research like in information-acquisition strategy Subscribe-InvestIFF*against*, which constitutes CAIS. While this was true for all shareholders in Proposition 2 in a certain parameter range, we now find that in much larger areas of the parameter space some, but not all, shareholders use this strategy. Based on SOM Lemmata 2.1, 2.2 and 2.3, the following result summarizes the number of informed and conditionally informed shareholders in asymmetric equilibria with a PA.

Proposition 2.2 (ASYM with PA). *Let Assumption PAF hold. Let costs $c > 0$ be arbitrarily small and let fee f be sufficiently smaller. Suppose there is a PA with information quality q_P such that*

$\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2}$. *Then there exists an equilibrium in which the number of shareholders who invest or conditionally invest in own research is* z_2 $(\geq \frac{N+1}{2})$, *with* $z_2 := N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$. *In particular, in the Pareto-efficient equilibria the following holds:*

i. If $\frac{\ell_B + \ell_P}{\ell_S} < \frac{N+1}{2}$, *then* $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor \approx 2\frac{\ell_P}{\ell_S}$ *shareholders conditionally invest in own research, playing strategy CAIS, and* $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ *shareholders unconditionally invest in own research, playing strategy UNIS.*

ii. If $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$, *then* $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ *shareholders conditionally invest in own research, playing strategy CAIS, and no shareholder unconditionally invests in own research.*

iii. Otherwise, i.e., if $\frac{N+1}{2} \leq \frac{|\ell_B - \ell_P|}{\ell_S}$, *no shareholder invests in own research.*

Proof. To prove SOM Proposition 2.2, we use SOM Lemma 2.3 established in Section 2.2.

Suppose $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2}$. Then either one of the cases i.-iv. of SOM Lemma 2.3 applies. In each of these, there is an equilibrium where the number of CAIS players plus the number of UNIS players equals $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$. Indeed, in cases i. and ii. of SOM Lemma 2.3, we have $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor + N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders who invest or conditionally invest in own research. In cases iii. and iv. of SOM Lemma 2.3 $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders play strategy CAIS and hence conditionally invest.

Now, we address part i. For $\frac{|\ell_B + \ell_P|}{\ell_S} < \frac{N+1}{2}$, either case i. or ii. of SOM Lemma 2.3 applies. In both cases, the assertion follows directly from the lemma. In the Pareto-efficient equilibria, $\lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor \approx 2\frac{\ell_P}{\ell_S}$ shareholders conditionally invest in own research, playing strategy CAIS, and $N - \lfloor \frac{\ell_B + \ell_P}{\ell_S} \rfloor$ shareholders unconditionally invest in own research, playing UNIS.

Second, we address part ii. For $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2} \leq \frac{\ell_B + \ell_P}{\ell_S}$, either case iii. or iv. of SOM Lemma 2.3 applies. In both cases, the assertion follows directly from the lemma. ($N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders conditionally invest in own research, playing strategy CAIS in the Pareto-efficient equilibrium.)

Finally, we address part iii. For $\frac{N+1}{2} \leq \frac{|\ell_B - \ell_P|}{\ell_S}$, case v. of Lemma 2.3 applies. The assertion of part iii. follows directly from it. □

The proposition states that there exists an equilibrium in which more than half of all shareholders invest or conditionally invest in own research, given that the PA's information quality is not too different from the board's. More precisely, the condition is $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2}$, which means that the difference between the information quality of the PA and the information quality of the board must not exceed the aggregated information quality of about half of all shareholders together. (Observe that the larger the number of shareholders N, the less demanding this assumption is.) Under this condition, $z_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor$ shareholders invest or conditionally invest in research. Compared to the number of shareholders who invest in research without a PA (SOM Proposition 2.2), we have $z_2 \geq z_1$, or $N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor \geq N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$, with a strict difference, e.g., if $q_P > q_S$. Hence, due to the presence of the PA, there are more shareholders who invest or conditionally invest under these conditions. The term $\frac{|\ell_B - \ell_P|}{\ell_S}$ measures the difference between the information qualities of the board and the PA relative to a single shareholder's. The larger this difference, the lower the number z_2 of shareholders who invest or conditionally invest. It starts with $z_2 = N$ when the difference is close to zero, i.e., $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor = 0$, and decreases down to $z_2 = \frac{N+1}{2}$ when $\lfloor \frac{\ell_B - \ell_P}{\ell_S} \rfloor = \frac{N-1}{2}$. Hence, the number of shareholders who invest or conditionally invest is maximized for $q_P \approx q_B$.

SOM Proposition 2.2 then further characterizes the Pareto-efficient equilibria distinguishing three cases. The lower panel of Figure 2.1 illustrates for each of the three cases, how many shareholders are investing in research, followed by how many are conditionally investing in research. The first two cases both satisfy assumption $\frac{|\ell_B-\ell_P|}{\ell_S} < \frac{N+1}{2}$, i.e., that the PA's and the board's signal quality are not too different. The cases differ in that the joint decision quality of PA and board is assumed to be smaller than the decision quality of half of all shareholders in case i., i.e., $\frac{\ell_B+\ell_P}{\ell_S} < \frac{N+1}{2}$, but not in case ii. In the lower panel of Figure 2.1, case i. corresponds to bottom left triangle and case ii. corresponds to the area between the two parallel lines.[13] In case i., a majority of $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ shareholders unconditionally invests in own research. In case ii., a larger majority of $N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ shareholders conditionally invests in own research (while in both cases in total $z_2 = N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ shareholders invest or conditionally invest).

Moreover, the upper panel of SOM Figure 2.1 illustrates how many shareholders are investing in own research in the Pareto-efficient equilibria for the benchmark setting in which the PA is not admitted, as established by SOM Proposition 2.1. We can observe the following effects when admitting a PA: in some areas (e.g., in the lower right), the number of shareholders who invest or conditionally invest in own research remains zero. In some other areas (e.g., in the upper right) this number increases from 0 to $N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ who conditionally invest in own research. In the most interesting regions (in the lower left and center), the number of shareholders changes from $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ who invest without a PA, to $N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ of whom either all conditionally invest or of whom $N - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ invest and the other $\approx 2\frac{\ell_P}{\ell_S}$ conditionally invest (cases i. and ii. of SOM Proposition 2.2). Finally, there is the area in the upper left, where the number of shareholders who invest reduces from $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ to 0.[14]

The change in research incentives can be seen even more specifically, when returning to Figure 3 of the main text. In the lower panel the three cases of SOM Proposition 2.2 are further split into six regions, according to whether $\ell_B > \ell_P$, i.e., whether we are below or above the 45-degree line. For instance, in the bottom left triangle, defined by $\frac{\ell_B+\ell_P}{\ell_S} < \frac{N+1}{2}$ and $\ell_B \geq \ell_P$ (case i. of SOM Lemma 2.3), in the Pareto-efficient equilibrium $\lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor$ play strategy Rubber-stamping, $\lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor - \lfloor \frac{|\ell_B-\ell_P|}{\ell_S} \rfloor \approx 2\frac{\ell_P}{\ell_S}$ play strategy CAIS, and $N - \lfloor \frac{\ell_B+\ell_P}{\ell_S} \rfloor$ play strategy UNIS. In comparison to the Pareto-efficient equilibrium without a PA, in which $\lfloor \frac{\ell_B}{\ell_S} \rfloor$ play strategy Rubber-stamping and $N - \lfloor \frac{\ell_B}{\ell_S} \rfloor$ play strategy UNIS, this means that due to the presence of the PA, we gain about $2\lfloor \frac{\ell_P}{\ell_S} \rfloor$ who play strategy CAIS, about half of which would play strategy Rubber-stamping without a PA and half of which would play strategy UNIS without a PA. Above the 45-degree line there can be shareholders who play Subscribe-NotInvest and always follow the recommendation. We call this strategy *Follow*, as these shareholders buy the PA's recommendation and follow it when voting. The only decreasing line incorporates the condition $\frac{\ell_B+\ell_P}{\ell_S} < \frac{N+1}{2}$, which is necessary and sufficient for having shareholders who unconditionally invest, i.e., play strategy UNIS.

Overall, the conditions for the existence of an equilibrium with information acquisition by a majority of shareholders are relaxed (SOM Proposition 2.2), compared with those for a symmetric equilibrium in which all shareholders acquire information (Proposition 2). In fact, we move from the requirement that the normalized difference in expertise between board and PA equals at most one shareholder in the symmetric case to the corresponding requirement for the asymmetric case that this difference equals at most half of all shareholders, approximately. In sum, the novel type of

[13]This triangle is not the small lower left triangle at $x = 1$ and $y = 1$ which in the case of symmetric equilibria admits UNIS and is excluded by Assumption BIB, but a much larger area of the parameter space, as it goes up to $\frac{N+1}{2}$.

[14]As Proposition 3 implies, the overall effect of the PA on decision quality is however still positive, even in this range.

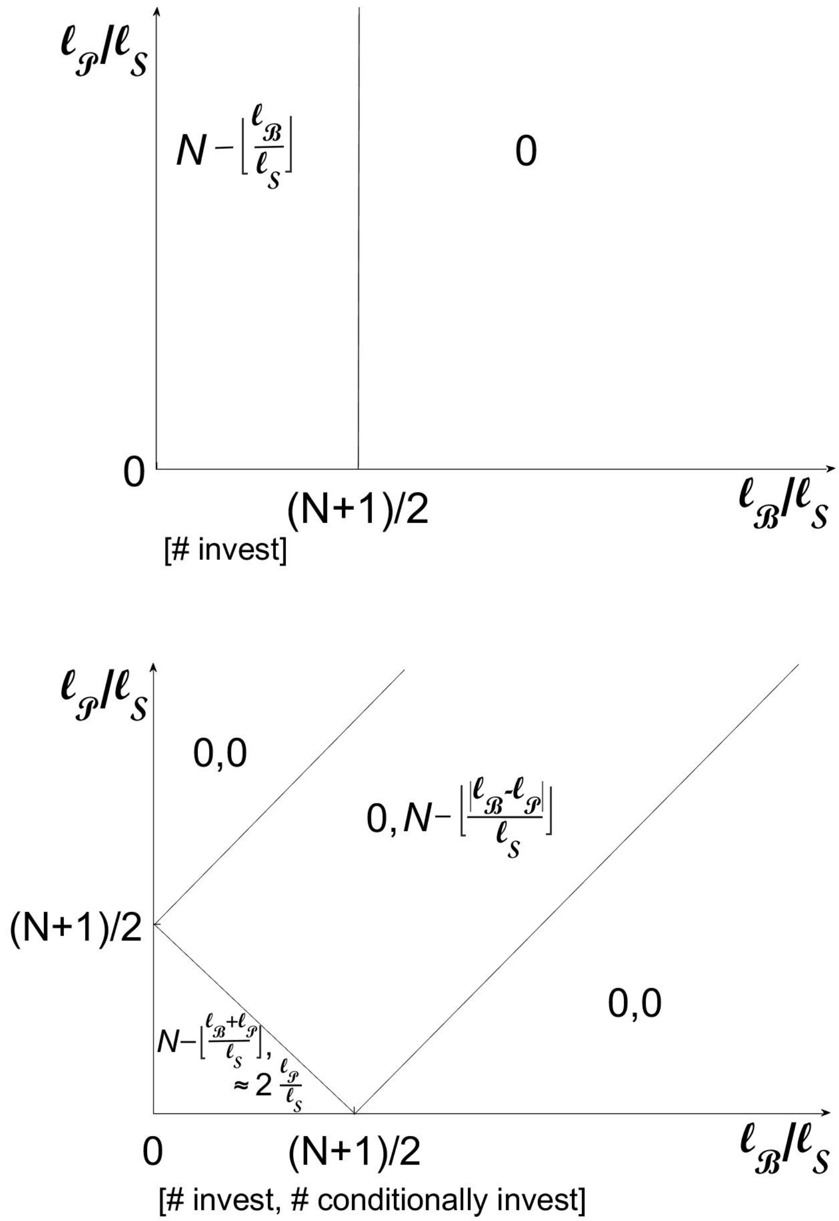

Figure 2.1: Number of shareholders who invest or conditionally invest. The upper panel depicts the game without a PA based on SOM Proposition 2.1. The lower panel a depicts the game with a PA based on SOM Proposition 2.2.

equilibrium behavior that we find in this paper exists in a broad range of the parameter space.

2.4 Decision Quality Improves with a Proxy Advisor

For asymmetric equilibria, Proposition 3 in the main text establishes generally that the introduction of a PA weakly improves decision quality. More specifically, SOM Proposition 2.1 provides the Pareto-efficient equilibria in the benchmark setting when no PA is admitted and SOM Lemma 2.3 provides the Pareto efficient equilibria with a PA.

To illustrate the quantitative difference in decision quality, we consider a simple numerical example.

Example 2.1 (Asymmetric Equilibria)**.** *Let $q_S = 0.6$, $q_B = 0.8$, and $q_P = 0.7$.*[15] *Then, $\ell_B/\ell_S = 3.4$ and $\ell_P/\ell_S = 2.1$, and the case distinction in SOM Proposition 2.1, which characterizes the benchmark scenario in which no PA is admitted, has a threshold at $N = 5.8$. Hence, for $N \leq 5$, part i. of SOM Proposition 2.1 applies, while for $N \geq 6$ part ii. applies. Thus, either all shareholders are uninformed or at most $z_1 = N - \lfloor \frac{\ell_B}{\ell_S} \rfloor = N - 3$ invest in own research when there is no PA.*

Concerning the game with a PA, the condition $\frac{|\ell_B - \ell_P|}{\ell_S} < \frac{N+1}{2}$ of SOM Proposition 2.2 is satisfied. Hence, we get existence of an equilibrium with $z_2 = N - \lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor = N - 1$ shareholders who invest, or conditionally invest, in an own signal. More precisely, for $N \leq 10$, part i. of SOM Proposition 2.2 and case i. of SOM Lemma 2.3 apply, while for $N \geq 11$, part ii. and case iii. apply, respectively. For $N \leq 10$, one shareholder plays strategy Rubber-stamping and $N - 1$ shareholders play strategy CAIS. For $N \geq 11$, again one shareholder plays strategy Rubber-stamping and four shareholders play strategy CAIS, while $N - 5$ play strategy UNIS. The consequences for decision quality are reported in Table 2.1. The example illustrates the main insight: Decision quality is weakly higher with a PA than without.

Setting	Decision quality	$N = 3$	$N = 5$	$N = 21$	$N = 101$	$N = 1,001$
No PA	Π^{no-PA}	0.8	0.8	0.867	0.983	1
With PA	$\Pi(\sigma^*)$	0.812	0.824	0.884	0.984	1

Table 2.1: Decision quality in Pareto-efficient asymmetric equilibria. Illustration of SOM Propositions 2.1 and 2.2 and SOM Lemma 2.3 for $q_B = 0.8$, $q_P = 0.7$, and $q_S = 0.6$, i.e., SOM Example 2.1.

While SOM Example 2.1 is restricted to decision qualities $q_B = 0.8$, and $q_P = 0.7$, we now vary them in the next numerical example, which is illustrated in SOM Figure 2.2. The upper left panel depicts the benchmark scenario that no proxy advisor is admitted, based on SOM Proposition 2.1. The upper right panel depicts decision quality in the game with a PA, based on SOM Lemma 2.3. The lower panel depicts the difference in decision quality, i.e., the effect of admitting a PA. There are several observations to make.

First, the difference is never negative, reflecting Proposition 3 in the main text that decision quality weakly improves by the introduction of a PA. Second, decision quality changes smoothly with the signal qualities of the board q_B and of the PA q_P. Notice in particular, that the smooth

[15]SOM Example 2.1 differs from Example 1 in that the board is better informed: $q_B = 0.8$, instead of 0.75. As a consequence, the symmetric strategy profile CAIS is no longer an equilibrium, as $\ell_P \leq \ell_B - \ell_S$ or $\lfloor \frac{|\ell_B - \ell_P|}{\ell_S} \rfloor \geq 1$.

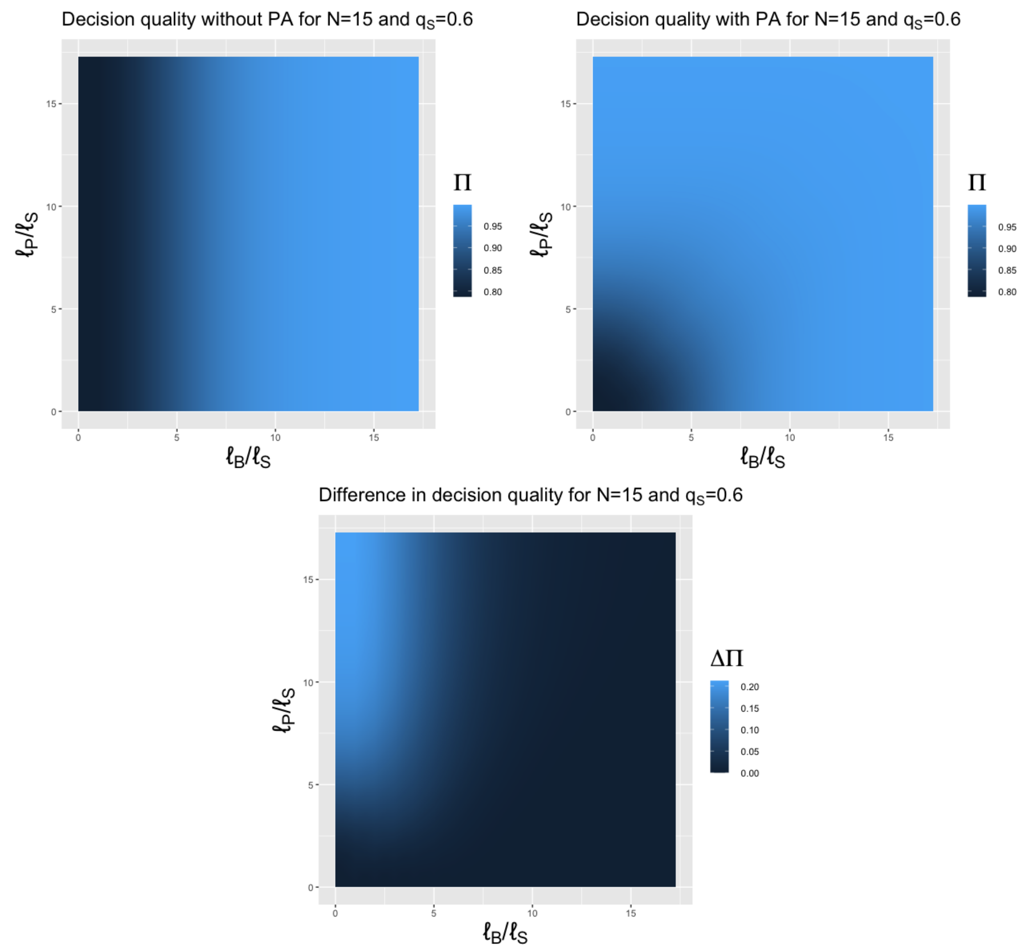

Figure 2.2: Decision quality in the Pareto-efficient strategy profile in the parameter space for $q_S = 0.6$ and $N = 15$. The first panel depicts decision quality when there is no PA based on SOM Prop. 2.1. The second panel depicts decision quality when there is a PA based on SOM Prop. 2.2 and SOM Lemma 2.3. The third panel depicts the difference in decision quality, i.e., the improvement in decision quality due to the PA.

profile in the right upper panel of SOM Figure 2.2 is based on the six cases of SOM Lemma 2.3, corresponding to the six regions in Figure 3. Indeed, decision quality changes continuously at every boundary between these six cases of SOM Lemma 2.3, as it can be generally checked when plugging in the condition for the boundary into the two expressions for decision quality for the corresponding cases. Third, as the figure suggests, decision quality turns out to be always weakly increasing in ℓ_P and ℓ_B. This is a notable difference to the analysis of symmetric equilibria, where non-monotonicity and discontinuities arise. For symmetric equilibria, a PA weakly improved decision quality under the Assumption BIB. Relaxing it led to the possibility that a PA worsens decision quality and also that increasing the quality of the PA worsens decision quality. Now, for asymmetric equilibria, decision quality is always weakly improved by the presence of a PA and it is monotonically and smoothly increasing in the quality of the PA. This also holds for the quality of the board.

Finally, considering that ℓ_P close to zero is similar to having no PA at all and that decision quality is weakly increasing in ℓ_P, it is intuitive that a PA weakly improves decision quality. We analytically prove this main result (Proposition 3 in the main text) in Appendix A. Overall, the analysis of asymmetric equilibria may differ from the analysis of symmetric equilibria when it comes to some comparative statics, but it supports the main results. First, the shareholders' incentive to invest in own research is fostered by the presence of a PA, with maximal research incentives when the PA's information quality is similar to the board's. Second, decision quality weakly improves due to the presence of a PA.